

## Multi-Agent Systems

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### Exercise Sheet 4

**Due: November 14, 2018, 16:00**

#### **Exercise 4.1** (Public Announcement Logic, 3+3+3+3)

We want to model and solve a simple logics puzzle using Public Announcement Logic (PAL).<sup>1</sup> Consider the following situation with two agents. Both have a number written on their forehead, which can only be seen by the other agent. It is also common knowledge between the agents that both numbers have to be consecutive integers between 0 and 9. Furthermore, let us assume that Agent 1 sees the number 4 on Agent 2's forehead. Now consider the following sequence of truthful announcements:

Agent 1: *"I don't know my number!"*

Agent 2: *"I don't know my number!"*

Agent 1: *"I don't know my number!"*

Agent 2: *"I don't know my number!"*

Agent 1: *"I know my number!"*

- (a) Model the initial situation (prior to the announcements) as epistemic model  $M$ . Identify the worlds that Agent 1 considers possible, given the problem description (but not the dialogue).
- (b) Model the announcements from the dialogue as epistemic formulas  $\alpha_1, \dots, \alpha_5$ .
- (c) Compute all the updated models  $M_{\alpha_1}, M_{\alpha_1\alpha_2}, \dots, M_{\alpha_1\dots\alpha_5}$ .
- (d) What is Agent 1's number? Will Agent 2 know his number in the end? Justify your answers!

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<sup>1</sup>Different versions of this puzzle and their origins are discussed in the book *One Hundred Prisoners and a Light Bulb* by Hans van Ditmarsch and Barteld Kooi, Springer, 2015.