Multi-Agent Systems

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Exercise Sheet 3

Due: November 07, 2018, 16:00

Exercise 3.1 (Tableaux, 2+2+2+2+2 (2 bonus))

Your task in this exercise is to use the tableau method from the lecture to construct some models and show some validities. For parts (b) and (e), also briefly discuss the intuitive appropriateness of the respective results taking into account that $K\phi$ reads "The agent knows that ϕ" and $O\phi$ reads "The agent ought to bring about that ϕ".

(a) Prove that $\phi_1 = K(p \vee q) \wedge K\neg p$ is S5-satisfiable.
Derive a pointed S5 model (M, w) from your tableaux that satisfies ϕ_1.

(b) Prove that $\phi_2 = K(p \wedge q) \rightarrow Kp$ is S5-valid.

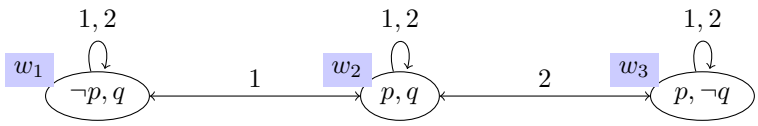
(c) Prove that $\phi_3 = (K_a p \vee K_a \neg p) \wedge \neg K_b(K_a p \vee K_a \neg p)$ is S5$_n$-satisfiable.
Derive a pointed S5 model (M, w) from your tableaux that satisfies ϕ_3.

(d) Prove the validity of $\phi_4 = O(Op \rightarrow p) \rightarrow (OOp \rightarrow Op)$ in KD.

(e) Prove the validity of $\phi_5 = OKp \rightarrow Op$ where O is a KD modality and K is a S5 modality.

Exercise 3.2 (Common and Distributed Knowledge, 4)

Consider the following $S5$ model M:



Which of the following are true statements?

ϕ	$M, w_2 \models K_1\phi$	$M, w_2 \models K_2\phi$	$M, w_2 \models C\phi$	$M, w_2 \models D\phi$	$M, w_1 \models C\phi$	$M, w_1 \models D\phi$
p	☐	☐	☐	☐	☐	☐
q	☐	☐	☐	☐	☐	☐
$p \wedge q$	☐	☐	☐	☐	☐	☐
$p \vee q$	☐	☐	☐	☐	☐	☐