Multi-Agent Systems

Multi-Agent Path Finding

Albert-Ludwigs-Universität Freiburg

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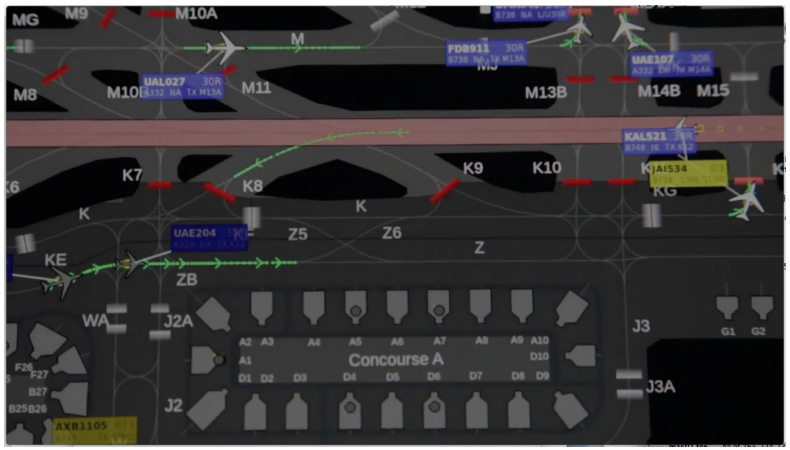
Motivation

Agents moving in a spatial environment

A central problem in many applications is the coordinated movement of agents/robots/vehicles in a given spatial environment.

Logistic robots (KARIS)



Airport ground traffic control (atrics)

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Multi-agent path finding (MAPF)

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Definition (Multi-agent path finding (MAPF) problem)

Given a set of *agents* A, an undirected, simple *graph* $G = (V, E)$, an *initial state* modelled by an injective function $\alpha_0 : A \to V$, and a *goal state* modelled by another injective function α_*, can α_0 be *transformed* into α_* by *movements of single agents* without collisions?

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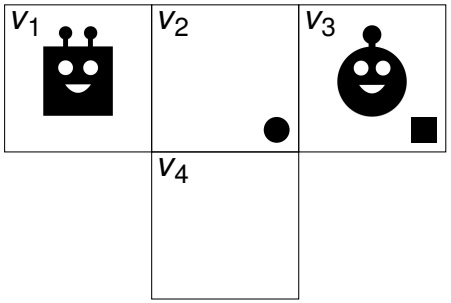
- *Plan generation problem*: Generate a plan.
- *Optimal plan generation problem*: Generate a shortest plan.

Example

Can we find a (central) plan to move the square robot S to v_3 and the circle robot C to v_2?



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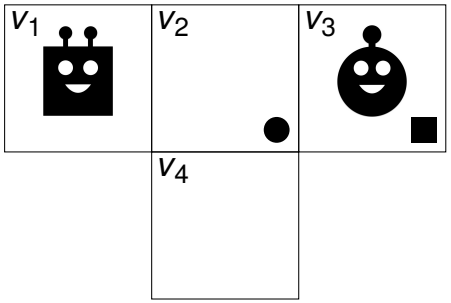

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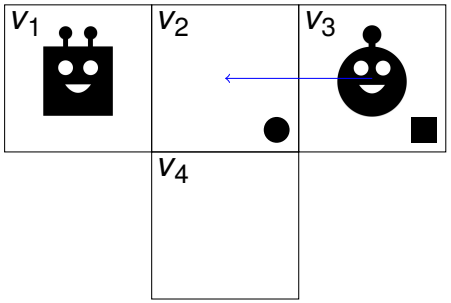

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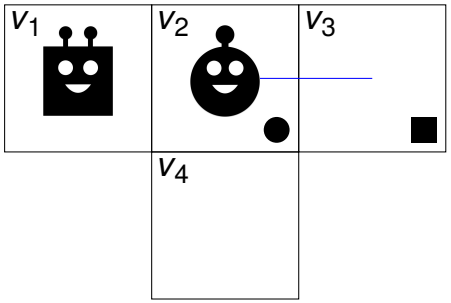

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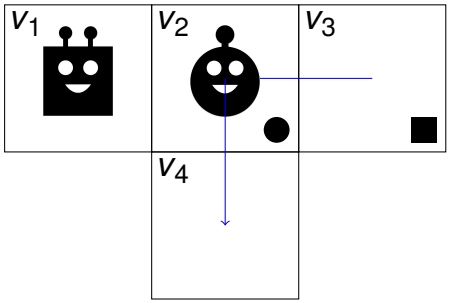

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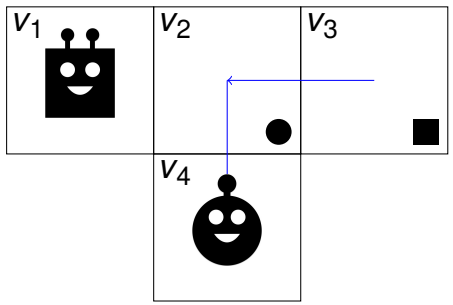

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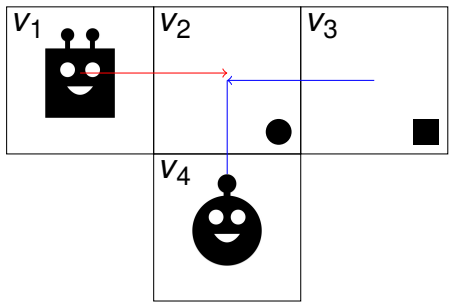

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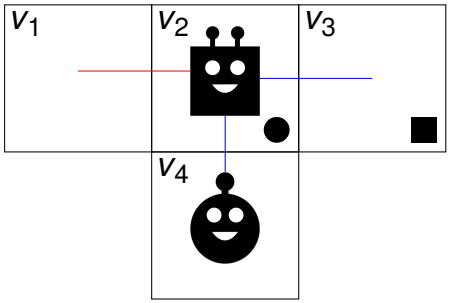

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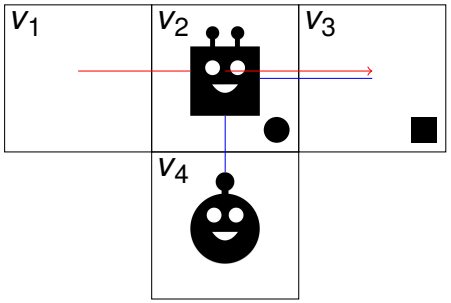

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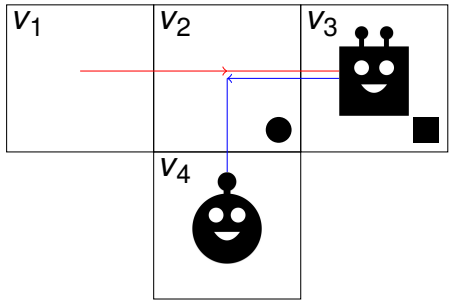

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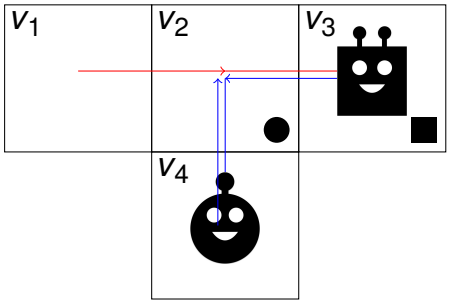

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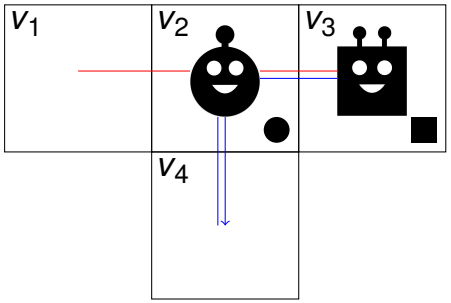

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A special case: 15-puzzle

Pictures from Wikipedia article on 15-Puzzle

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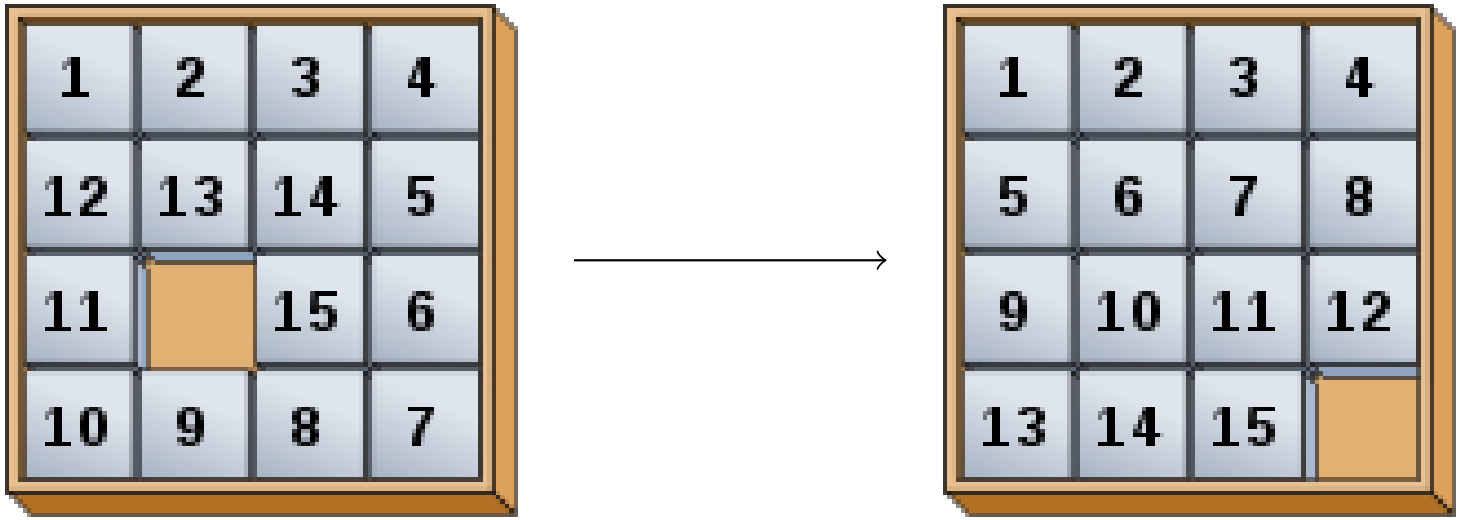

Pictures from Wikipedia article on 15-Puzzle

Lecture plan

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- MAPF: variations, algorithms, complexity
- Distributed MAPF (each agent plans on it own): DMAPF
- Distributed MAPF with destination uncertainty: MAPF/DU

Sequential MAPF

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- *Sequential MAPF* (or pebble motion on a graph) allows only one agent to move per time step.
- An agent $a \in A$ can move in one step from $s \in V$ to $t \in V$ transforming α to α', if
  - $\alpha(a) = s$,
  - $\langle s, t \rangle \in E$,
  - there is no agent b such that $\alpha(b) = t$.
- In this case, α' is determined as follows:
  - $\alpha'(a) = t$,
  - for all agents $b \neq a : \alpha(b) = \alpha'(b)$,
- One usually wants to minimize the number of single movements (= *sum-of-cost* over all agents)

Parallel MAPF

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- *Parallel MAPF* allows many agents to move in parallel, provided they do not collide.
- Two models:
  - *Parallel*: A chain of agents can move provided the first agent can move on a an unoccupied vertex.
  - *Parallel with rotations*: A closed cycle in move synchronously.
- In both cases, one is usually interested in the number of parallel steps (= *make-span*).
- However, also the sum-of-cost is sometimes considered.

Anonymous MAPF

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- There is a set of agents and a set of targets (of the same cardinality as the agent set).
- Each target must be reached by one agent.
- This means one first has to assign a target and then to solve the original MAPF problem.
- Interestingly, the problem as a whole is easier to solve (using flow-based techniques).

Types of MAPF algorithms

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- *Rule-based algorithms*: *Kornhauser's algorithm*, *Push-and-Rotate*, **BIBOX**, . . . (complete on a given class of graphs, but suboptimal)

A*-based algorithm

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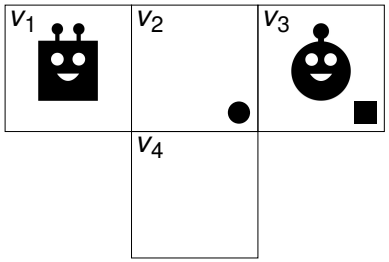
- Define state space:
  - A state is an assignment of agents to vertices (modelled by a function α)
  - There is a transition from one state α to α' iff there is a legal move from α to α' according to the appropriate semantics (sequential, parallel, or parallel with rotations)
- Search in this state space using the A* algorithm.
- Possible *heuristic estimator*: Sum or maximum over the length of the individual movement plans (ignoring other agents).
- **Problem**: Large *branching factor* because of many agents that can move.

Example: State space for A* algorithm



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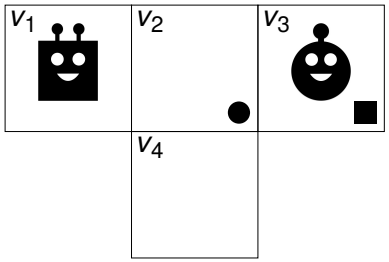

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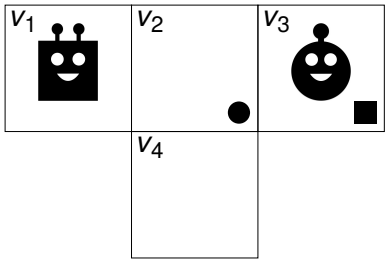

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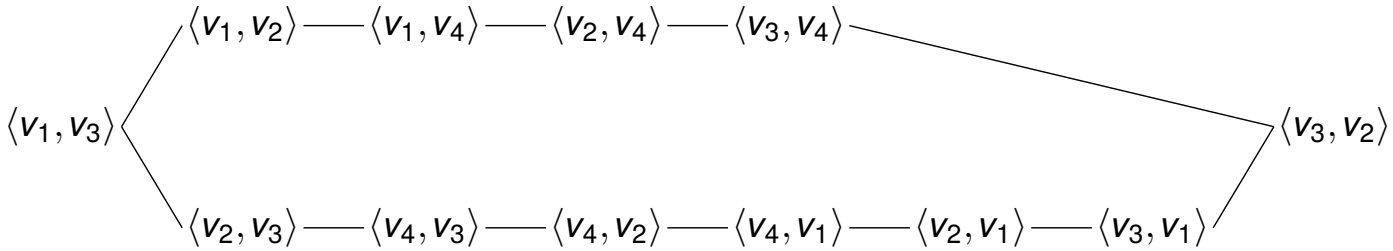

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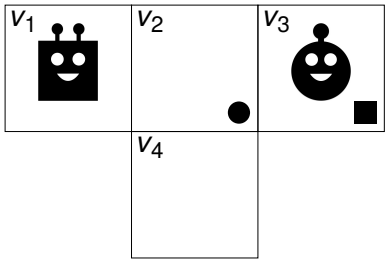

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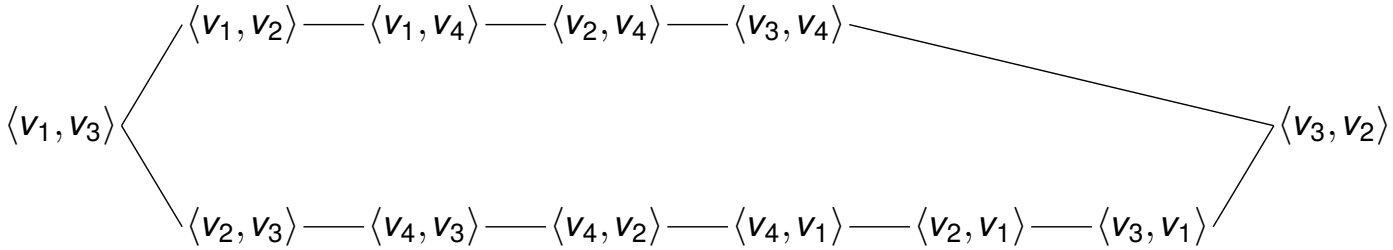

Question: Heuristic value for states $\langle v_1, v_2 \rangle$ and $\langle v_2, v_3 \rangle$ under the sum-aggregation?

CA*

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CA*

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- Problems with A^* on MAPF state space:
  - super-exponential state space, i.e., $m!/(m-n)!$ with m nodes and n agents;
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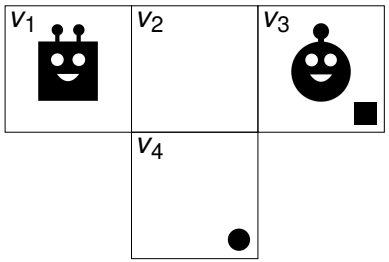
  - Our small example is not solvable with this method!

Example CA* run



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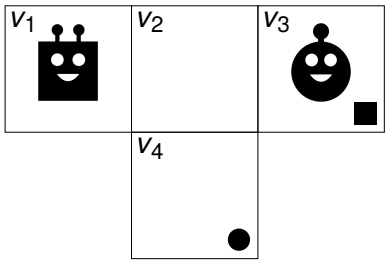

- Linear order: $\langle C, S \rangle$

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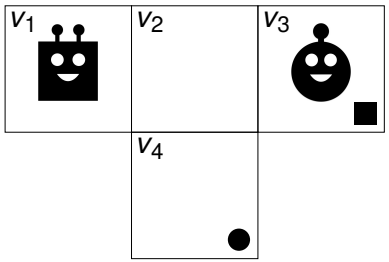

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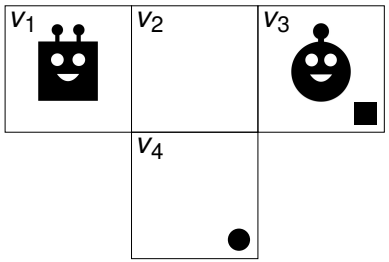

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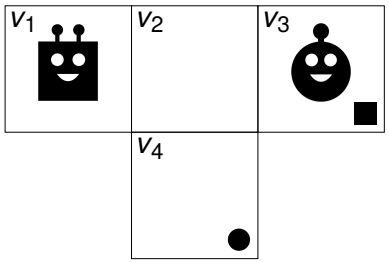

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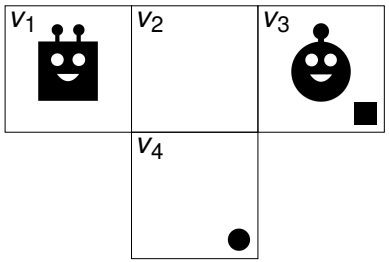

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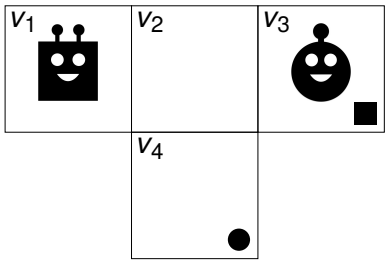

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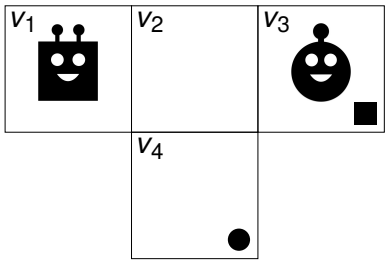

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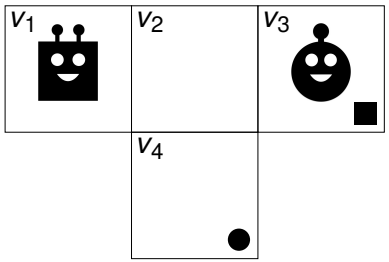

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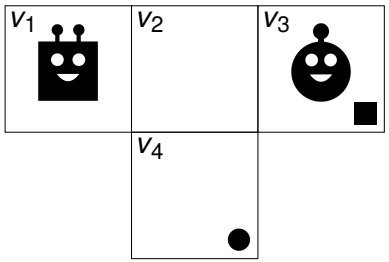

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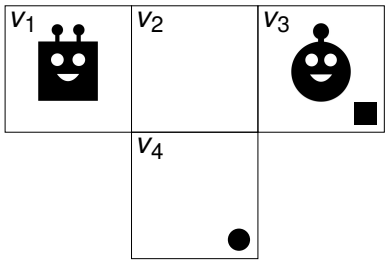

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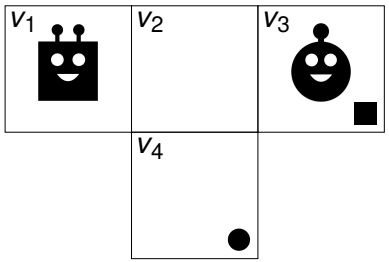

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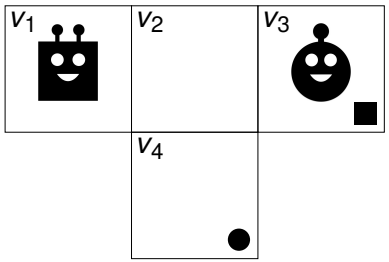

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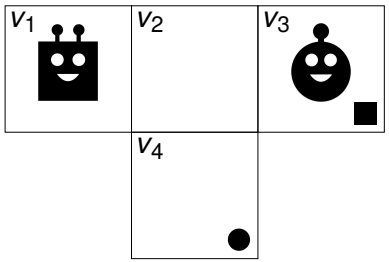

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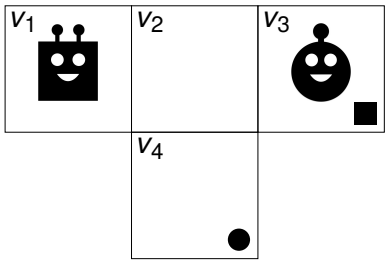

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- Not solvable with different order!

BIBOX

UNI FREIBURG

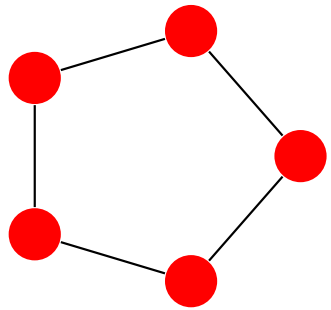
BIBOX is a rule-based algorithm that is complete on all *bi-connected* graphs with at least two unoccupied nodes in the graph.

Definition

A graph $G = (V, E)$ is *connected* iff $|V| \geq 2$ and there is *path* between each pair of nodes $s, t \in V$. A graph is *bi-connected* iff $|V| \geq 3$ and for each $v \in V$, the graph $(V - \{v\}, E')$ with $E' = \left\{ \{x, y\} \in E \,|\, x, y \neq v \right\}$ is connected.

Loop decomposition

Every bi-connected graph can be constructed from a *cycle* by adding *loops* iteratively.



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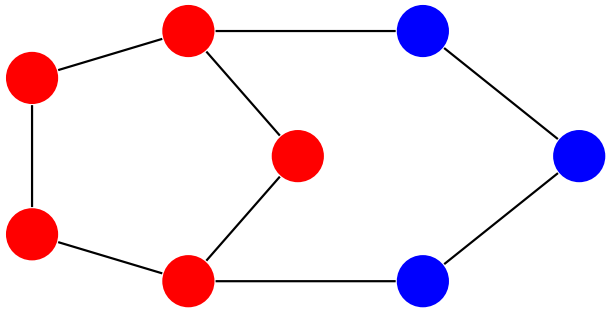

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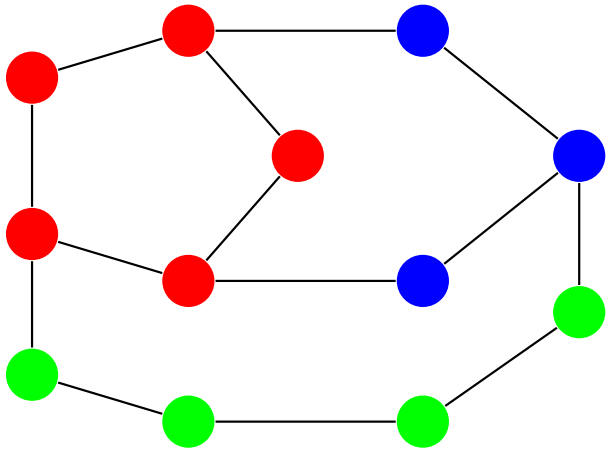

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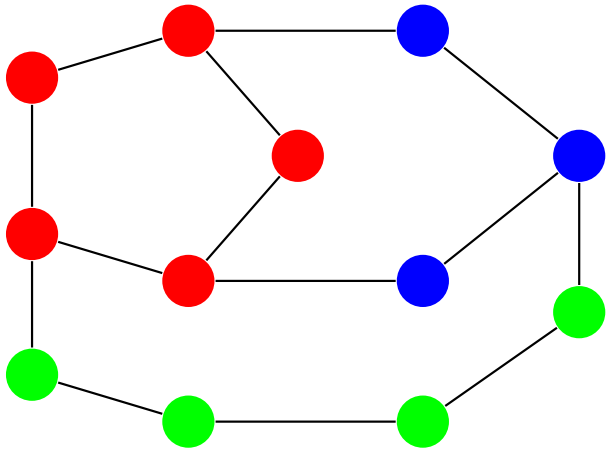

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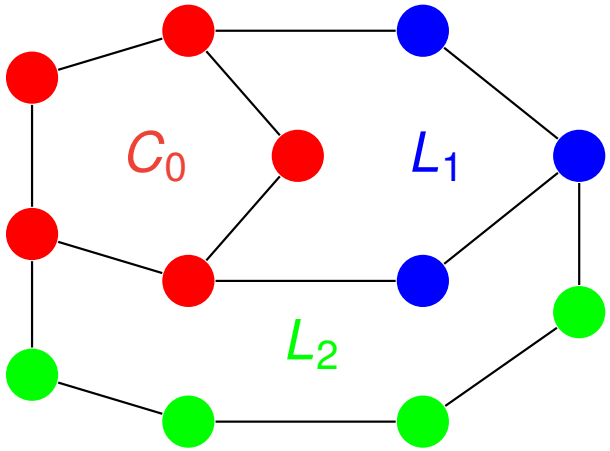
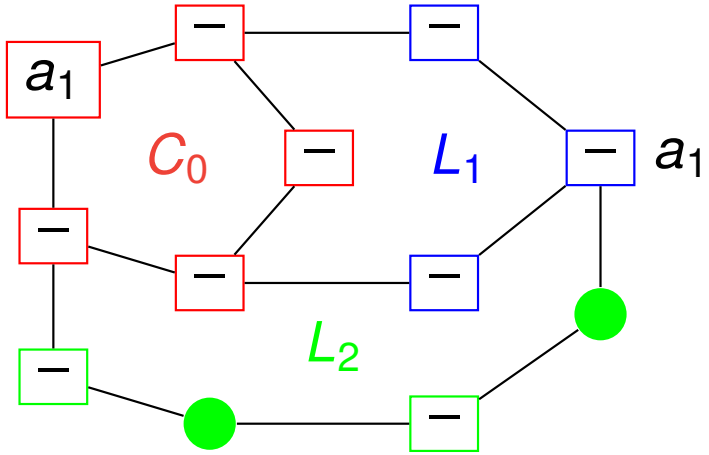


A *loop decomposition* into a basic cycle and additional loops can be done in time $O(|V|^2)$.
Let us name them C_0, L_1, L_2, ..., where the index depends on the time when the loop is added.

Moving unoccupied nodes and agents around



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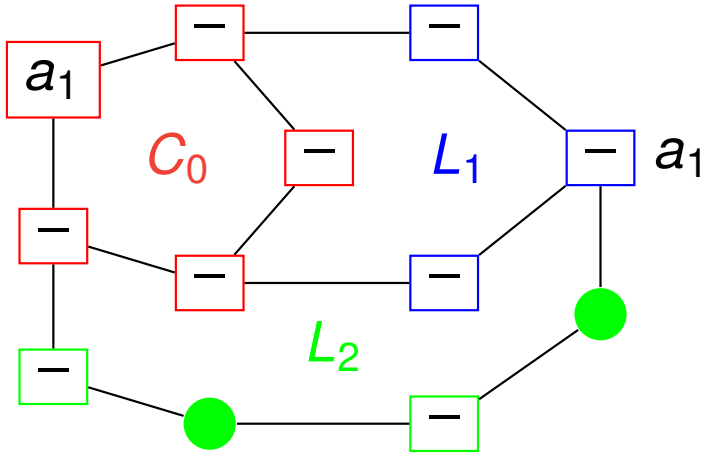

- An unoccupied place can be sent to any node.

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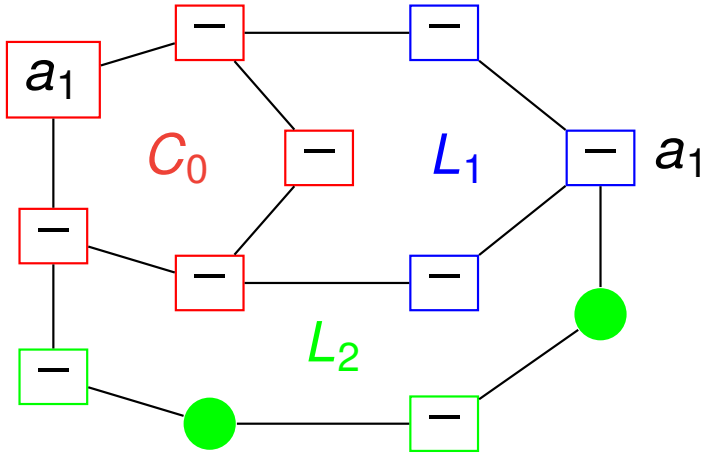

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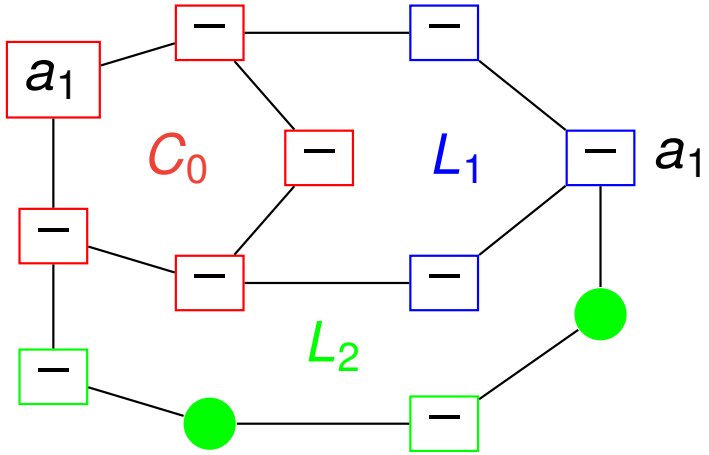

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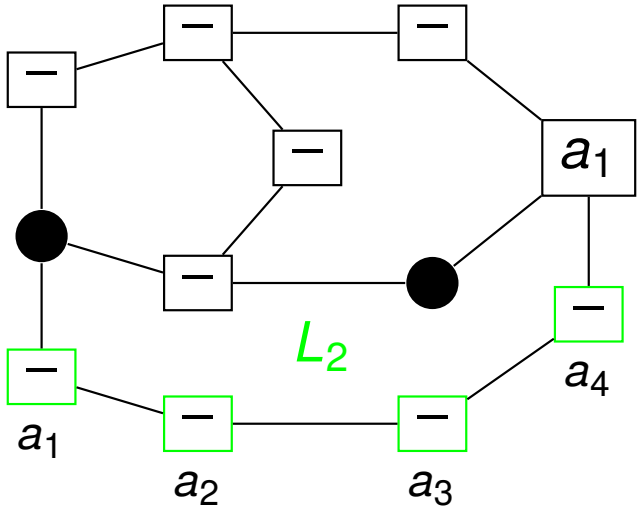
- This can be done without disturbing loops with a higher index than the one the agent starts and finishes in.

Filling loops

- Starting with highest-index loop: Move agents to destination loop, then shift agents to their destinations.
- Special case: When agents are already in the destination loop, they have to be rotated out of the loop.



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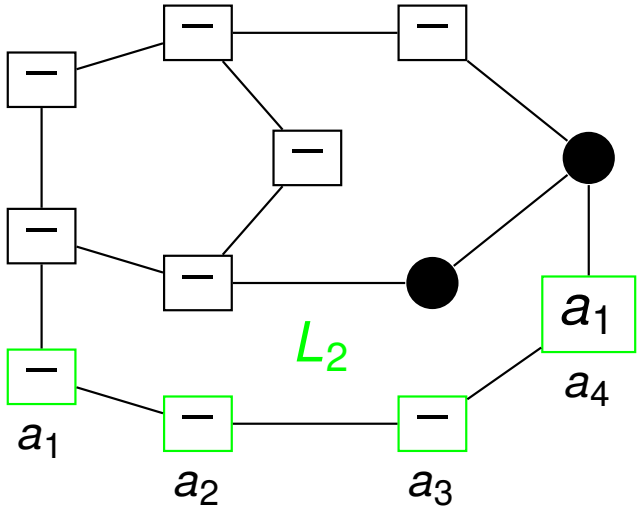

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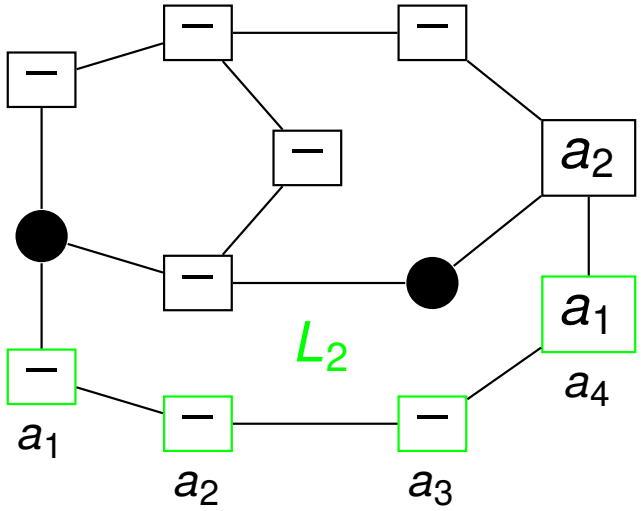

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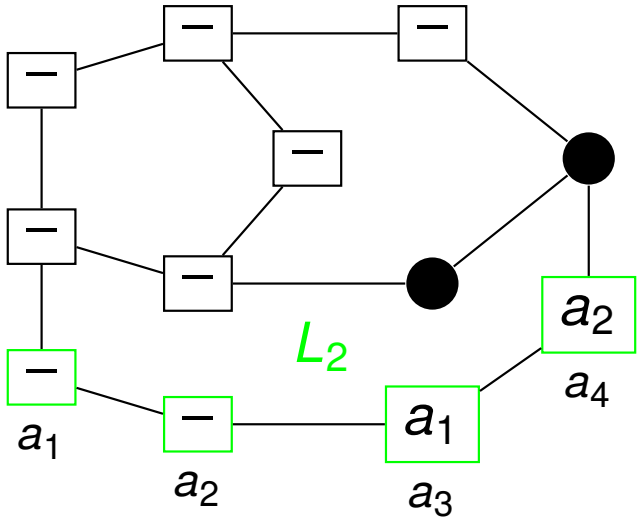

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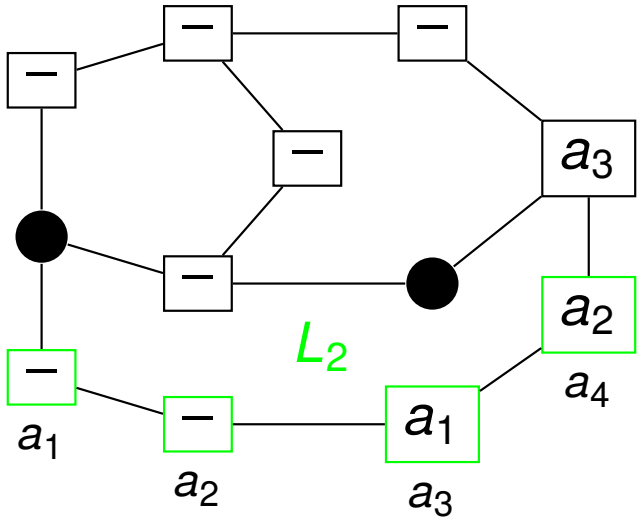

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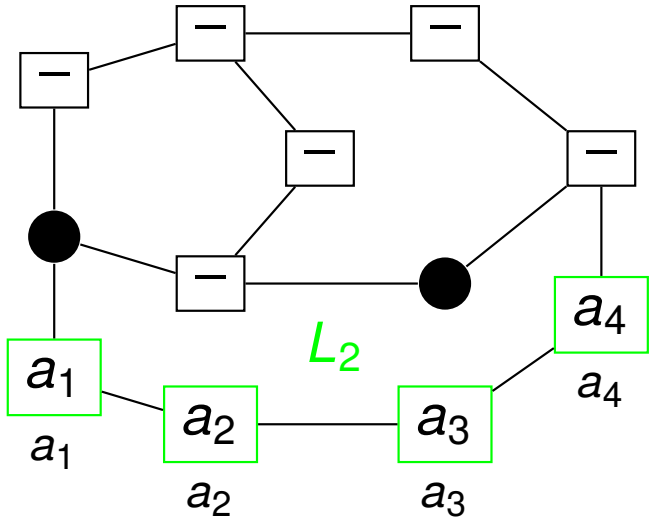

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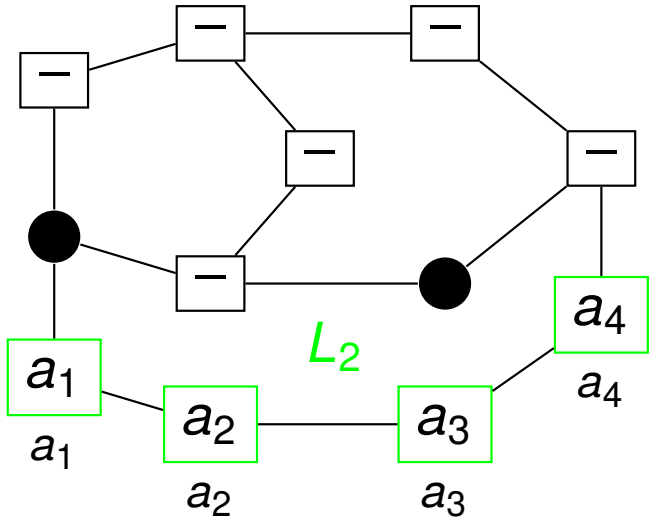
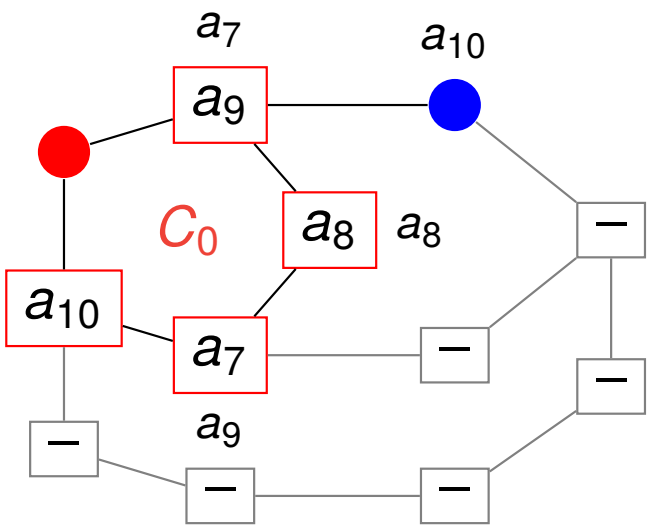


- When done with one loop, repeat for next one with next lower index.

Reordering agents in the cycle

- Assumption: The destinations for the empty places are in the cycle C_0 (can be relaxed).
- If the agents are in the right order, just rotate them to their destinations.
- Otherwise reorder by successively take one out and re-insert.



Runtime and plan length estimation

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- Question: Is the problem also NP-hard?

The Exact Cover By 3-Sets (X3C) Problem

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Definition (Exact Cover By 3-Sets (X3C) Problem)

Given a set of elements U and a collection of subsets $C = \{s_j\}$ with $s_j \subseteq U$ and $|s_j| = 3$. Is there a sub-collection of subsets $C' \subseteq C$ such that $\bigcup_{s \in C'} s = U$ and all subsets in C' are pairwise disjoint, i.e., $s_a \cap s_b = \emptyset$ for each $s_a, s_b \in C'$ with $s_a \neq s_b$?

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NP-hardness of MAPF: Reduction from X3C

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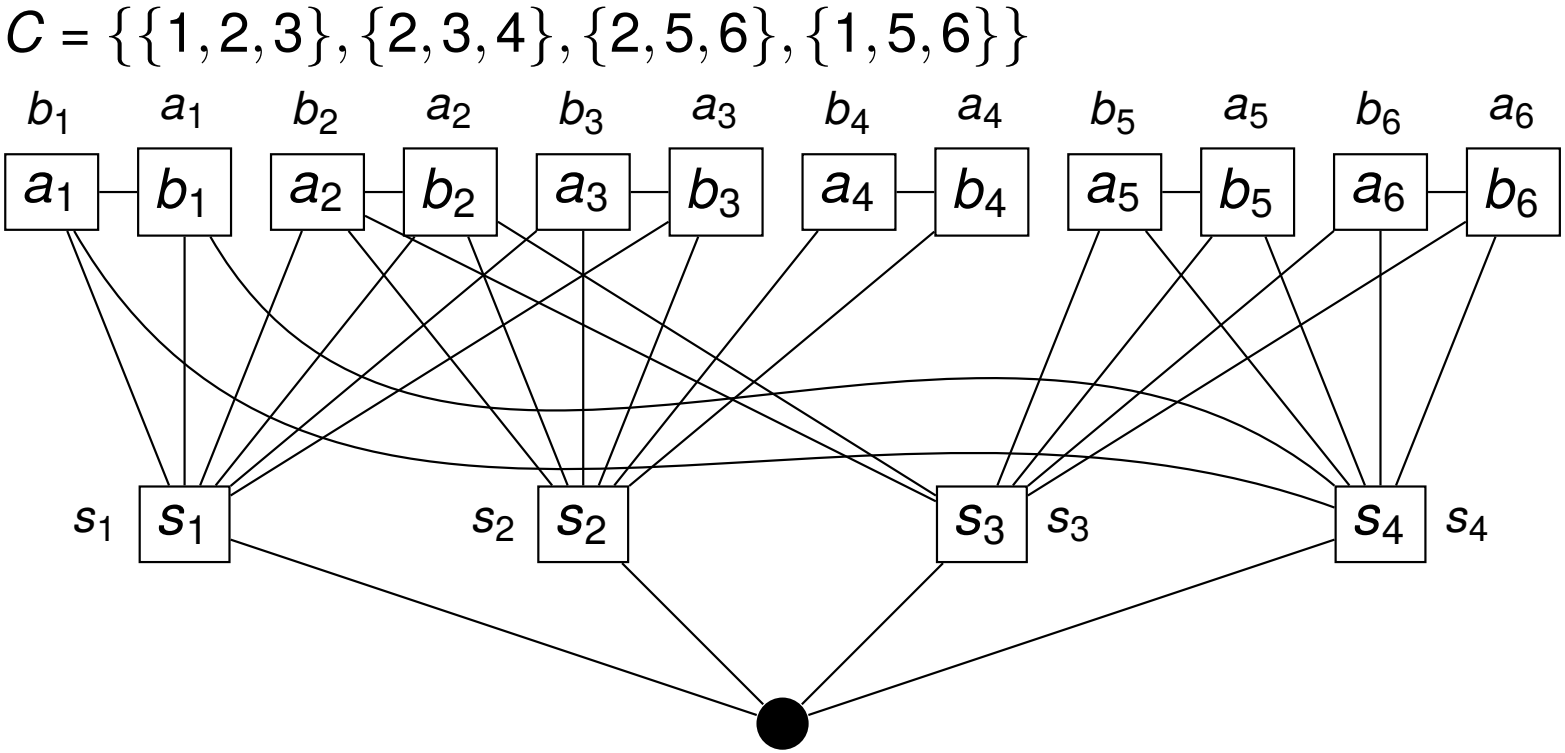

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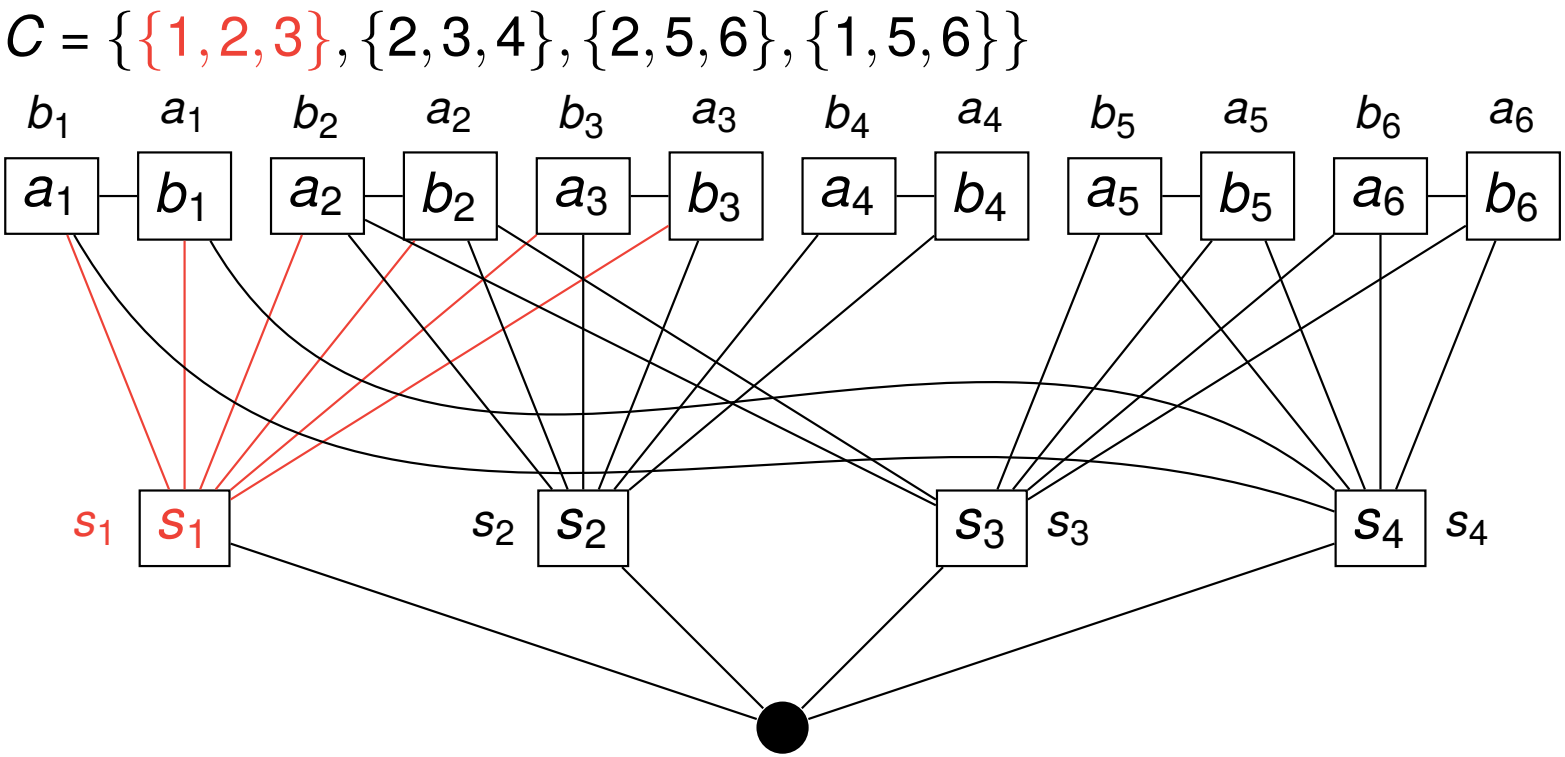

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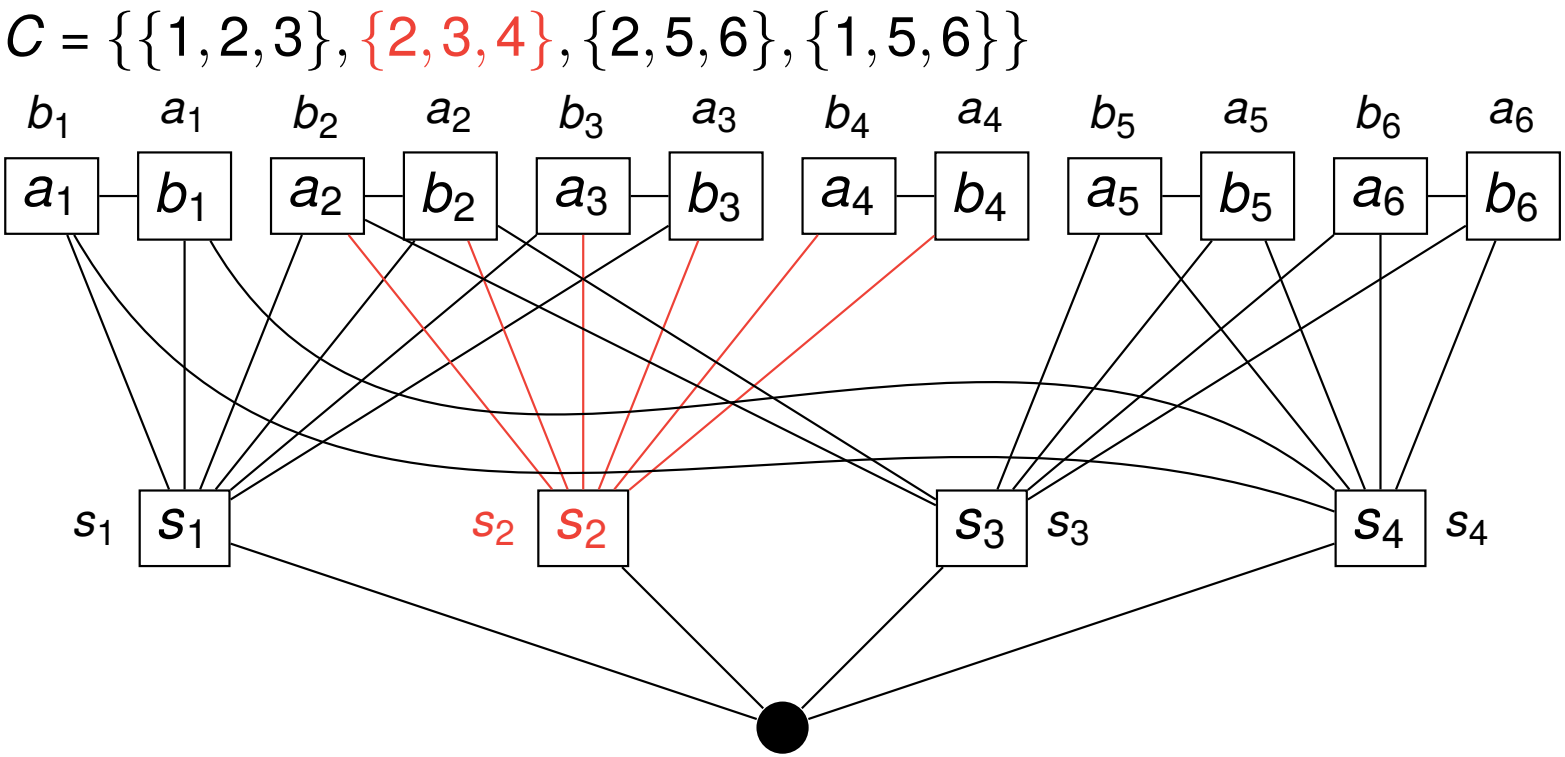

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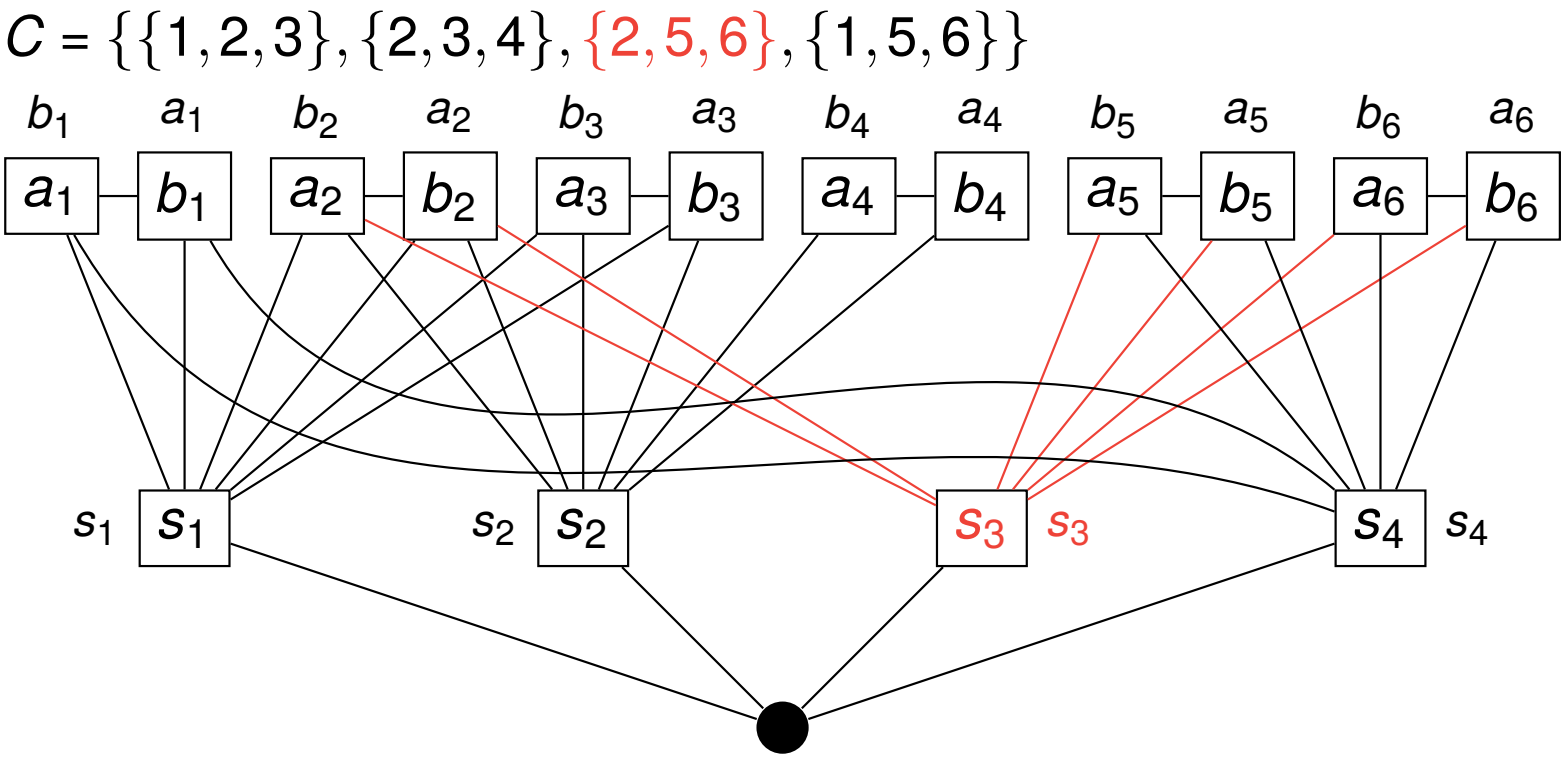

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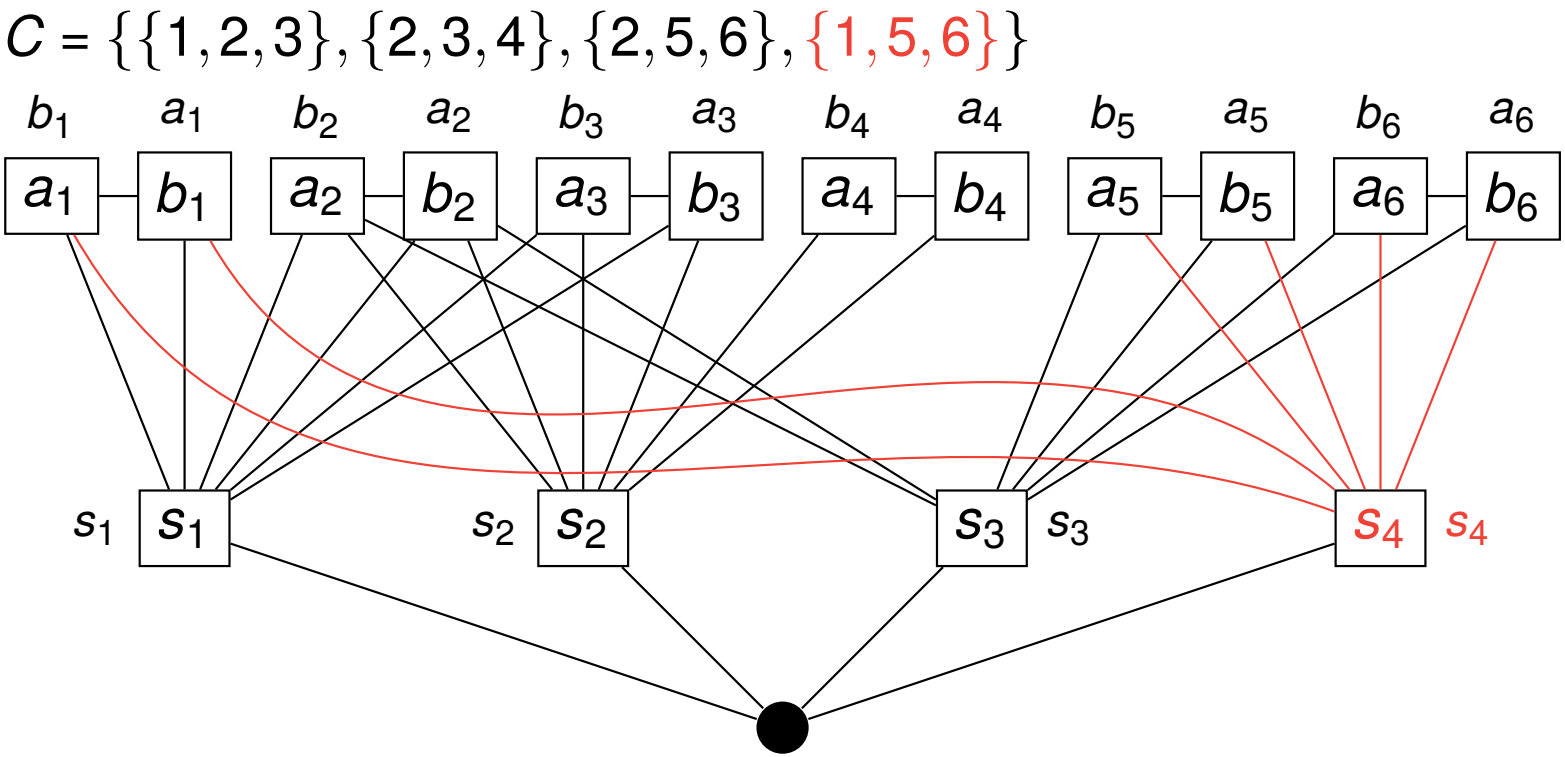

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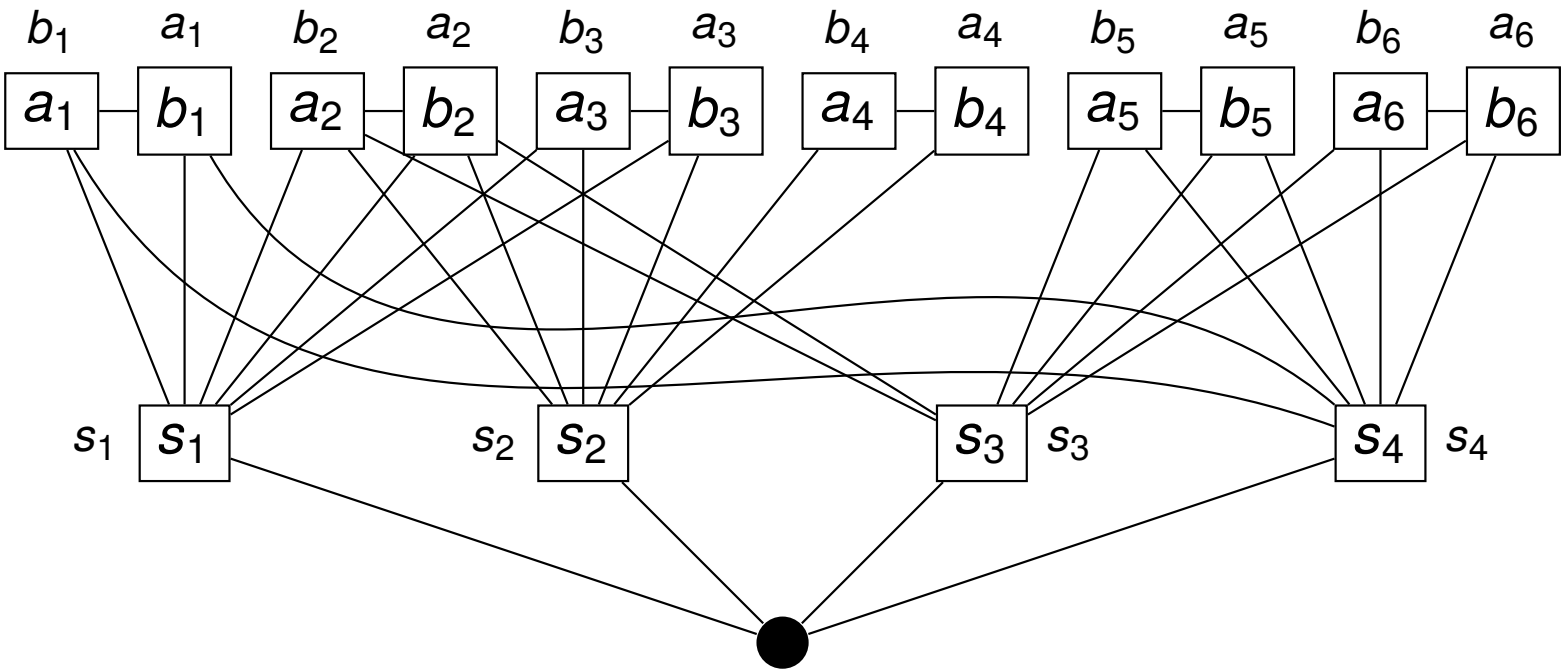

Claim: There is an exact cover by 3-sets iff the constructed MAPF instance can be solved in at most $k = 11/3|U|$ moves.

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Distributed MAPF

Going beyond MAPF

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- How do we define joint execution of such conflicting plans?

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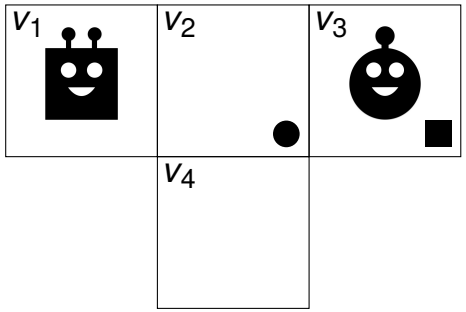

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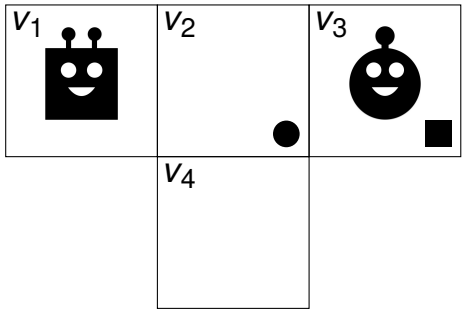

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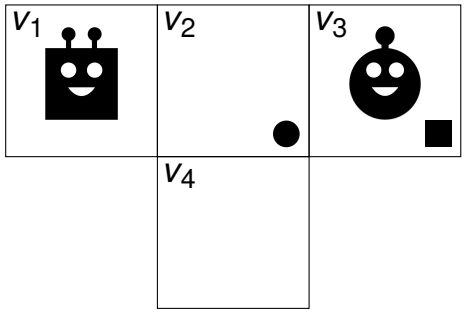

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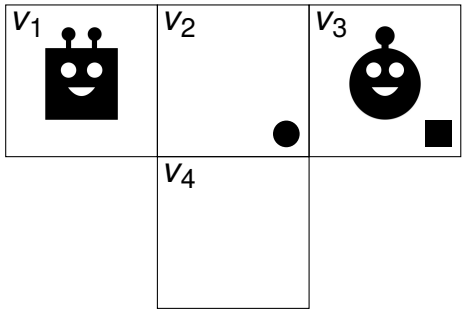

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Joint execution

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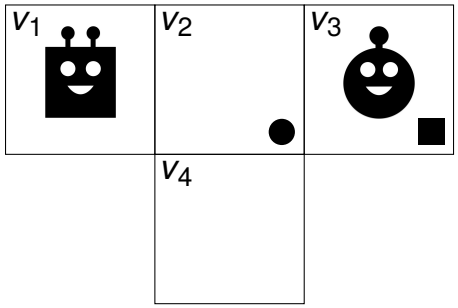
- Let us assume, all agents have planed and a subset of them came up with a *family of plans* $(\pi_i)_{i \in A}$.
- Among the agents that have a plan with their own action as the next action to execute, one is chosen.
- The action of the chosen agent is executed.
- Agents, which have anticipated the action, track that in their plans.
- All other agents have to *replan* from the new state.
- Since everybody has a successful plan, no acting agent will ever execute an action that leads to a dead end.

Example execution



Planning, executing, and replanning:

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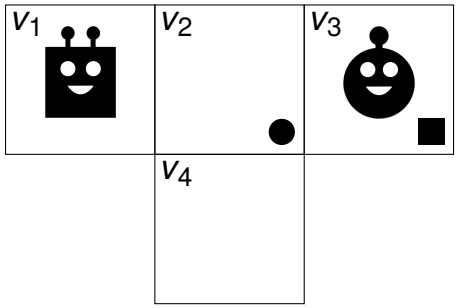

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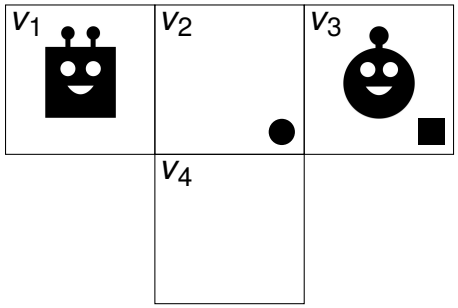

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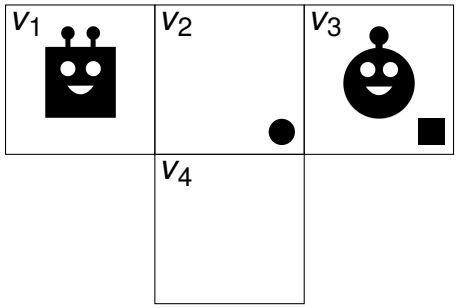

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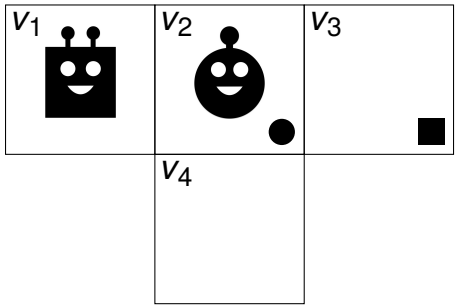

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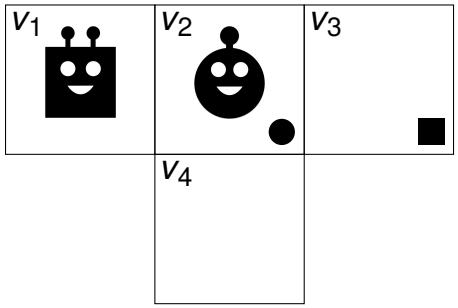

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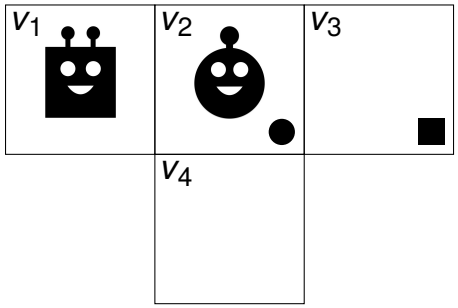
$$S: \quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), (S, v_2, v_3), (C, v_1, v_2) \rangle$$

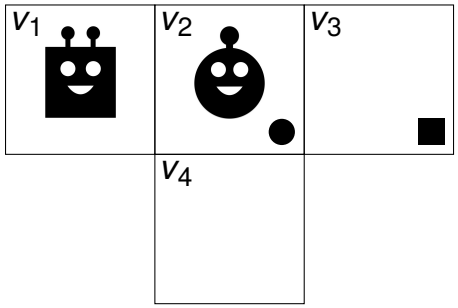
Example execution



Planning, executing, and replanning:

$$C: \quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$
$$S: \quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), (S, v_2, v_3), (C, v_1, v_2) \rangle$$
$$C: \quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
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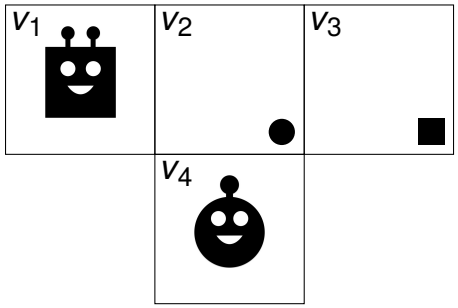
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
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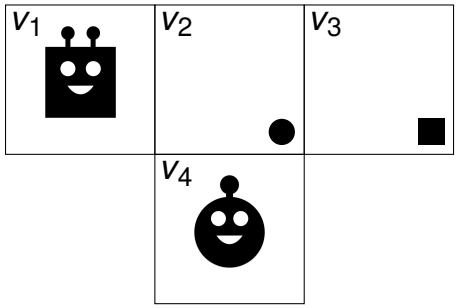
&\quad (S, v_2, v_3), (C, v_1, v_2) \rangle \\
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
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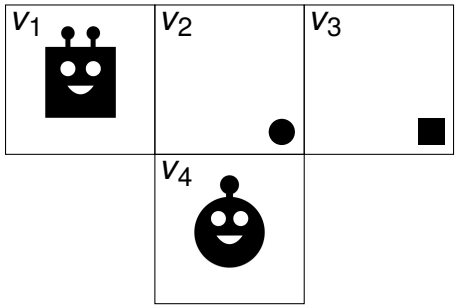
\end{aligned}
$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
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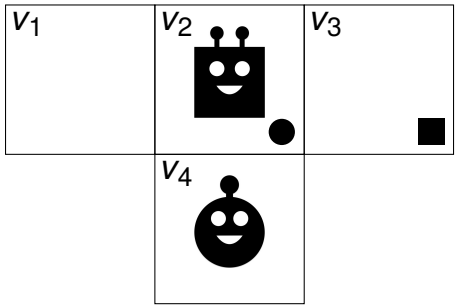
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

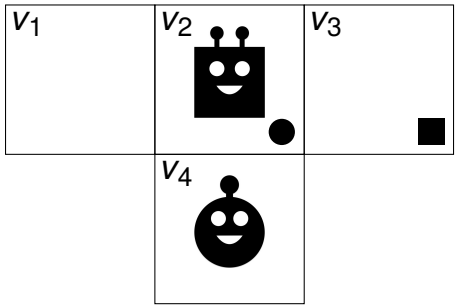
Example execution



Planning, executing, and replanning:

$$
\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
S: &\quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), \\
&\quad (S, v_2, v_3), (C, v_1, v_2) \rangle \\
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
S: &\quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), \\
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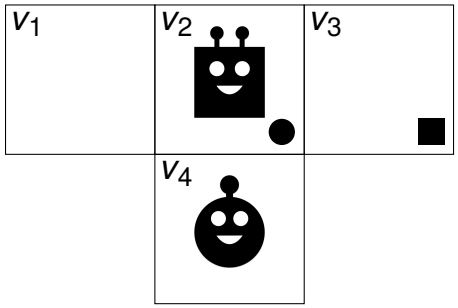
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

Example execution



Planning, executing, and replanning:

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\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
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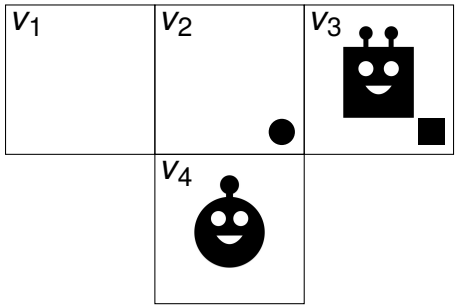
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

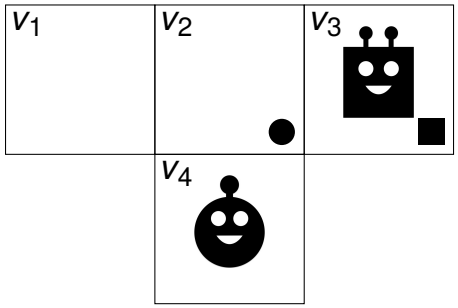
Example execution



Planning, executing, and replanning:

$$C: \quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$
$$S: \quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), (S, v_2, v_3), (C, v_1, v_2) \rangle$$
$$C: \quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$

Example execution



Planning, executing, and replanning:

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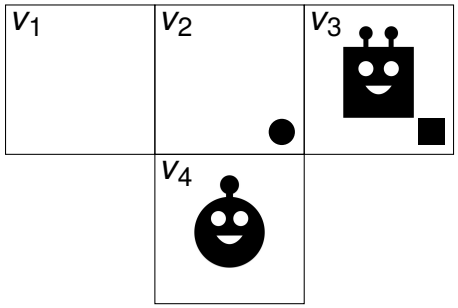
\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
S: &\quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), \\
&\quad\ (S, v_2, v_3), (C, v_1, v_2) \rangle \\
C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

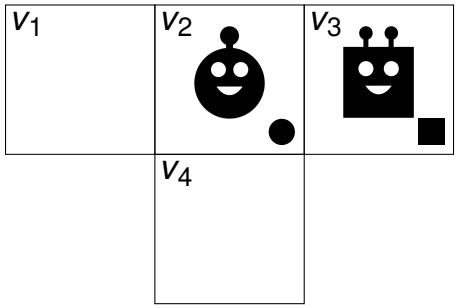
Example execution



Planning, executing, and replanning:

$$C: \quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$
$$S: \quad \langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), (S, v_2, v_3), (C, v_1, v_2) \rangle$$
$$C: \quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$$

Example execution



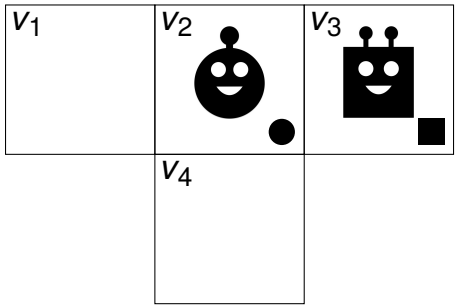
Planning, executing, and replanning:

C : $\langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$

S : $\langle (S, v_1, v_2), (S, v_2, v_4), (C, v_3, v_2), (C, v_2, v_1), (S, v_4, v_2), (S, v_2, v_3), (C, v_1, v_2) \rangle$

C : $\langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle$

Example execution



Planning, executing, and replanning:

$$
\begin{aligned}
C: &\quad \langle (C, v_3, v_2), (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle \\
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C: &\quad \langle (C, v_2, v_4), (S, v_1, v_2), (S, v_2, v_3), (C, v_4, v_2) \rangle
\end{aligned}
$$

Done!

Lazy and eager agents

UNI FREIBURG

What can go wrong?

Lazy and eager agents

UNI FREIBURG

What can go wrong?

- Agents could be *lazy*: Sometimes they choose a plan where they expect that another agent should act, although they could act.

→ Agents may wait forever for each other to act (dish washing dilemma).

Lazy and eager agents

UNI FREIBURG

What can go wrong?

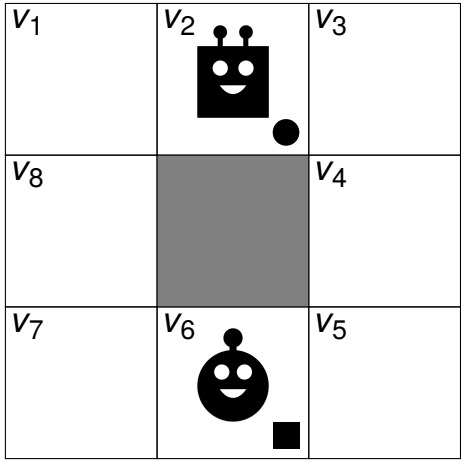
- Agents could be *lazy*: Sometimes they choose a plan where they expect that another agent should act, although they could act.

→ Agents may wait forever for each other to act (dish washing dilemma).

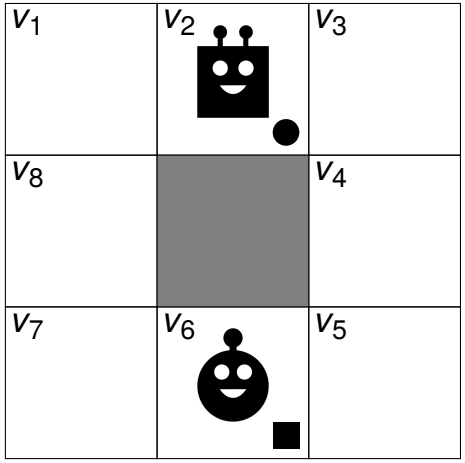
- Agents could be *eager*: If agents could act (without creating a cycle or a dead end), they choose to act.

→ Agents might create cyclic executions (without creating plans that are cyclic), leading to *infinite executions*.

Example for infinite execution

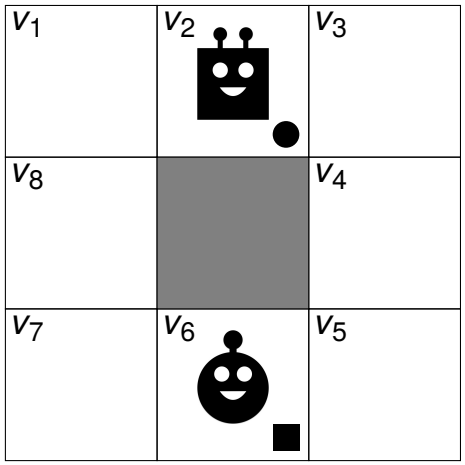


Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

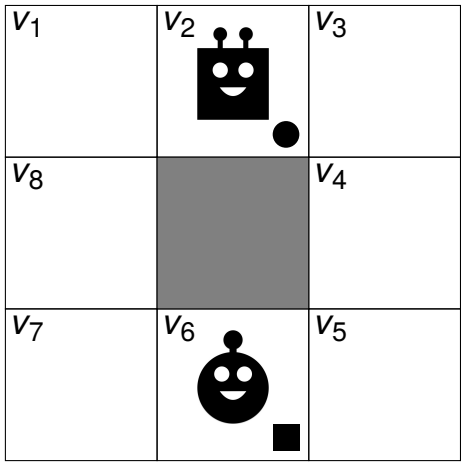
Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

π_2 (C initially): $\langle (C, v_6, v_5), (C, v_5, v_4), (C, v_4, v_3), (S, v_2, v_1), \ldots \rangle$

Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

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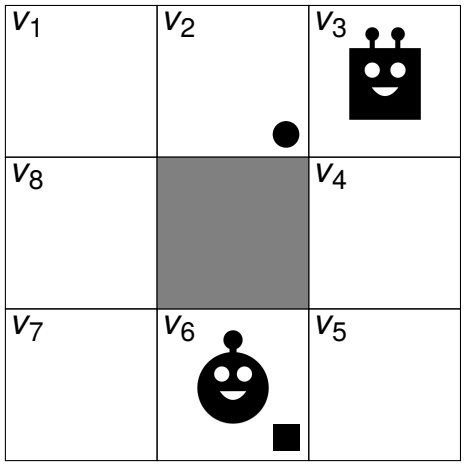
Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

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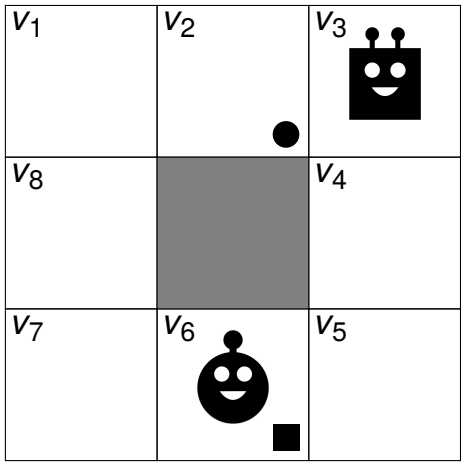
Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

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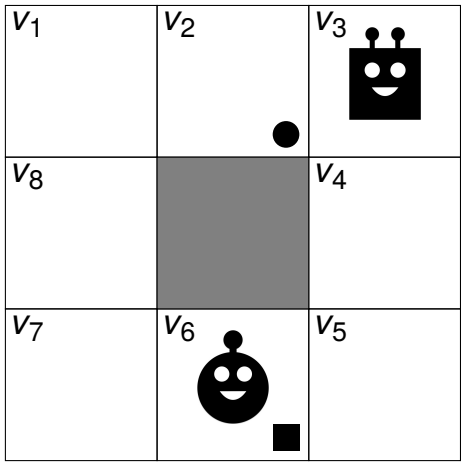
Example for infinite execution

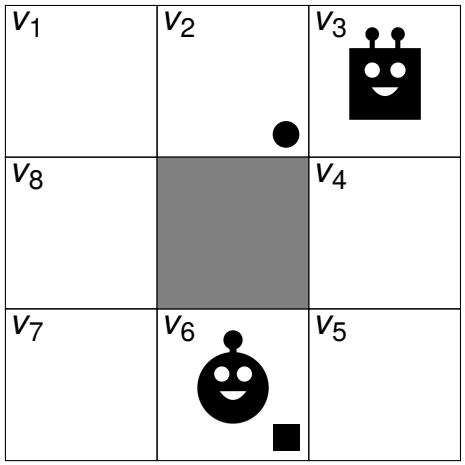


π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

π_2 (C initially): $\langle (C, v_6, v_5), (C, v_5, v_4), (C, v_4, v_3), (S, v_2, v_1), \ldots \rangle$

π_3 (C after (S, v_2, v_3)): $\langle (C, v_6, v_5), (C, v_5, v_4), \underline{(S, v_3, v_2)}, (C, v_4, v_3), \ldots \rangle$

Example for infinite execution

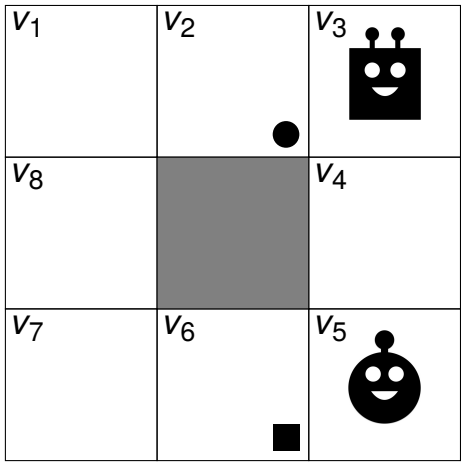


π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

π_2 (C initially): $\langle (C, v_6, v_5), (C, v_5, v_4), (C, v_4, v_3), (S, v_2, v_1), \ldots \rangle$

π_3 (C after (S, v_2, v_3)): $\langle (C, v_6, v_5), (C, v_5, v_4), \underline{(S, v_3, v_2)}, (C, v_4, v_3), \ldots \rangle$

Example for infinite execution



π_1 (S initially): $\langle (S, v_2, v_3), (S, v_3, v_4), (S, v_4, v_5), (C, v_6, v_7), \ldots \rangle$

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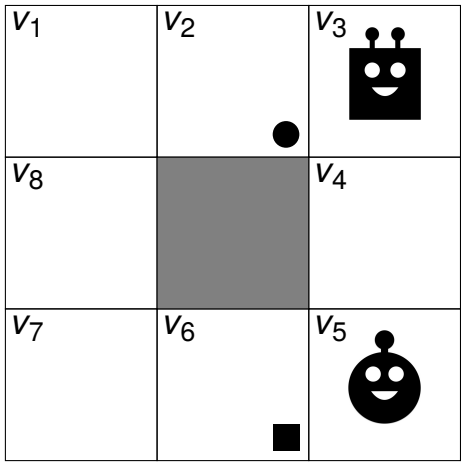
Example for infinite execution



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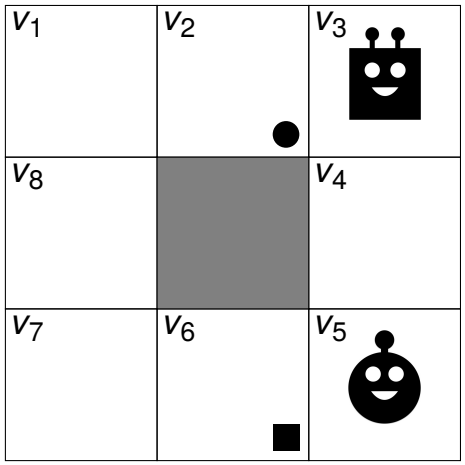

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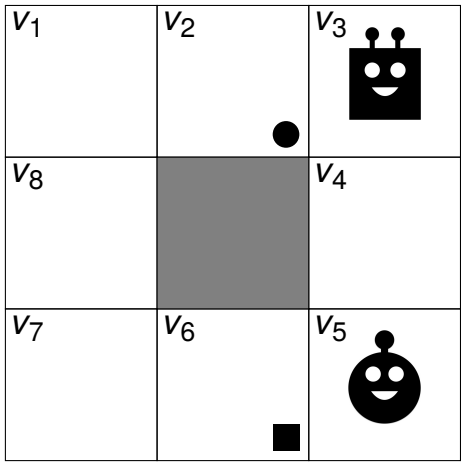

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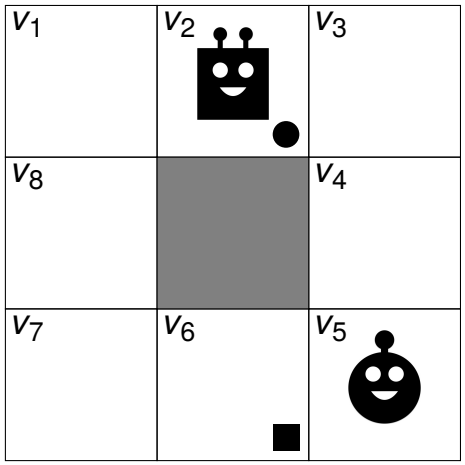

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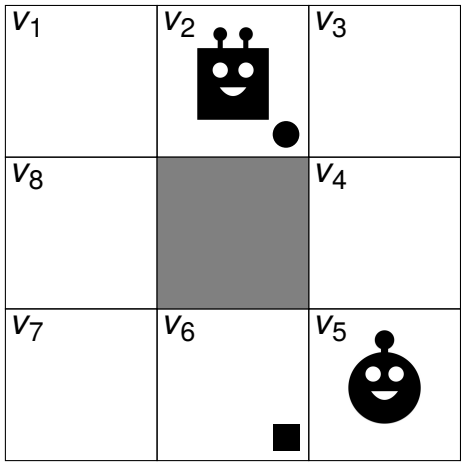

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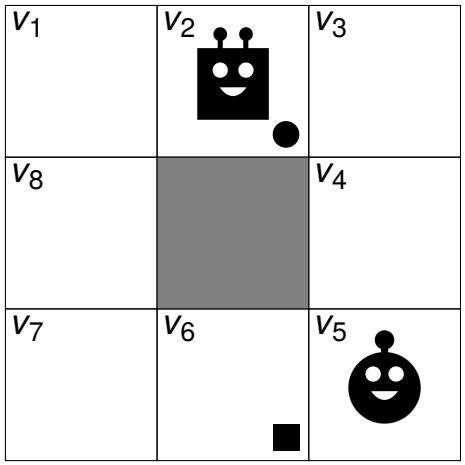

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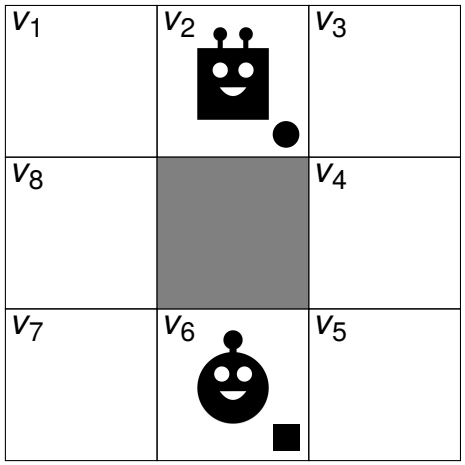

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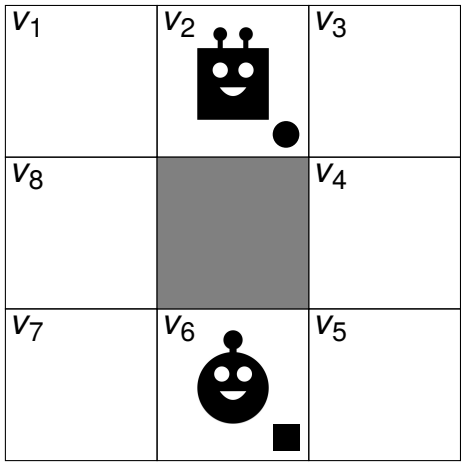

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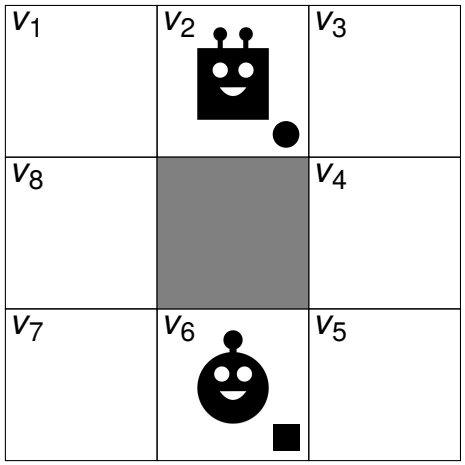

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Optimally eager agents

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- Eager agents avoid *deadlocks*, however they are *hyper-active*.
- They might even move away from their destination!
- So, let force them to be smart: They should generate only optimal plans . . . and among those optimal plans they should also be eager.
- In our previous example: After the square agent moved right, the circle agent will choose to move left!

→ Does it always work out?

Optimally eager agents are always successful

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Theorem

Optimally eager agents are always successful on all solvable DMAPF instances.

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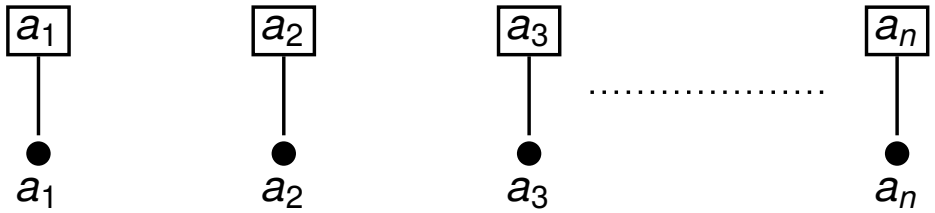

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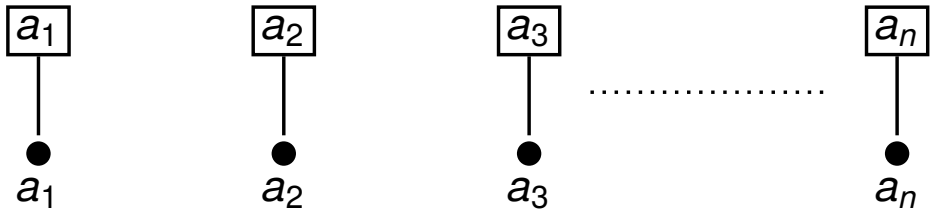

- Assume agents are selected for execution following a pattern similar to a Gray counter.

Other ways to coordinate?

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- If the agents may use different algorithms, then it is not clear how to avoid cyclic executions.
- Conservative replanning is not helpful in this context, because the executed actions might not be a prefix of a valid plan!

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→ Models multi-robot interactions without communication

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→ Since MAPF/DU is a special case of epistemic planning (initial state uncertainty which is monotonically decreasing), we can use concepts and results from this area.

MAPF/DU representation & state space

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- In addition to the sets of agents A, the graph $G = (V, E)$, and the assignment of agents to nodes α, we need a function to represent the *possible destinations* $\beta : A \to 2^V$.

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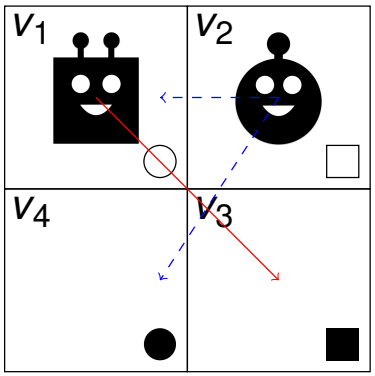
- A *MAPF/DU instance* is given by $\langle A, G, s_0, \alpha_* \rangle$, where $s_0 = \langle \alpha_0, \beta_0 \rangle$.

MAPF/DU: Implicitly coordinated branching plans

- Square agent S wants to go to v_3 and knows that circle agent C wants to go to v_1 or v_4.



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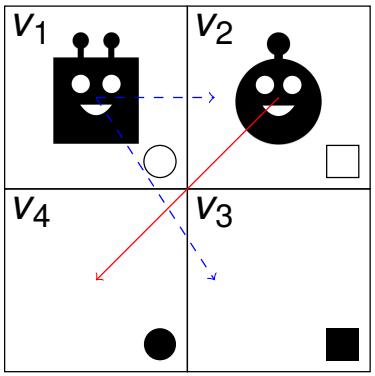

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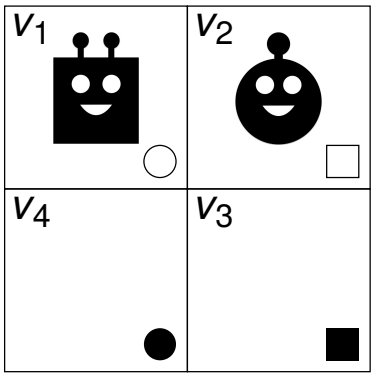

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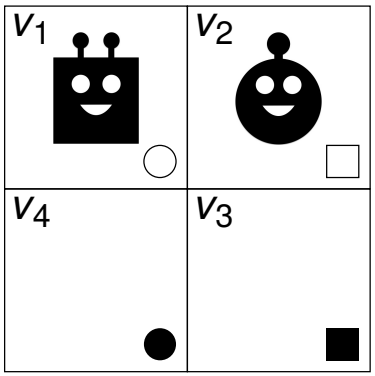

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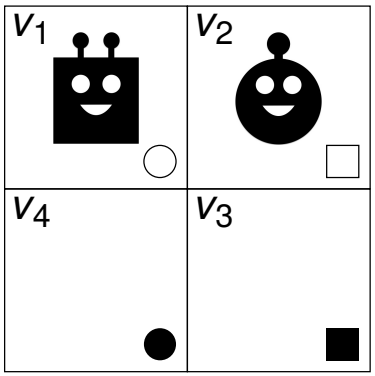

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- S needs *shifting its perspective* in order to plan for all possible destinations of C (*branching on destinations*).
- Planning for C, S must *forget* about its own destination.

Branching plans: Building blocks

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Branching plans consist of:

- *Movement actions*: $(\langle agent \rangle, \langle sourcenode \rangle, \langle targetnode \rangle)$, i.e., a movement of an agent
- *Success announcement*: $(\langle agent \rangle, \mathcal{S})$, after that all agents know that the agent has reached its destination and it cannot move anymore
- *Perspective shift*: $[\langle agent \rangle : \ldots]$, i.e., from here on we assume to plan with the knowledge of agent $\langle agent \rangle$. This can be unconditional or conditional on $\langle agent \rangle$'s destinations.
- *Branch on all destinations*: $(?\langle dest_1 \rangle\{\ldots\}, \ldots, ?\langle dest_n \rangle\{\ldots\})$, where all destinations of the current agent have to be listed. For each case we try to find a successful plan to reach the goal state.

Semantics of branching plans

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- Movement actions modify α in the obvious way.

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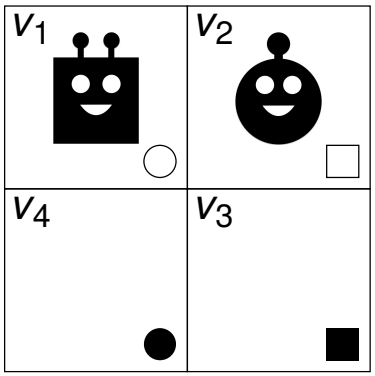
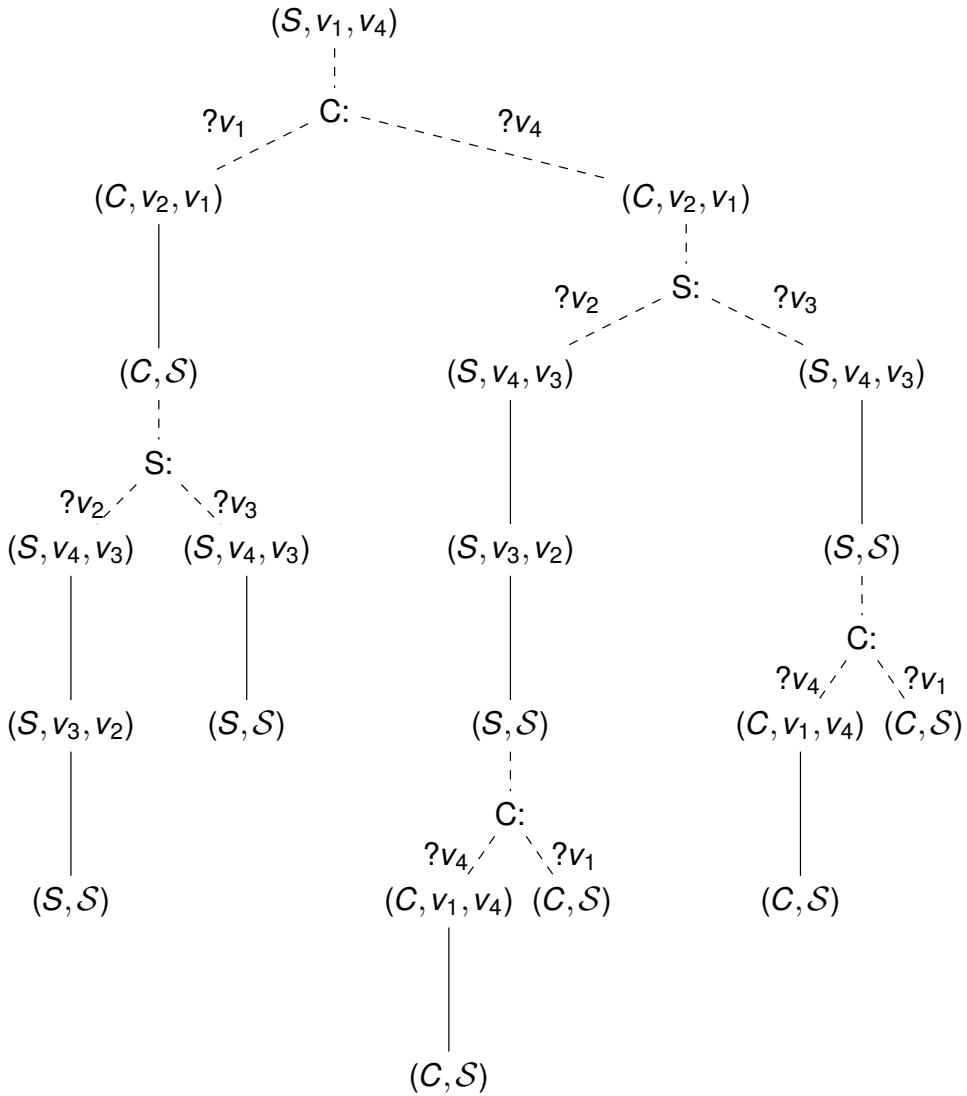
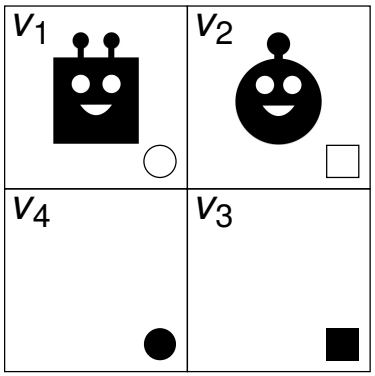
- A perspective shift from i to j without subsequent branching on destinations induces the same transformation, but enforces that the subsequent plans are the same for all states subjective states s^{j_k}.

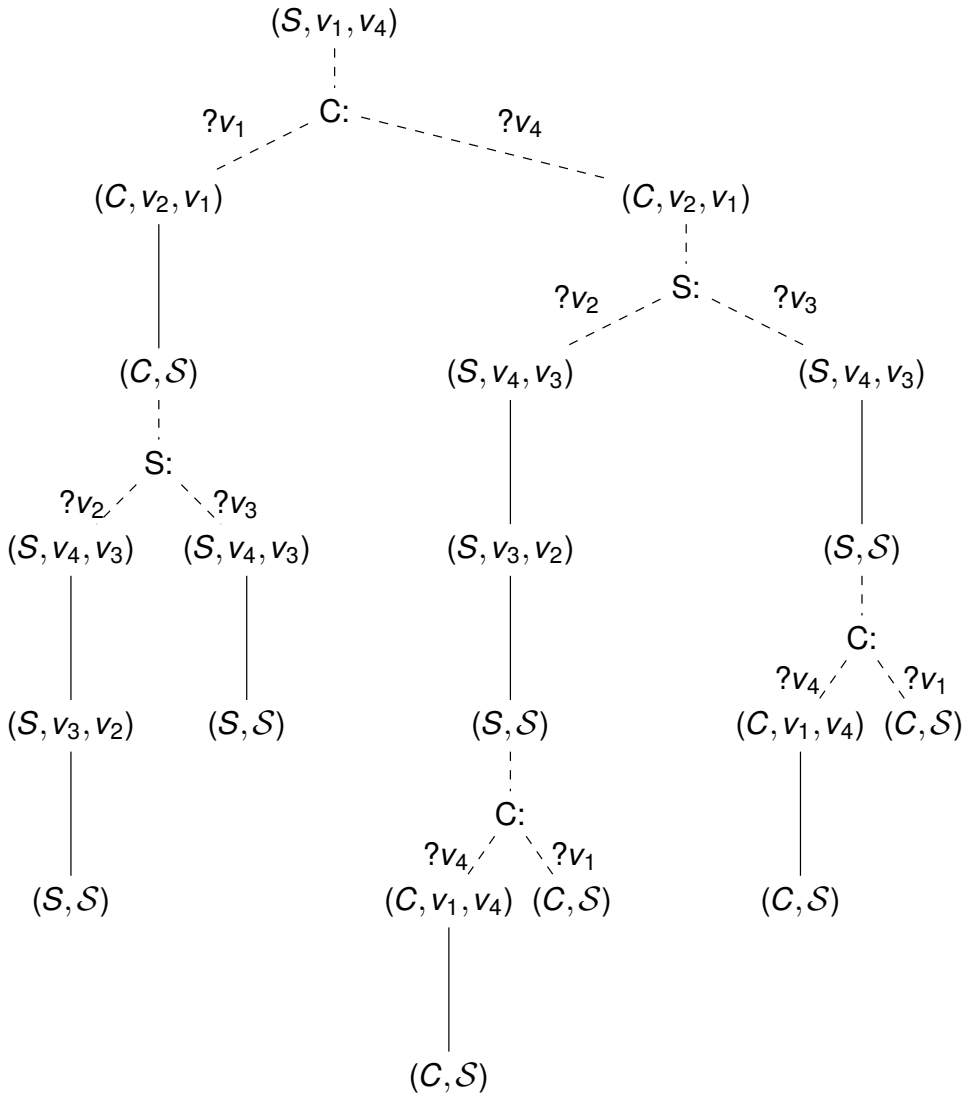
Branching plan: Example





Branching plan: Subjective execution example





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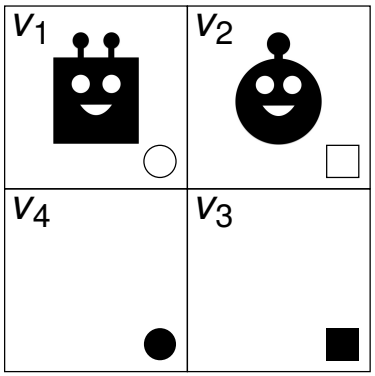

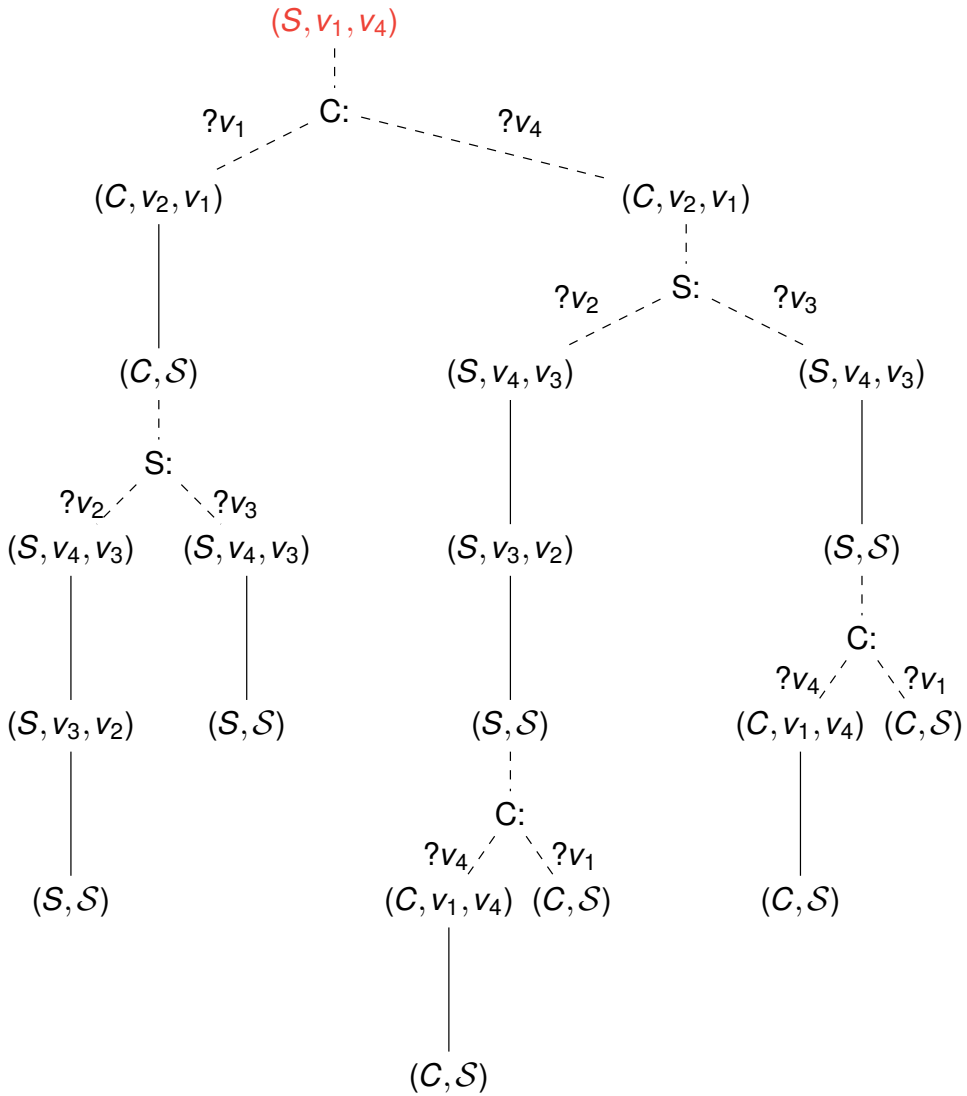

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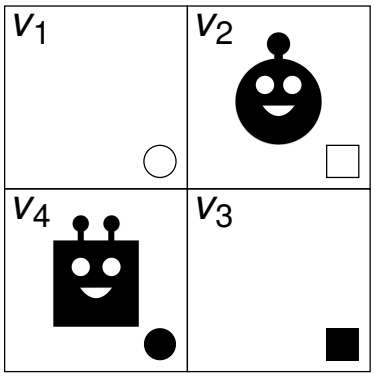

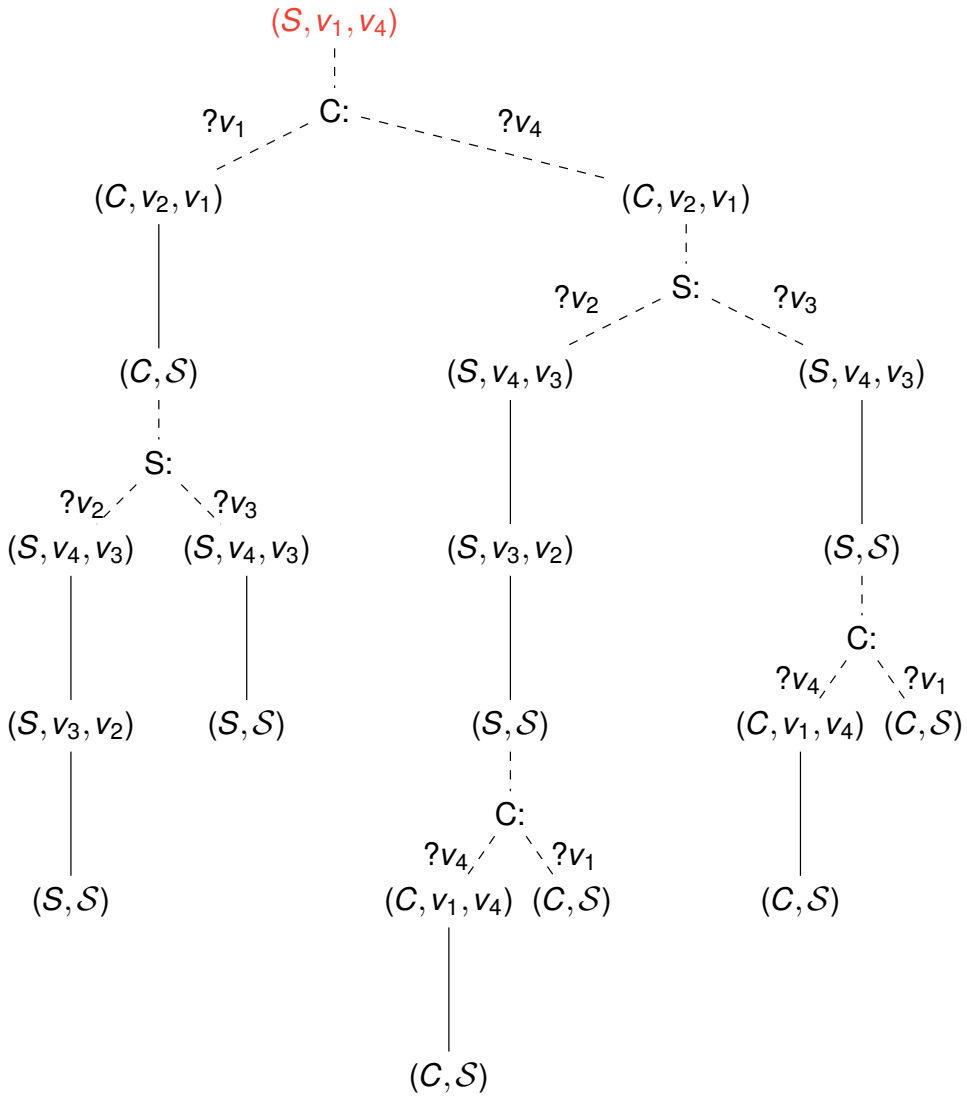

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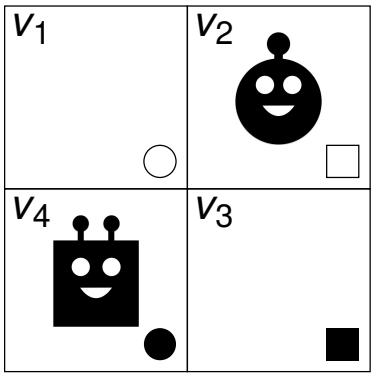

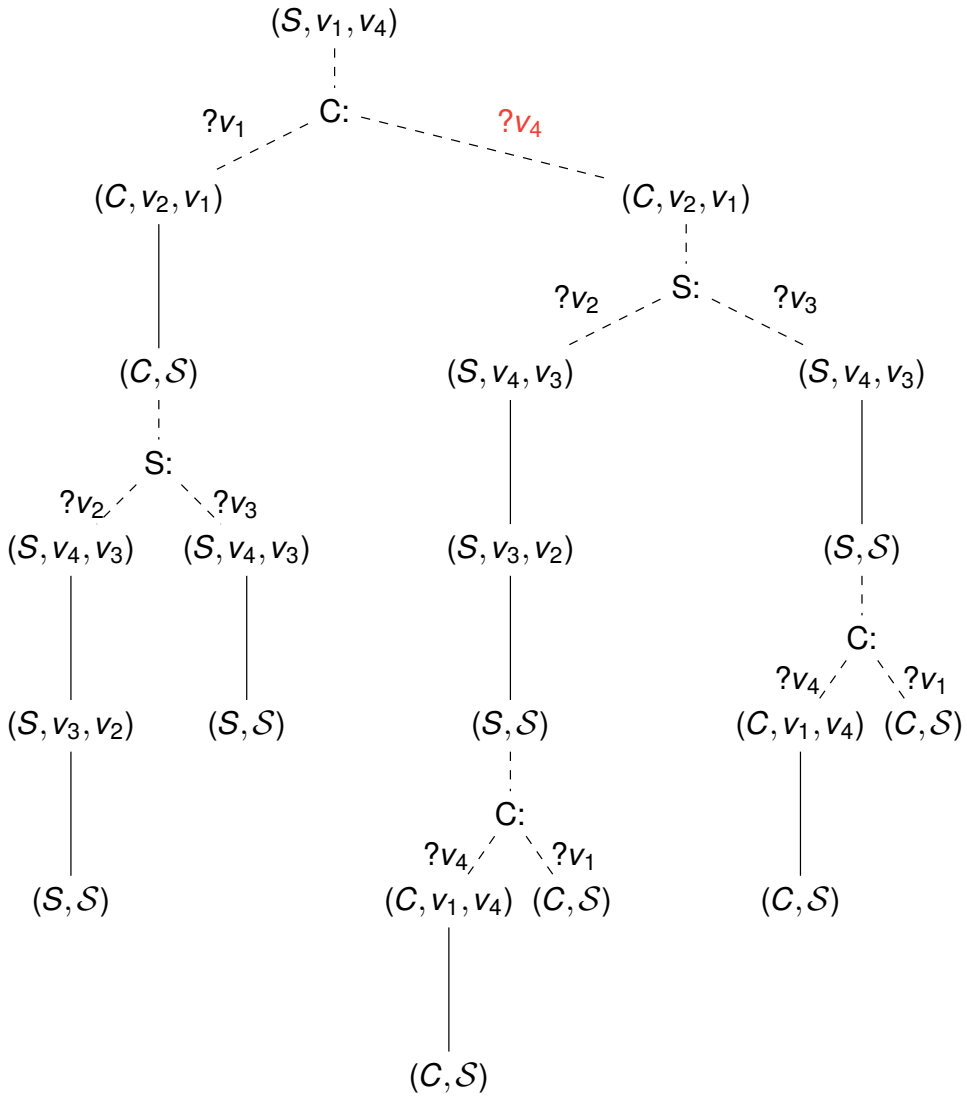

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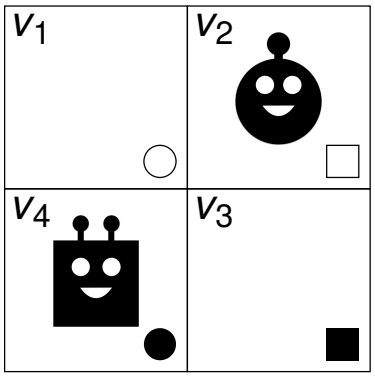

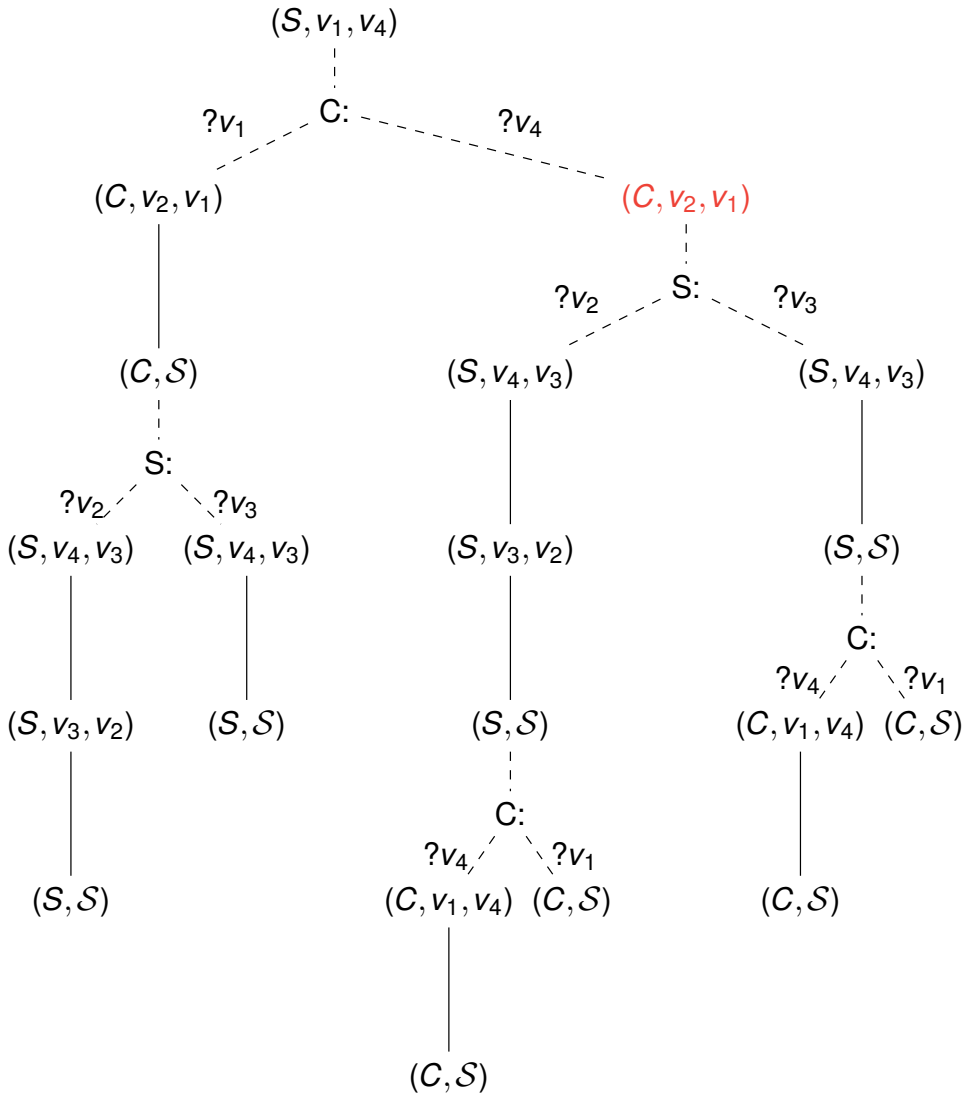

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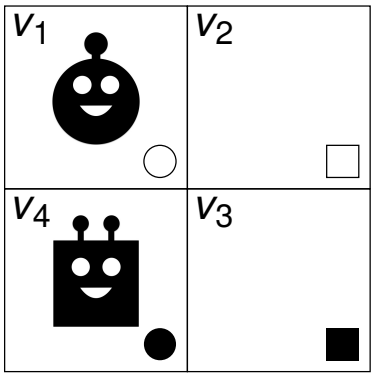

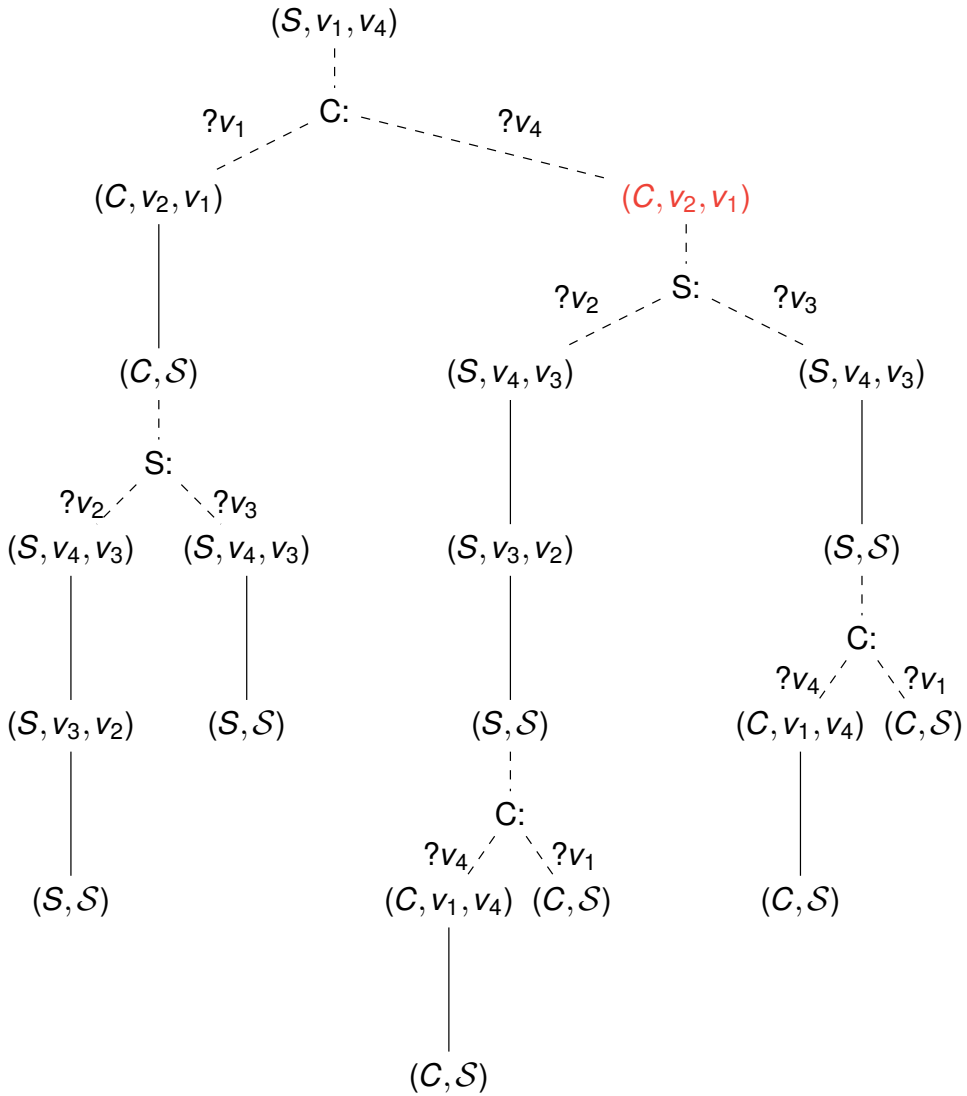

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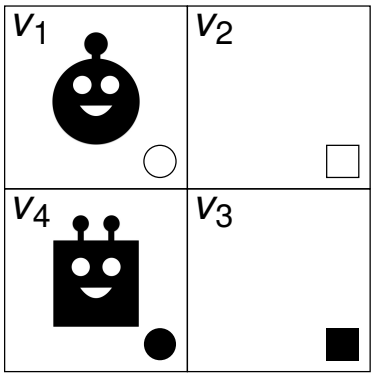

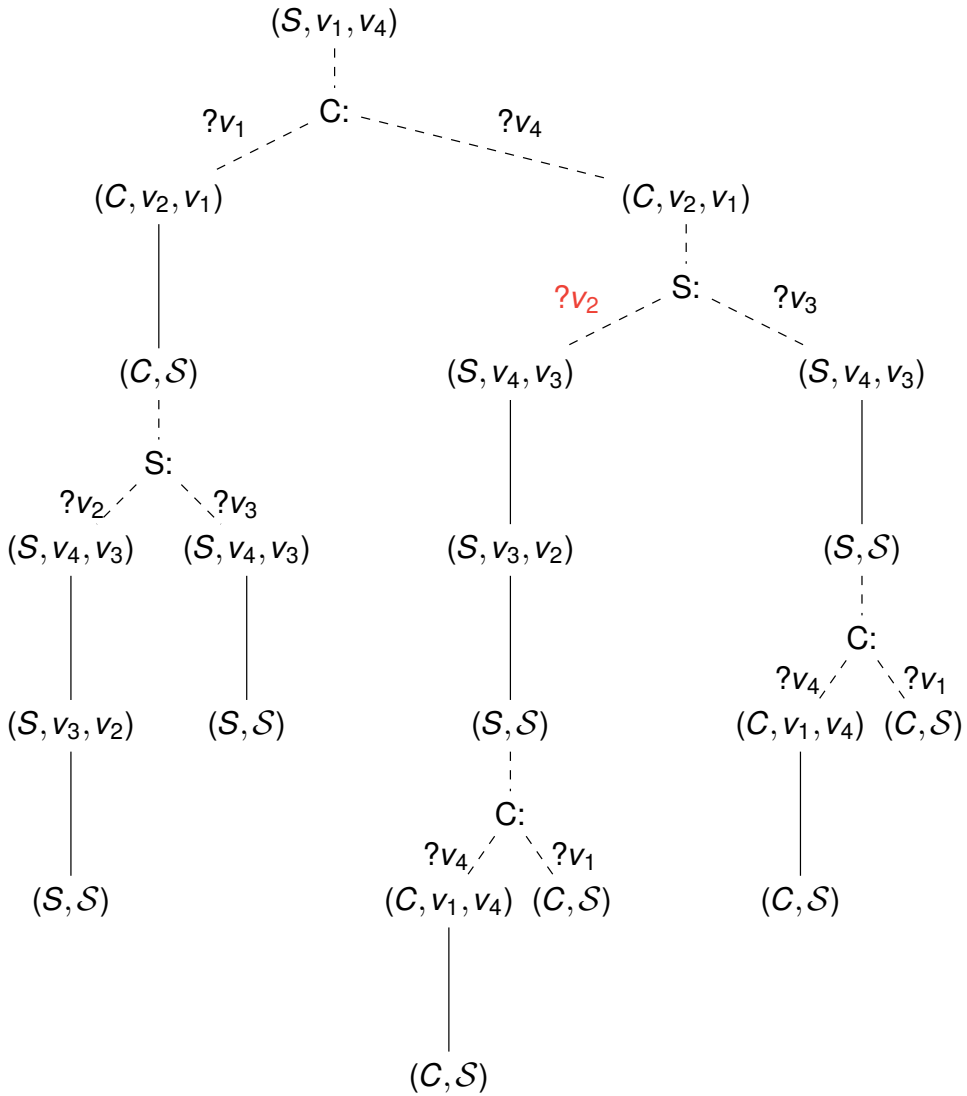

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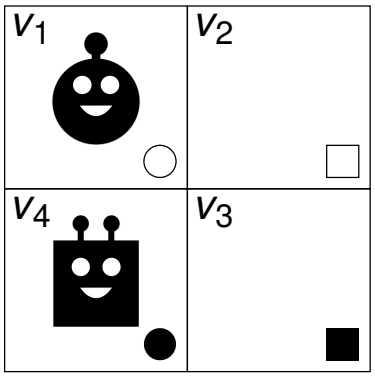

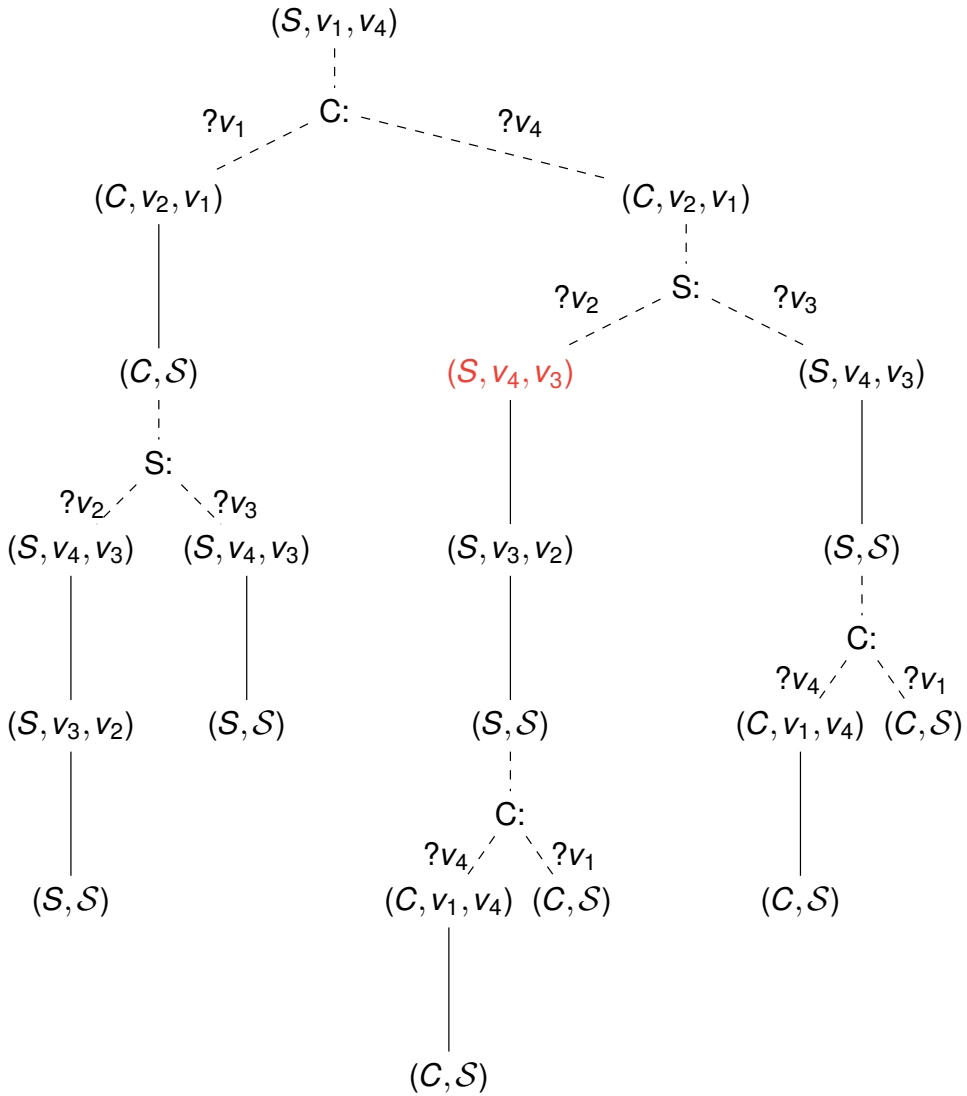

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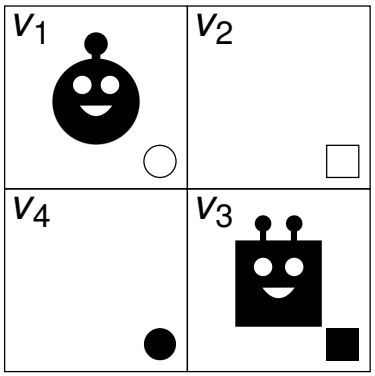

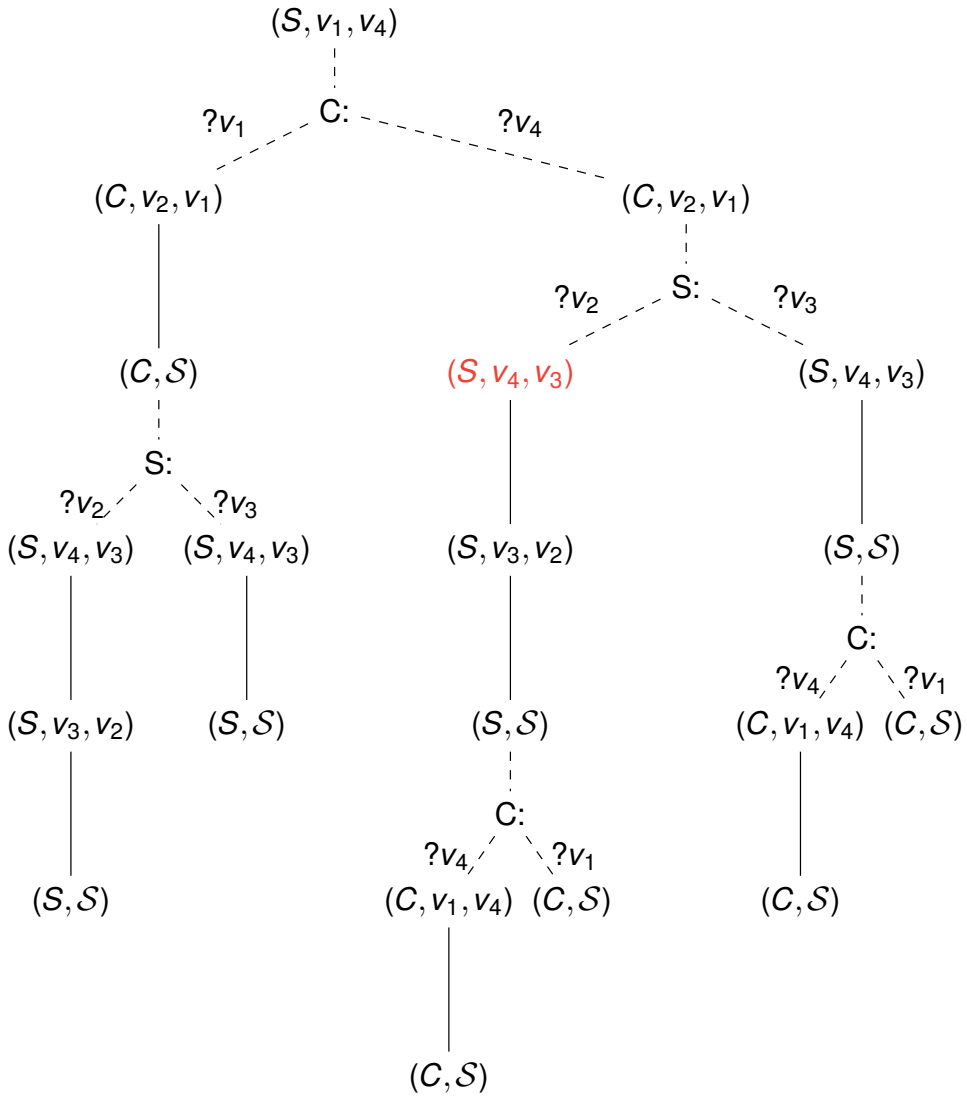

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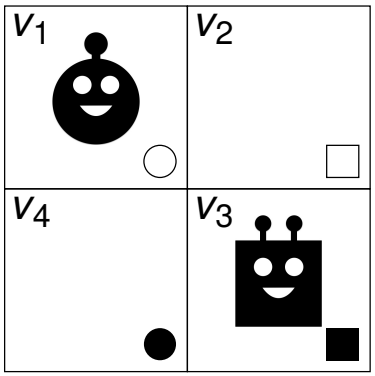

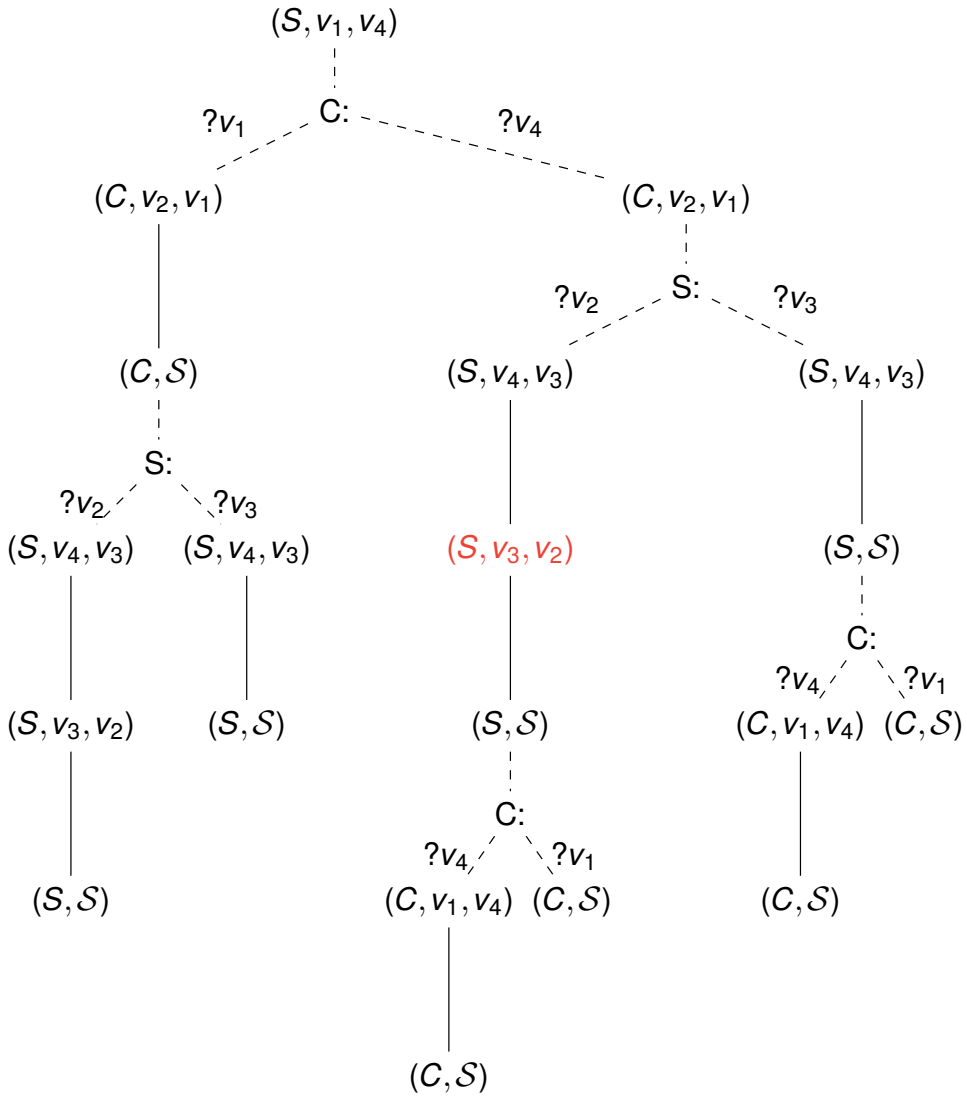

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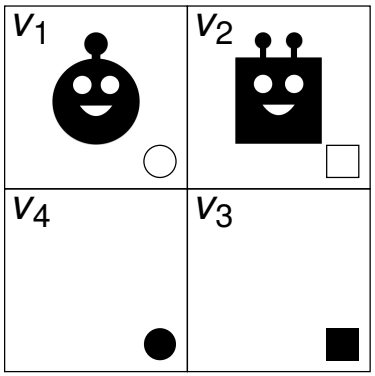

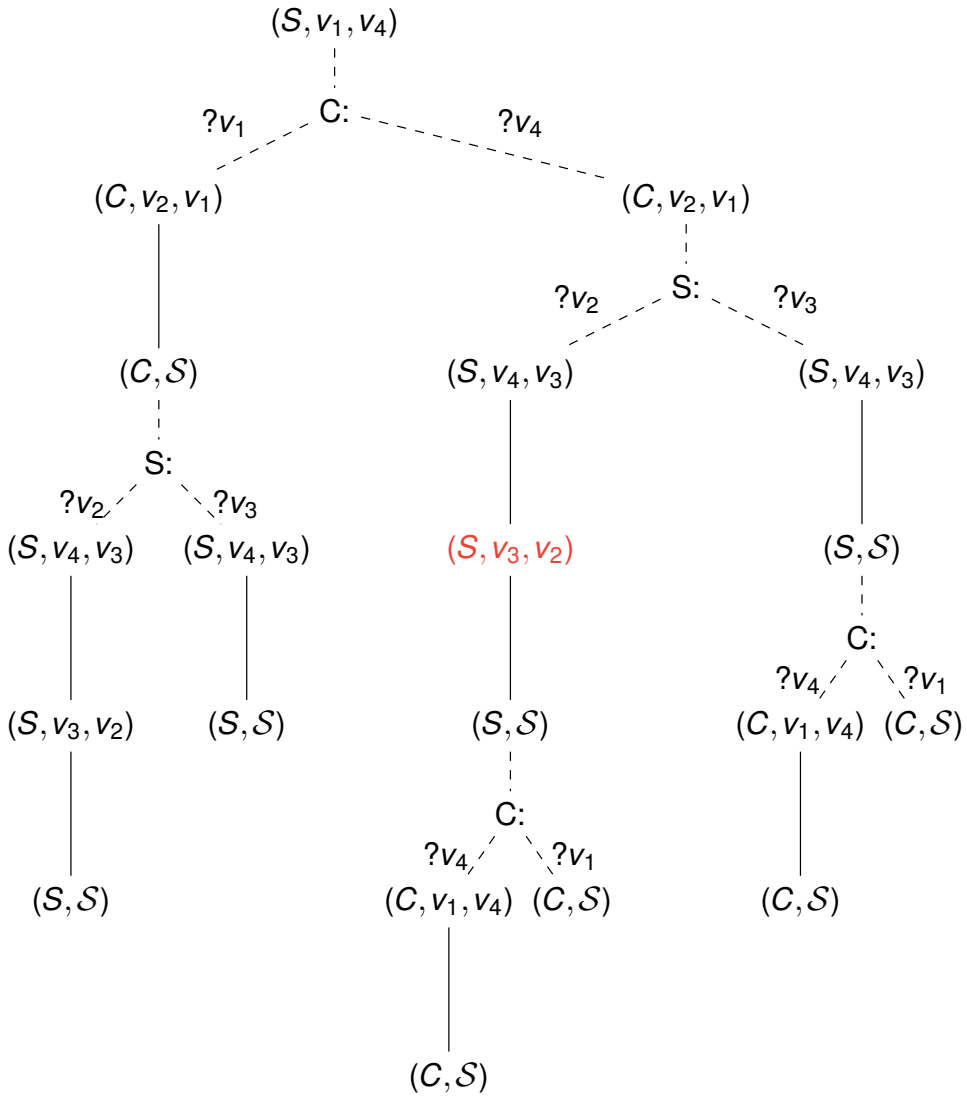

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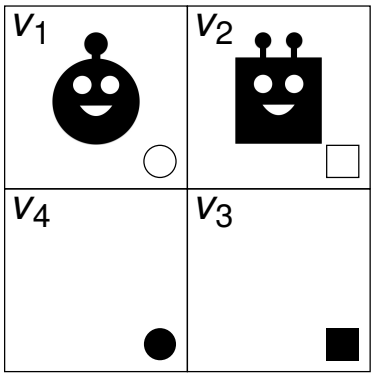

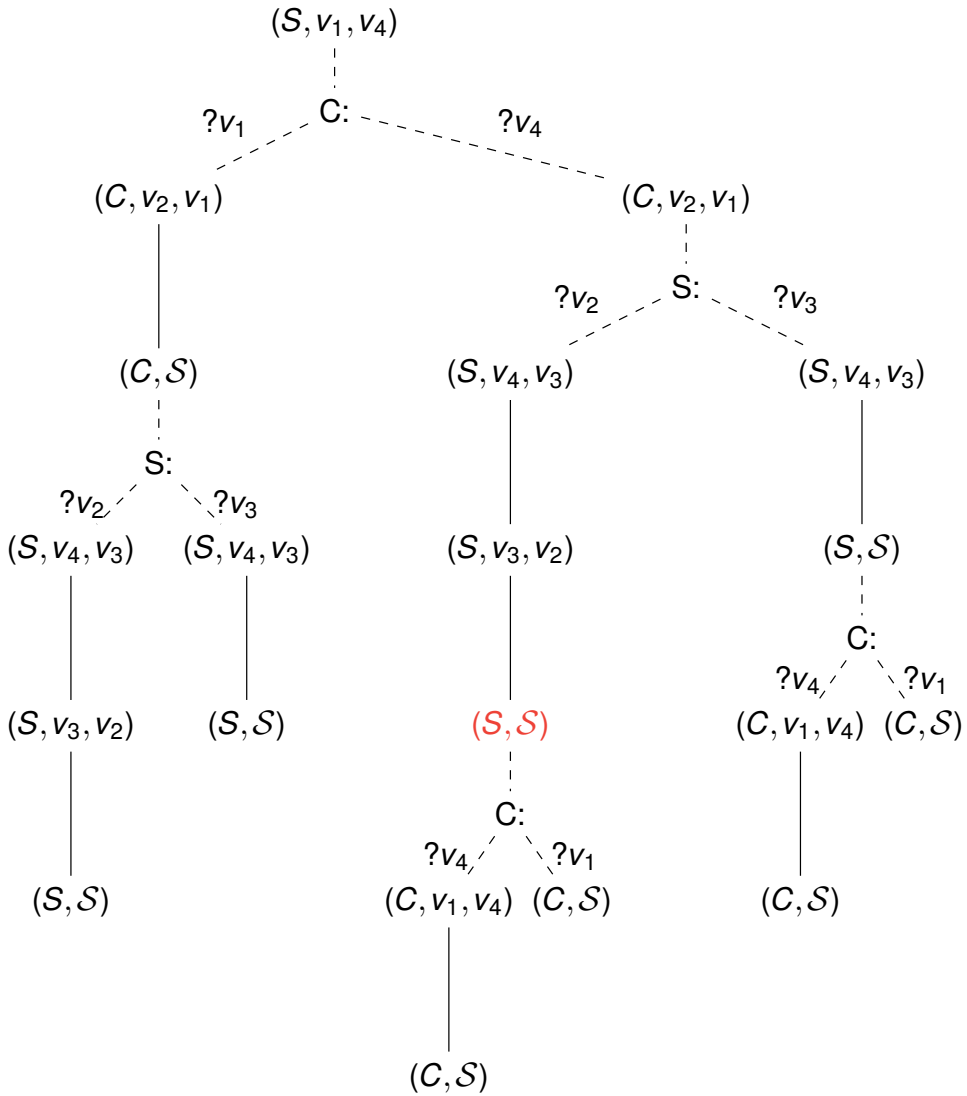

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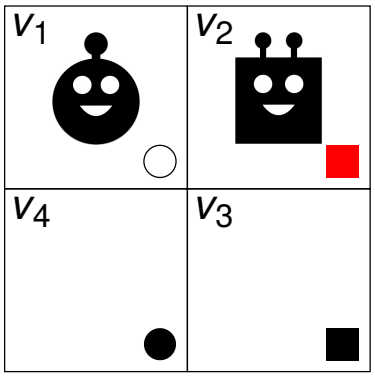

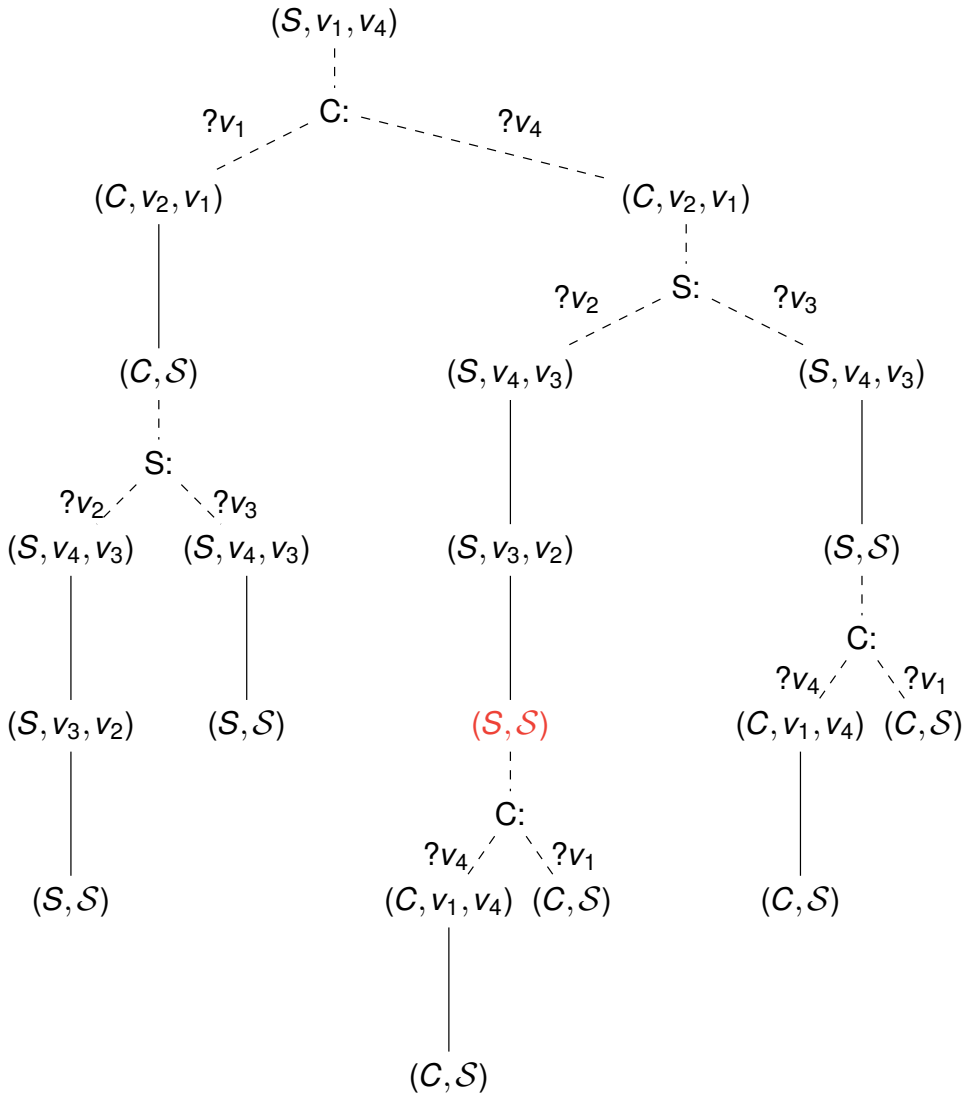

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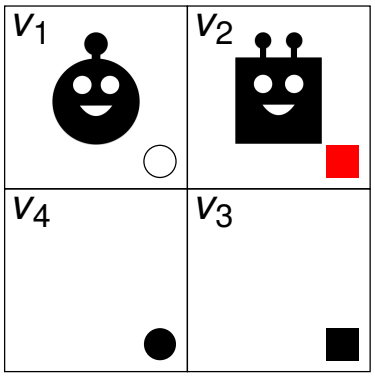

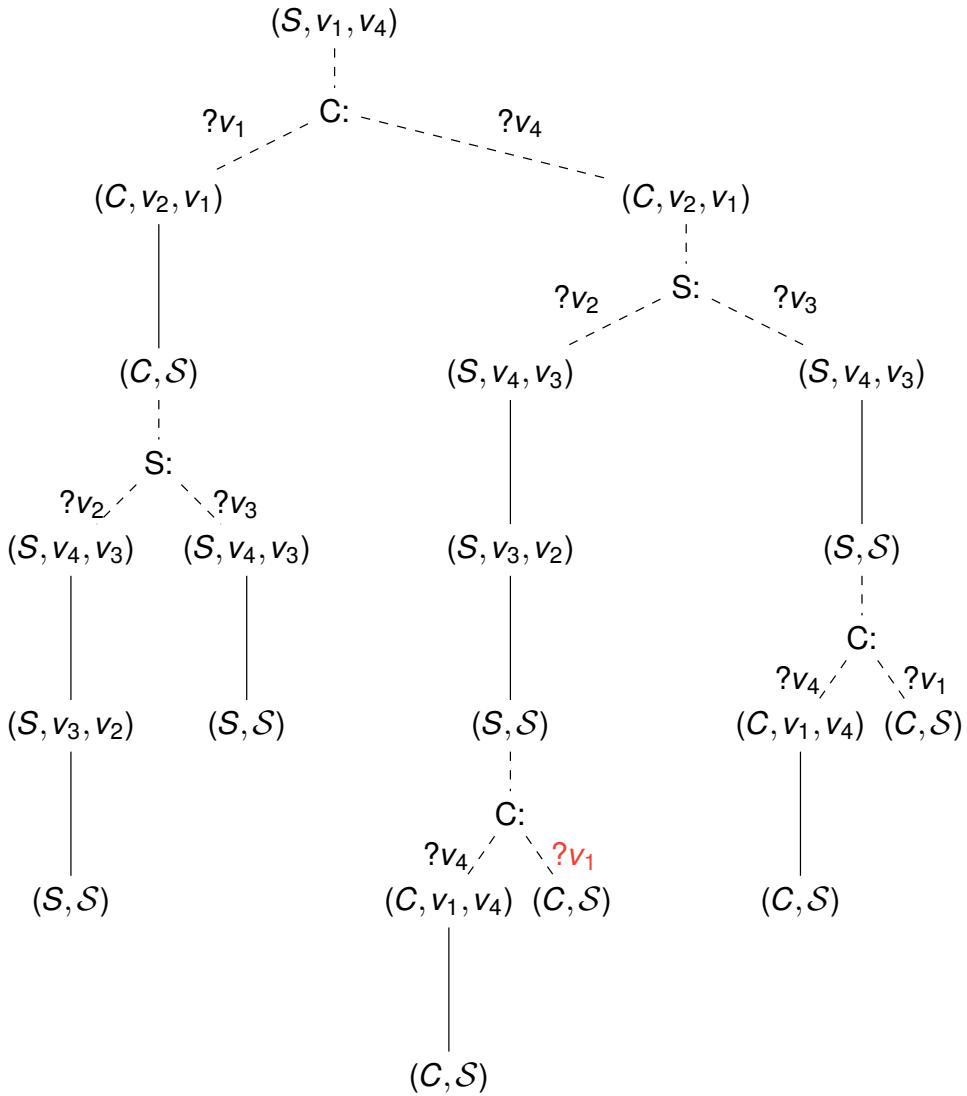

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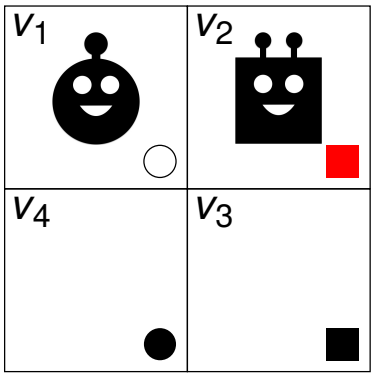

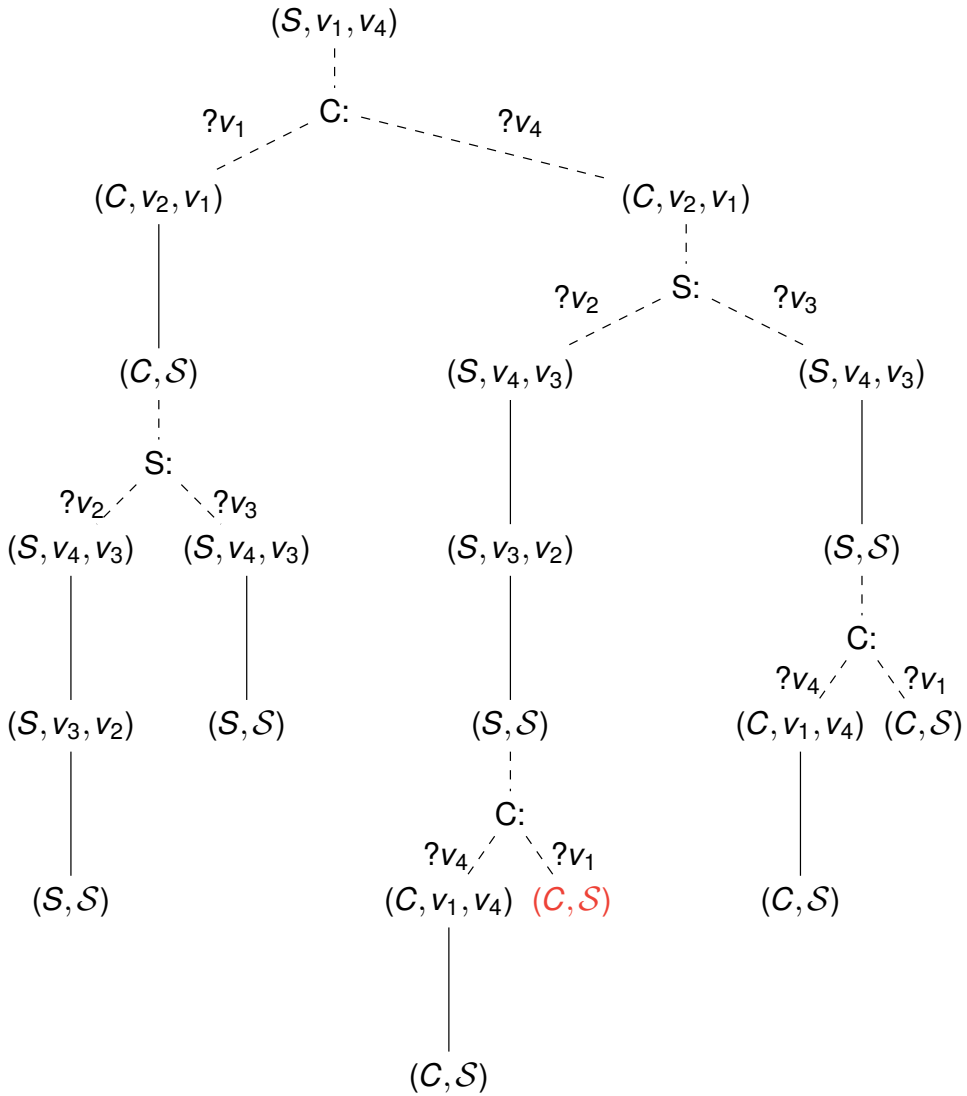

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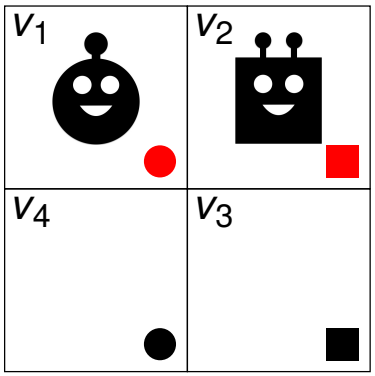

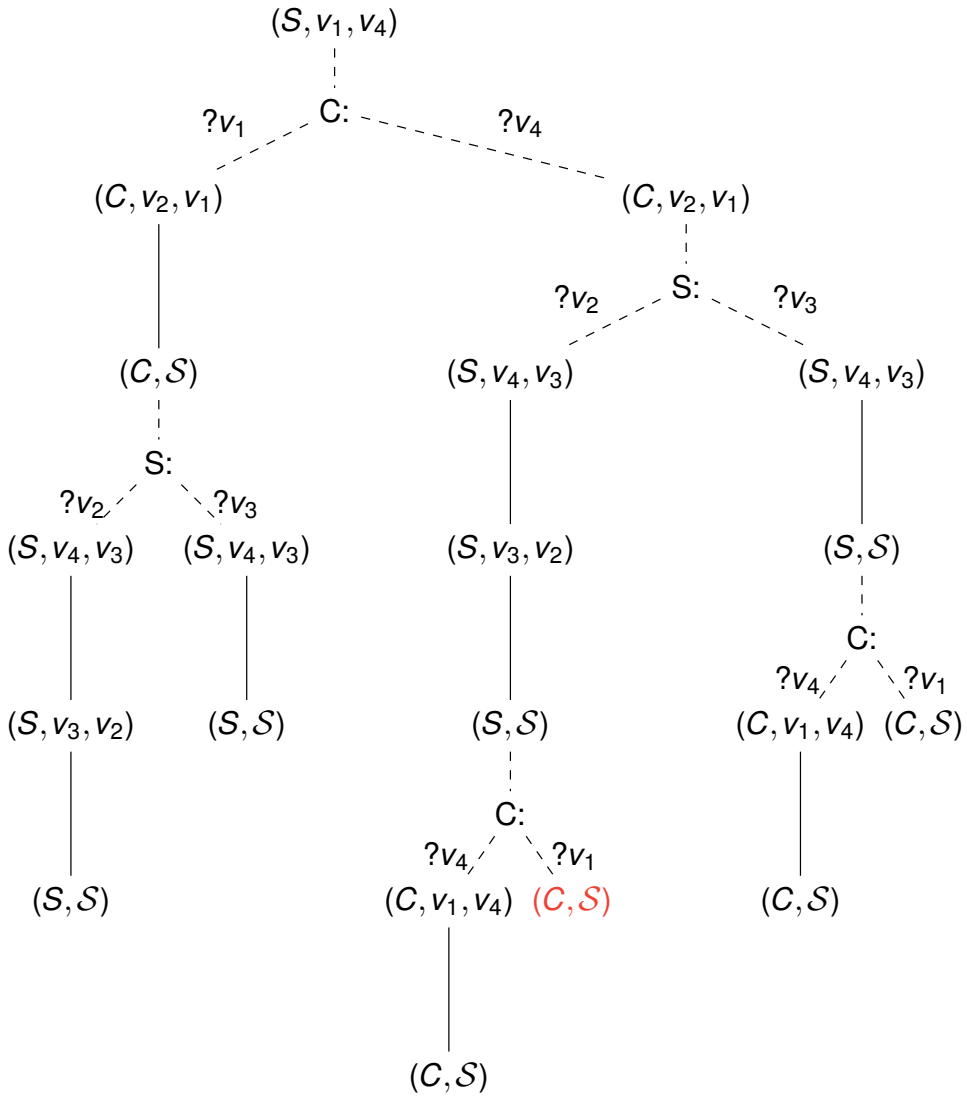

Strong plans

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Similar to the notion of strong plans in non-deterministic single-agent planning, we define *i-strong plans* for an agent *i* to be:

- *cycle-free*, i.e., not visiting the same objective state twice;
- *always successful*, i.e. always ending up in a state such that all agents have announced success;
- *covering*, i.e., for all combinations of possible destinations of agents different from *i*, success can be reached.

Subjectively and objectively strong plans

- A plan is called *subjectively strong* if it is *i*-strong for some agent *i*.
- A plan is called *objectively strong* if it is *i*-strong for each agent *i*.
- An instance is *objectively* or *subjectively solvable* if there exists an objectively or subjectively strong plan, respectively.

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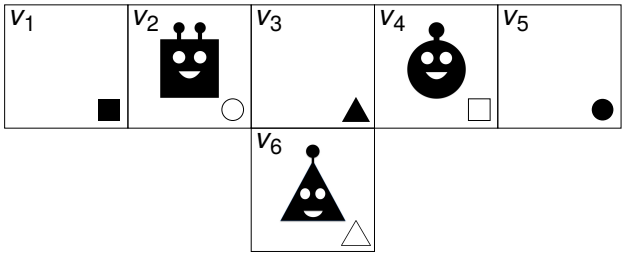

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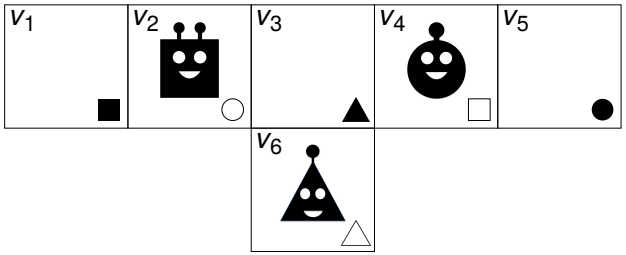

→ There does not exist a T-strong plan, but an S- and a C-strong plan.

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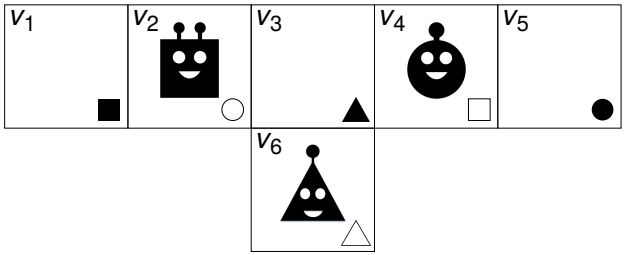
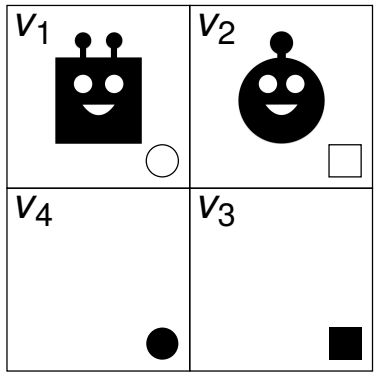


→ There does not exist a T-strong plan, but an S- and a C-strong plan.

- Difference between subjective and objective solvability concerns only the first acting agent!

Structure of strong plans: Stepping stones

- A *stepping stone* for agent i is a state in which i can move to each of its possible destinations, announcing success, and afterwards, for each possible destination, there exists an i-strong plan to solve the resulting states.



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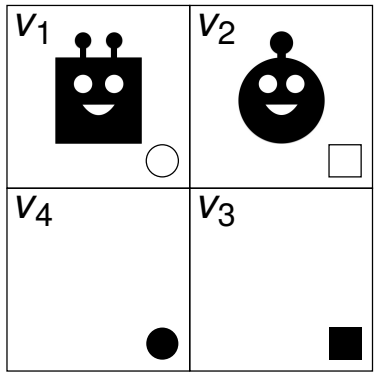

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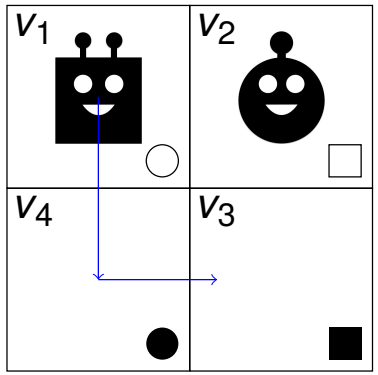

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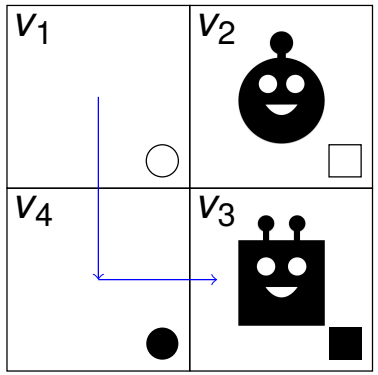

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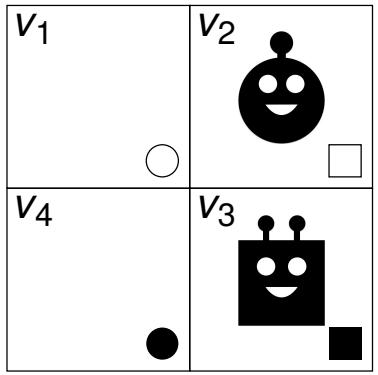

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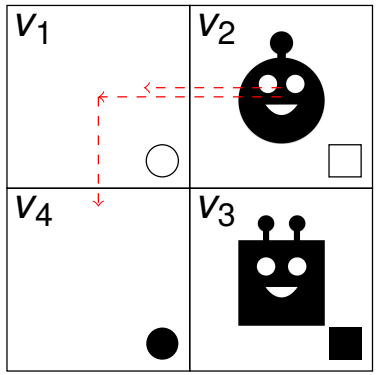

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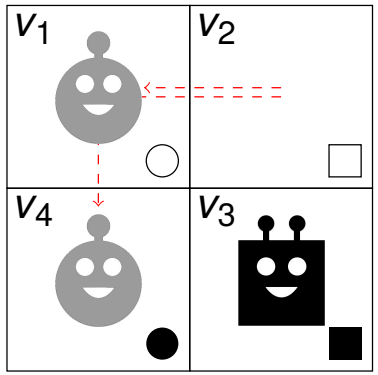

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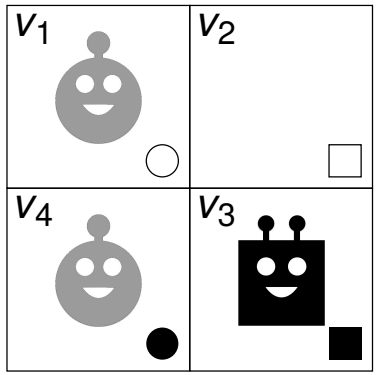

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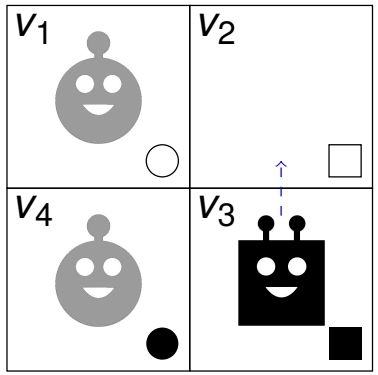

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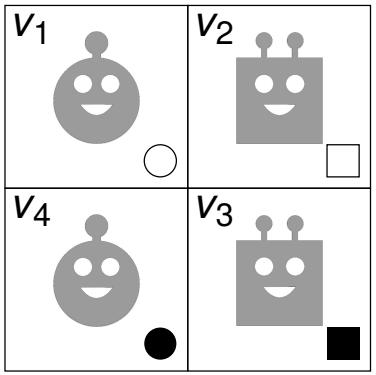

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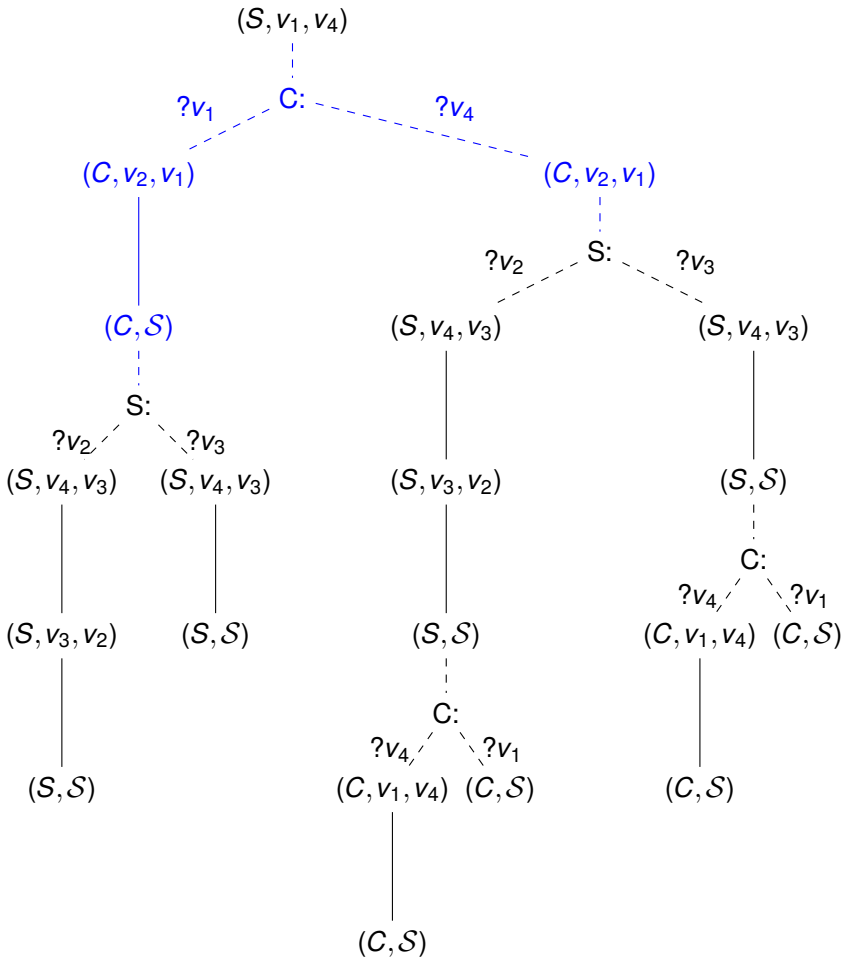

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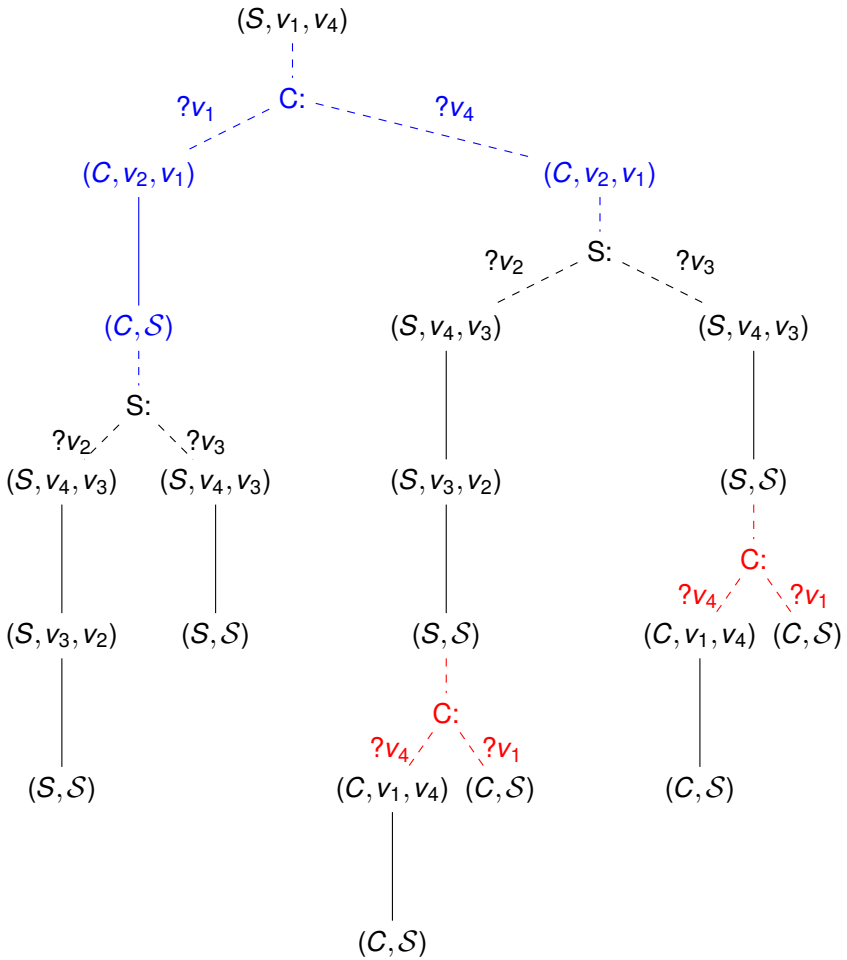

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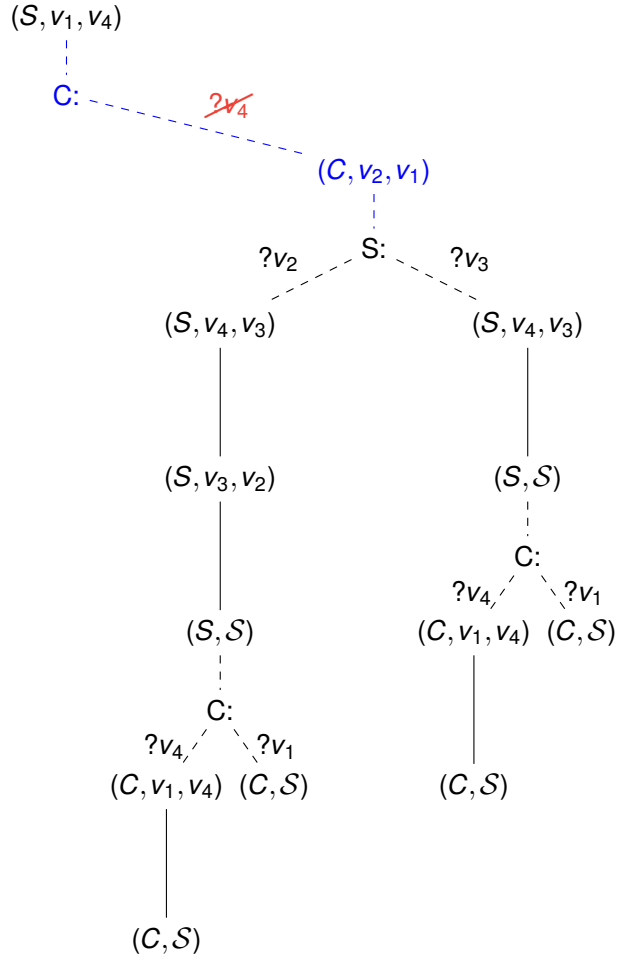

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Direct consequence of the stepping stone theorem and the maximal number of movements in the MAPF problem. □

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- We do not have uniform knowledge . . . and indeed, execution cycles cannot be excluded.

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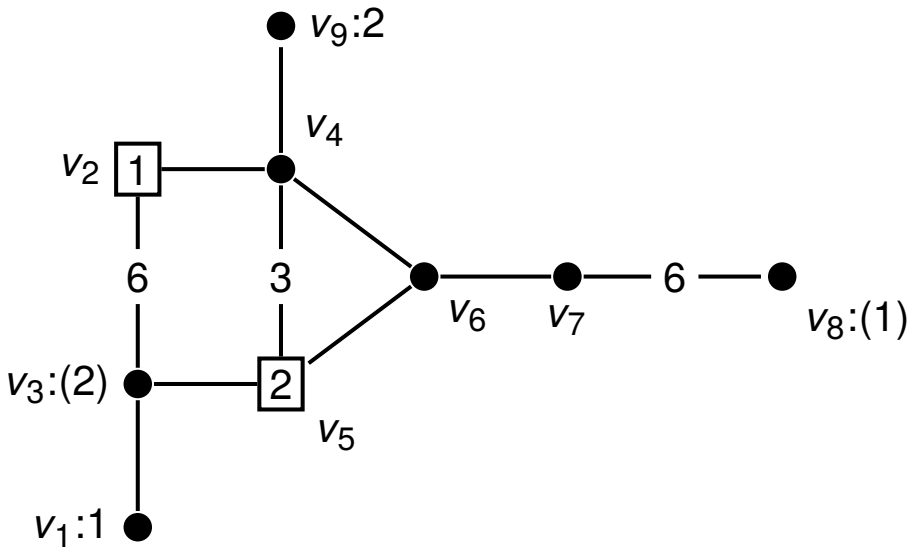

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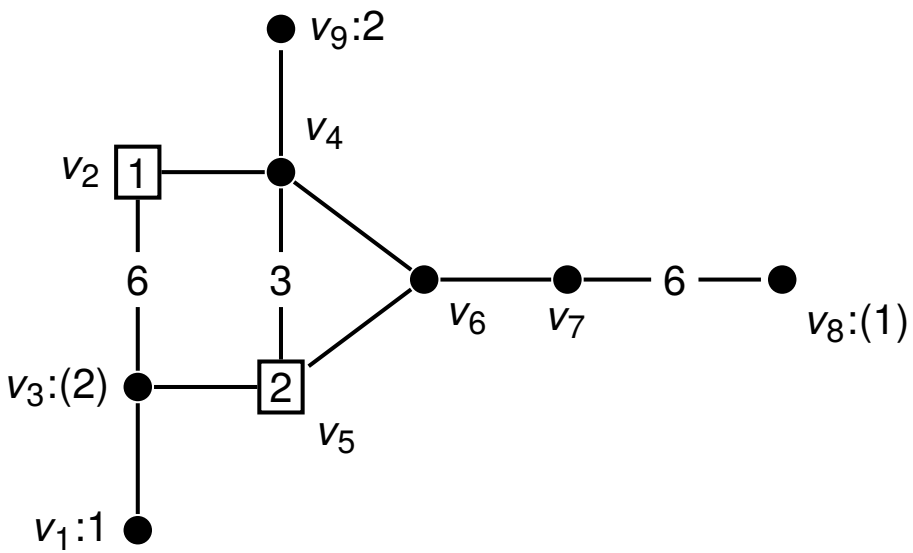

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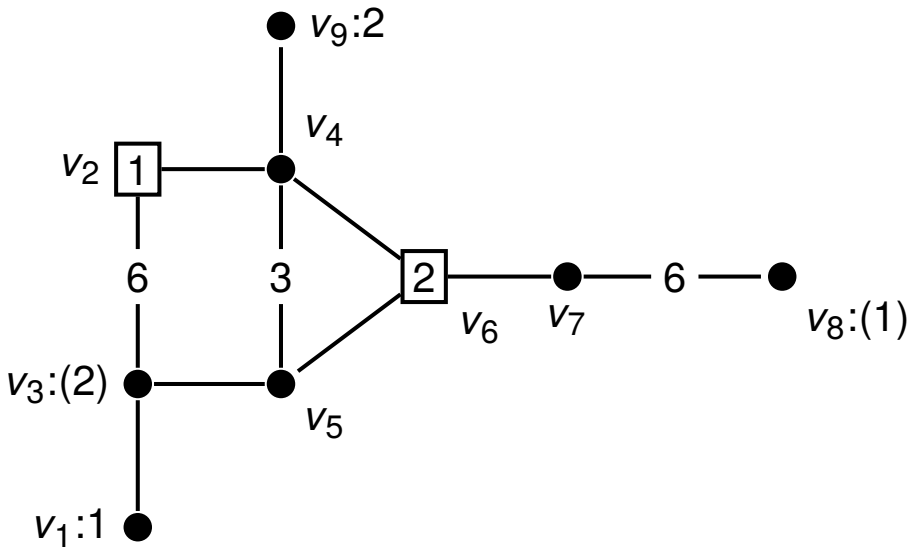

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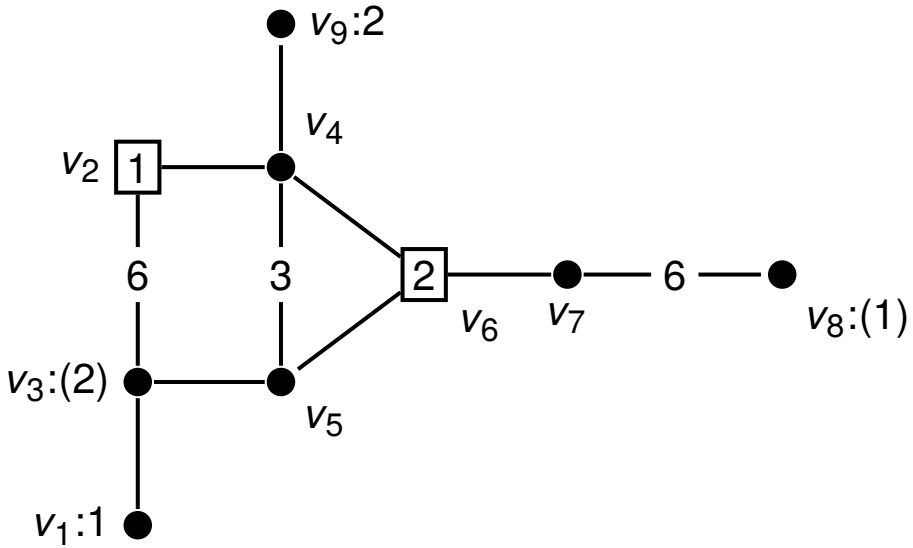

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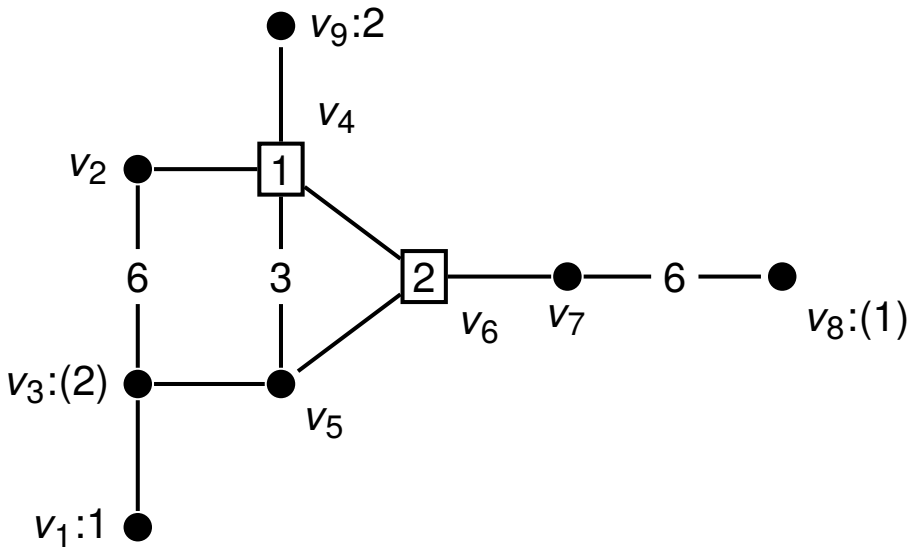

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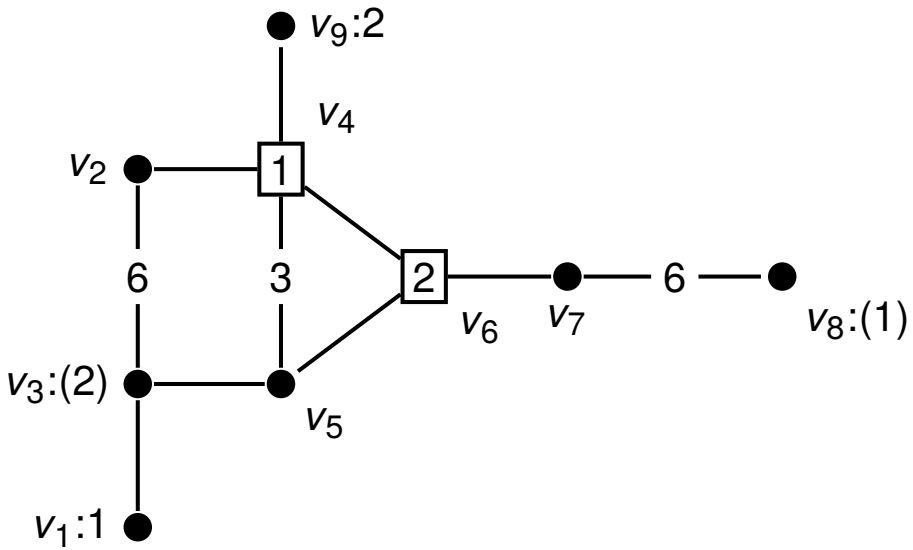

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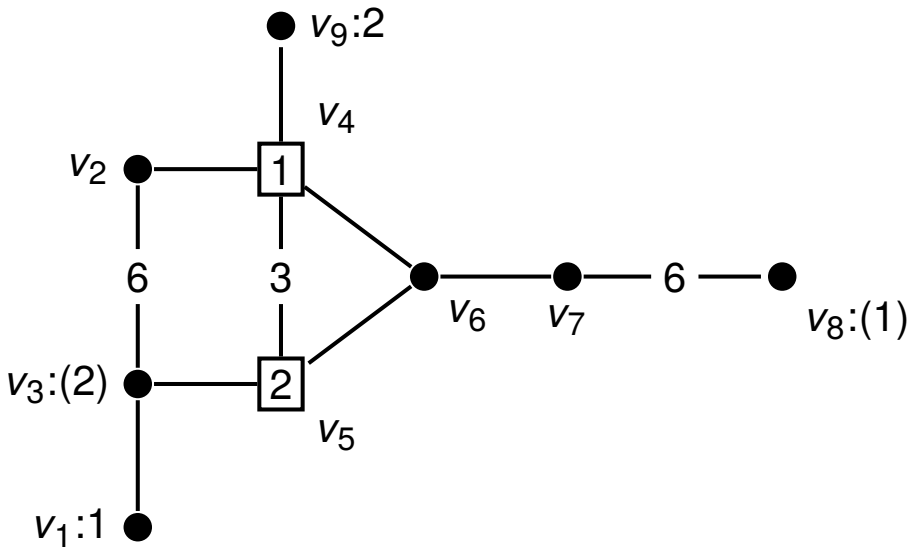

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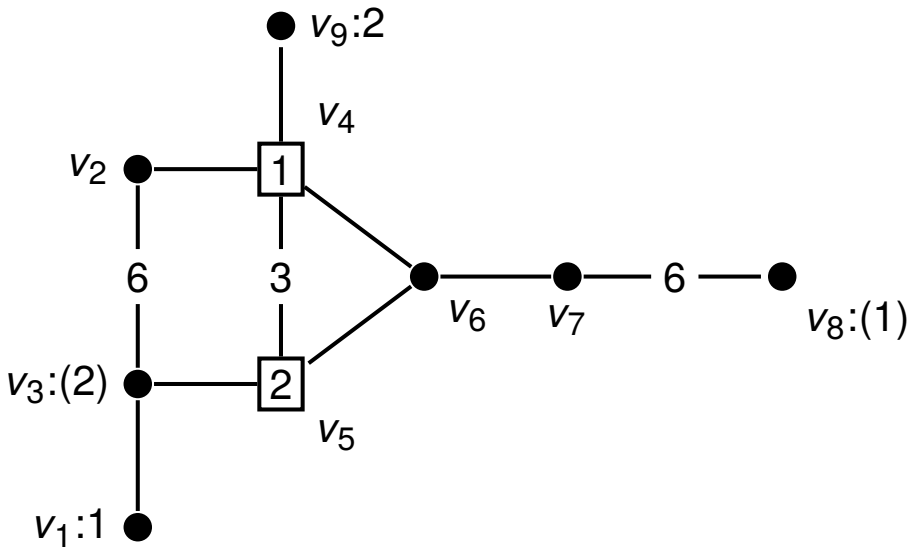

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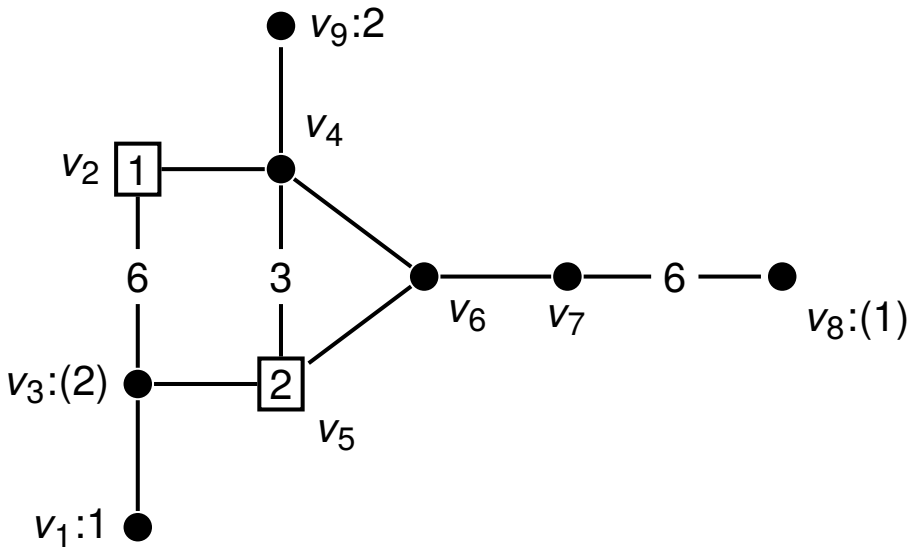

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Conservatism

- Perhaps conservatism can help!
- Similarly to DMAPF, conservative replanning means that the already executed actions are used as a prefix in the plan to be generated.
- Differently from DMAPF, we assume that after a success announcement, the initial state is modified so that the *real destination* of the agent is known in the initial state.
- Otherwise we could not solve instances that are only subjectively solvable.

Conservative, optimally eager agents

- Conservative, eager agents are always successful, but might visit the entire state space before terminating.

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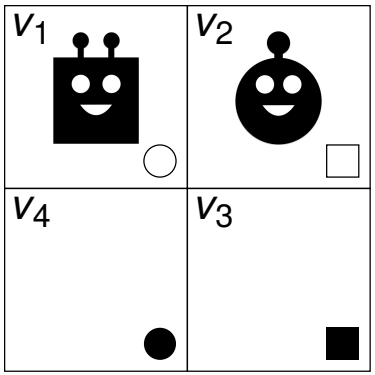
Proof idea.

After the second agent starts to act, all agents have an identical perspective and for this reason produce objectively strong plans with the same execution costs, which can be shown to be bounded polynomially using the stepping stone theorem. □

Conservative replanning example

- Assume S moves first to v_4.



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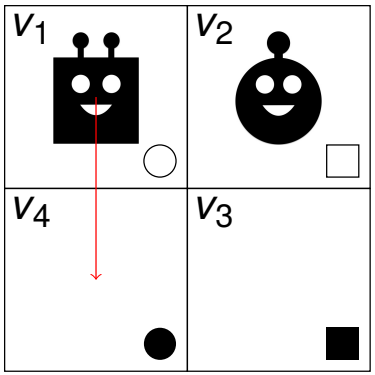

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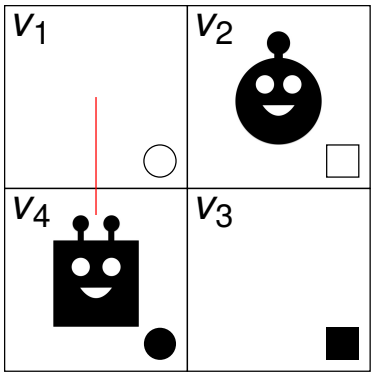

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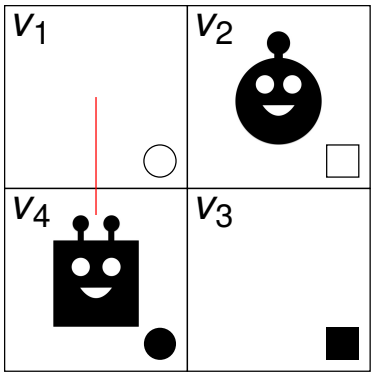

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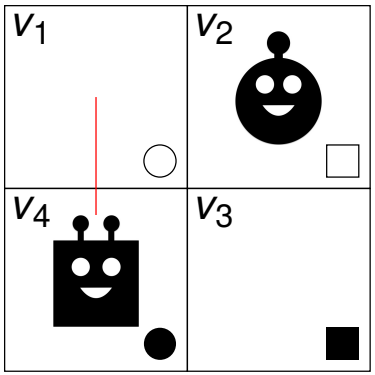

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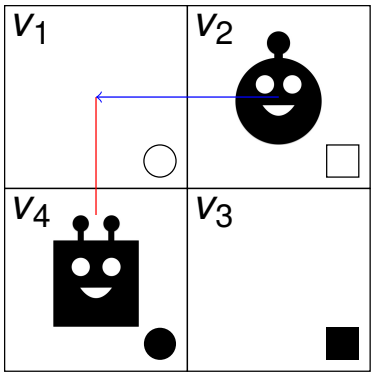

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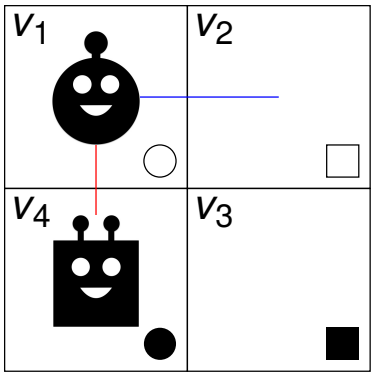

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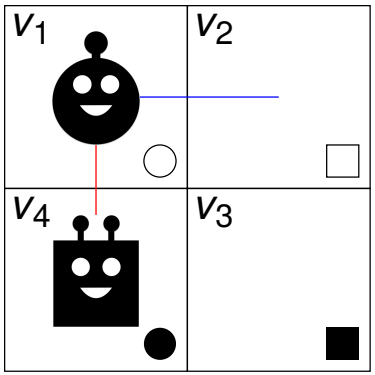
Conservative replanning example



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- Assume that C moves now to v_1.
- From now on, also S has to make a perspective shift to C, effectively "forgetting" its own destination, i.e., it also has to create a objectively strong plan.

Computational Complexity: Algorithms and Turing machines

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- We use Turing machines as formal models of algorithms
- This is justified, because:
  - we assume that Turing machines can compute all computable functions
  - the resource requirements (in term of time and memory) of a Turing machine are only polynomially worse than other models
- The regular type of Turing machine is the deterministic one: DTM (or simply TM)
- Often, however, we use the notion of nondeterministic TMs: NDTM

Computational Complexity: Problems, solutions, and complexity

UNI FREIBURG

- A problem is a set of pairs (I, A) of strings in $\{0,1\}^*$.
  I: instance; A: answer
  If all answers $A \in \{0,1\}$: decision problem

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$$T : \mathbb{N} \to \mathbb{N},$$

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- Complexity of a problem: complexity of the most efficient algorithm that solves this problem.

Computational Complexity: Complexity classes P and NP

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Problems are categorized into complexity classes according to the requirements of computational resources:

- The class of problems decidable on deterministic Turing machines in polynomial time: P
  - Problems in P are assumed to be efficiently solvable (although this might not be true if the exponent is very large)
  - In practice, a reasonable definition
- The class of problems decidable on non-deterministic Turing machines in polynomial time, i.e., having a poly. length accepting computation for all positive instances: NP
- More classes are definable using other resource bounds on time and memory

Computational Complexity: Upper and lower bounds

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  - the technical tool here is the polynomial reduction (or any other appropriate reduction)
  - show that some hard problem can be reduced to the problem at hand

Computational Complexity: Polynomial reduction

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- Given languages L_1 and L_2, L_1 can be polynomially reduced to L_2, written $L_1 \leq_p L_2$, if there exists a polynomial time-computable function f such that

$$x \in L_1 \iff f(x) \in L_2.$$

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- L is complete for C (C-complete) if L is C-hard and $L \in C$.

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- A problem is NP-complete iff it is NP-hard and in NP.

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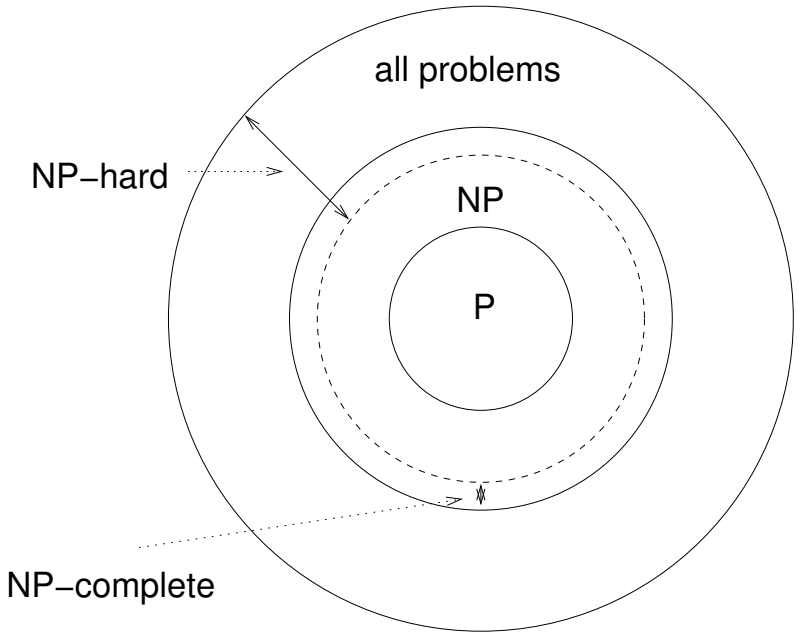

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- Examples: UNSAT, TAUT $\in$ co-NP!
- *Note:* P is closed under complement, in particular,

$$\text{P} \subseteq \text{NP} \cap \text{co-NP}$$

Computational Complexity: PSPACE

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There are problems even more difficult than NP and co-NP...

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Definition ((N)PSPACE)

PSPACE (NPSPACE) is the class of decision problems that can be decided on deterministic (non-deterministic) Turing machines using only polynomially many tape cells.

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Some facts about PSPACE:

- PSPACE is closed under complements (... as all other deterministic classes)
- PSPACE is identical to NPSPACE (because non-deterministic Turing machines can be simulated on deterministic TMs using only quadratic space: Savitch's Theorem)
- NP$\subseteq$PSPACE (because in polynomial time one can "visit" only polynomial space, i.e., NP$\subseteq$NPSPACE)

Computational Complexity: PSPACE-completeness

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An example for a PSPACE-complete problem is the NDFA equivalence problem:

Instance: Two non-deterministic finite state automata A_1 and A_2.

Question: Are the languages accepted by A_1 and A_2 identical?

Computational complexity of MAPF/DU bounded plan existence

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Theorem

Deciding whether there exists an eager MAPF/DU i-strong or objectively strong plan with execution cost k or less is PSPACE-complete.

Proof sketch.

Since plans have polynomial depth, all execution traces can be generated non-deterministically and tested using only polynomial space, i.e., PSPACE-membership.

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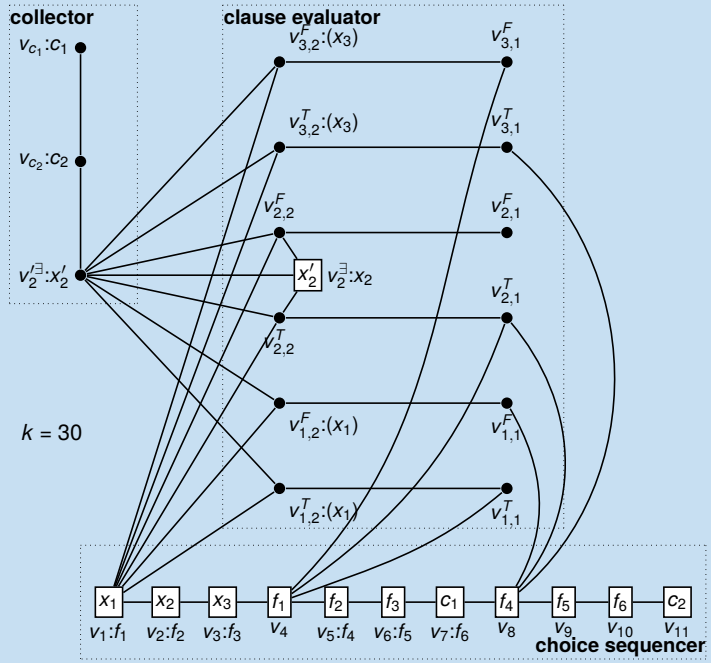
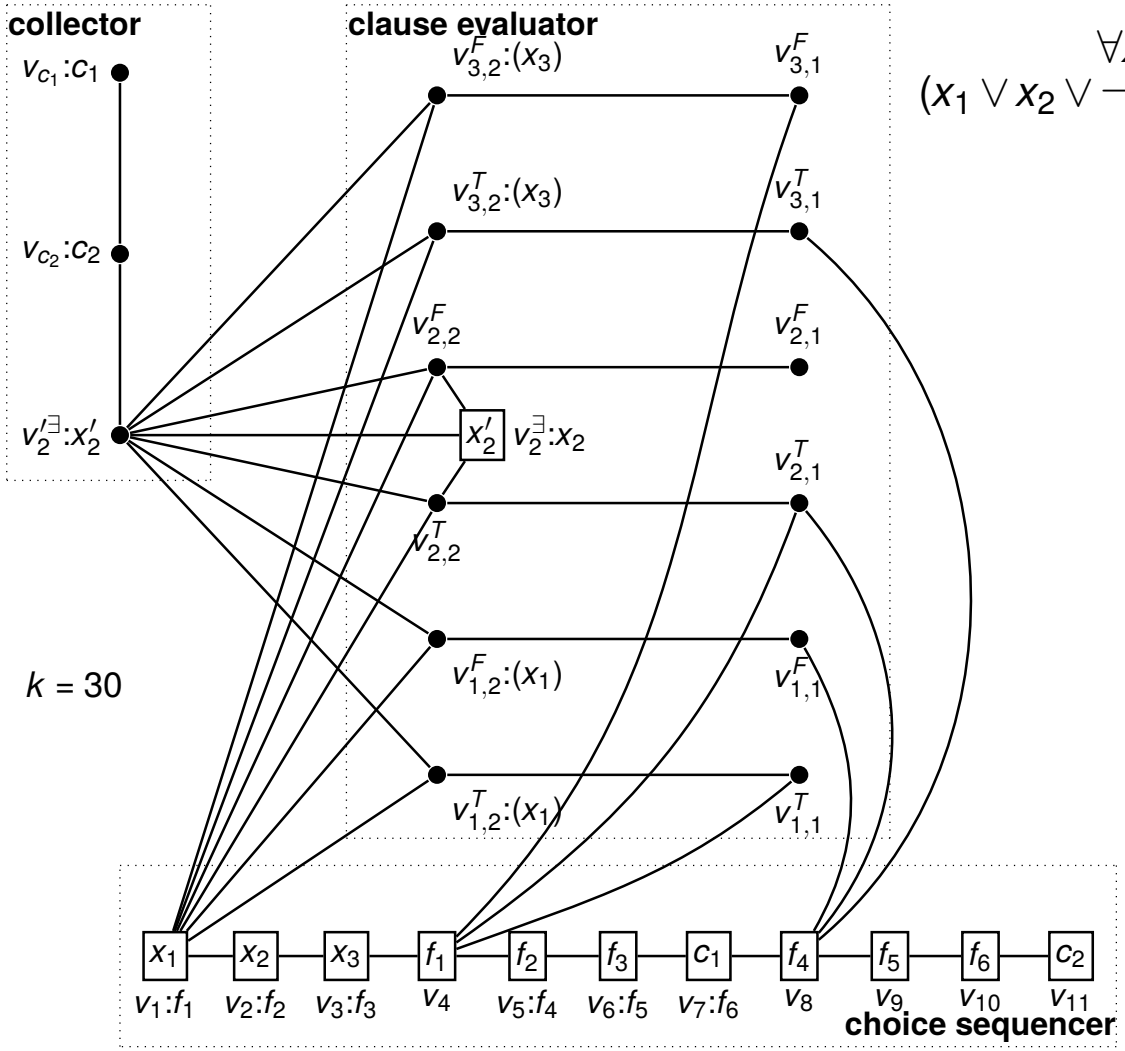


The reduction enlarged



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Complexity with a fixed number of agents

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These results probably imply that the technique could not be used online.

For a fixed number of agents, however, the bounded planning problem is polynomial.

Theorem

For a fixed number c of agents, deciding whether there exists a MAPF/DU i-strong or objectively strong plan with execution cost of k or less can be done in time $O(n^{c^2+c})$.

That means, for two agents, it takes "only" $O(n^6)$ time – but in practice it should be faster.

An algorithm for generating an objective MAPF/DU plan for two agents

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1. Determine in the *state space of all node assignments* the distance to the initial state using Dijkstra: $O(|V|^4)$ time.

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2. For each of the $O(|V|^2)$ configurations check, whether it is a *potential stepping stone* for one agent, i.e., whether all potential destinations of this agent are reachable using Dijkstra on the modified graph, where the other agent blocks the way: $O(|V|^4)$ time.

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3. For all $O(|V|^2)$ potential stepping stones, check whether for each of the $O(|V|)$ possible destination of the first agent, the second agent can reach its possible destinations and use Dijkstra to compute the shortest path: altogether $O(|V|^5)$ time.

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3. For all $O(|V|^2)$ potential stepping stones, check whether for each of the $O(|V|)$ possible destination of the first agent, the second agent can reach its possible destinations and use Dijkstra to compute the shortest path: altogether $O(|V|^5)$ time.
4. Consider all stepping stones and minimize over the maximum plan depth. Among the minimal plans select those that are eager for the planning agent.

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Summary & Outlook

Summary

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- DMAPF generalizes the MAPF problem by dropping the assumption that plans are generated centrally and then communicated.
- MAPF/DU generalizes the MAPF problem further by dropping the assumptions that destinations are common knowledge.
- A solution concept for this setting are *i*-strong branching plans corresponding to implicitly coordinated policies in the area of epistemic planning.
- The backbone of such plans are stepping stones.
- Joint execution can be guaranteed to be successful and polynomially bounded if all agents are conservative and optimally eager.
- While plan existence in general is PSPACE-complete, it is polynomial for a fixed number of agents.

Outlook

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- → Do the results still hold for planar graphs?
  - Is MAPF/DU plan existence also PSPACE-complete?
  - How would more general forms of describing the common knowledge about destinations affect the results?
- → Overlap of destinations or general Boolean combinations
  - Can we get similar results for other execution semantics?
- → Concurrent executions of actions
  - Can we be more aggressive in expectations about possible destinations?
- → Use forward induction, i.e., assume that actions in the past were rational.
  - Are other forms of implicit coordination possible?
- → More communication? Coordination in competitive scenarios?

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Literature

Literature (1)


D. Kornhauser, G. L. Miller, and P. G. Spirakis.
Coordinating pebble motion on graphs, the diameter of permutation groups, and applications.
In *25th Annual Symposium on Foundations of Computer Science (FOCS-84)*, pages 241–250, 1984.


O. Goldreich.
Finding the shortest move-sequence in the graph-generalized 15-puzzle is NP-hard.
In *Studies in Complexity and Cryptography. Miscellanea on the Interplay between Randomness and Computation*, pages 1–5. 2011.

Literature (2)

H. Ma, S. Koenig, N. Ayanian, L. Cohen, W. HÃűnig, T. K. Satish Kumar, T. Uras, H. Xu, C. A. Tovey, G. Sharon:
Overview: Generalizations of Multi-Agent Path Finding to Real-World Scenarios.
CoRR abs/1702.05515, 2017.

A. Felner, R. Stern, S. E. Shimony, E. Boyarski, M. Goldenberg, G. Sharon, N. R. Sturtevant, G. Wagner, and P. Surynek.
Search-Based Optimal Solvers for the Multi-Agent Pathfinding Problem: Summary and Challenges.
In *Proceedings of the Tenth International Symposium on Combinatorial Search (SOCS-17)*, pages 29–37, 2017.

P Surynek.
A novel approach to path planning for multiple robots in bi-connected graphs.
In *Proc. 2009 IEEE International Conference on Robotics and Automation, ICRA 2009*, pages 3613–3619, 2009.

Literature (3)

B. Nebel, T. Bolander, T. Engesser and R. Mattmüller.
Implicitly Coordinated Multi-Agent Path Finding under Destination Uncertainty.
Accepted to be published in *Journal of Artificial Intelligence research*.

T. Bolander, T. Engesser, R. Mattmüller and B. Nebel.
Better Eager Than Lazy? How Agent Types Impact the Successfulness of Implicit Coordination.
In *Proceedings of the Sixteenth Conference on Principles of Knowledge Representation and Reasoning (KR-18)*, pages 445-453. 2018.

T. Engesser, T. Bolander, R. Mattmüller, and B. Nebel.
Cooperative epistemic multi-agent planning for implicit coordination.
In *Proceedings of the Ninth Workshop on Methods for Modalities (M4MICLA-17)*, pages 75–90, 2017.