

Principles of AI Planning

18. Planning as search: Partial-Order Reduction

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- **Worst case:** Heuristic search may explore **exponentially** more states than necessary, even if heuristic is **almost perfect** (Helmert and Röger, 2008).
- **Example:** A* search in GRIPPER domain explores all permutations of ball transportations if heuristic is off only by a small constant.
- **Idea:** Complement heuristic search with **orthogonal technique(s)** to reduce size of explored state space.
- **Desired properties of this technique:** preservation of **completeness** and, if possible, **optimality**.

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Partial-Order Reduction



Idea:

- Enforce particular ordering among operators.
- Ignore all other orderings.

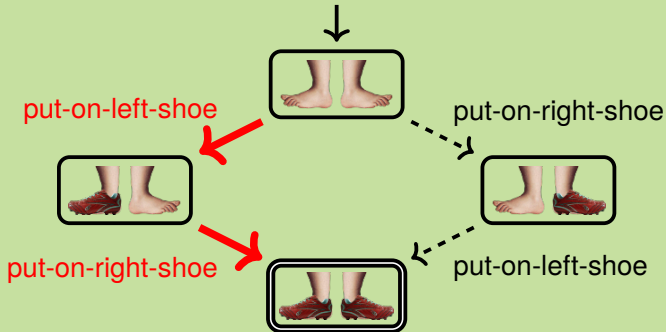
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Assumption: For the rest of the chapter, we assume that all planning tasks are SAS⁺ planning tasks $\Pi = (V, I, O, \gamma)$.

For convenience, we assume that operators have the form $o = \langle pre(o), eff(o) \rangle$, where $pre(o)$ and $eff(o)$ are both **partial states** over V , i.e., partial functions mapping variables v to values in \mathcal{D}_v . Similarly, we assume that γ is a partial state describing the goal.

Example

Operator $o = \langle pre(o), eff(o) \rangle$ with

- $pre(o) = \{v_1 \mapsto d_1, v_5 \mapsto d_5\}$ and
- $eff(o) = \{v_2 \mapsto d_2, v_3 \mapsto d_3\}$

corresponds to $o = \langle \chi, e \rangle$ with

$\chi = (v_1 = d_1 \wedge v_5 = d_5)$ and $e = (v_2 := d_2 \wedge v_3 := d_3)$.

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Definition (Operators)

Let $\Pi = (V, I, O, \gamma)$ be a SAS⁺ planning task and $o \in O$ an operator. Then

- $\text{prevars}(o) := \text{vars}(\text{pre}(o))$ are the variables that occur in the precondition of o .
- $\text{effvars}(o) := \text{vars}(\text{eff}(o))$ are the variables that occur in the effect of o .
- o **reads** $v \in V$ iff $v \in \text{prevars}(o)$.
- o **modifies** $v \in V$ iff $v \in \text{effvars}(o)$.

Variable $v \in V$ is **goal-related** iff $v \in \text{vars}(\gamma)$.

Assumption: $\text{effvars}(o) \neq \emptyset$ for all $o \in O$.

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Definition (Operator dependencies)

Let $\Pi = \langle V, O, I, \gamma \rangle$ be a planning task and $o, o' \in O$.

- 1 o **disables** o' iff there exists $v \in \text{effvars}(o) \cap \text{prevars}(o')$ such that $\text{eff}(o)(v) \neq \text{pre}(o')(v)$.
- 2 o **enables** o' iff there exists $v \in \text{effvars}(o) \cap \text{prevars}(o')$ such that $\text{eff}(o)(v) = \text{pre}(o')(v)$.
- 3 o and o' **conflict** iff there is $v \in \text{effvars}(o) \cap \text{effvars}(o')$ such that $\text{eff}(o)(v) \neq \text{eff}(o')(v)$.
- 4 o and o' **interfere** iff o disables o' , or o' disables o , or o and o' conflict.
- 5 o and o' are **commutative** iff o and o' do not interfere, and neither o enables o' , nor o' enables o .

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Example

$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = f, \text{left} := t \rangle$
 $\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = f, \text{right} := t \rangle$
 $\text{go-to-uni} = \langle \text{left} = t \wedge \text{right} = t, \text{pos} := \text{uni} \rangle$
 $\text{go-to-gym} = \langle \text{left} = t \wedge \text{right} = t, \text{pos} := \text{gym} \rangle$

Then:

- go-to-uni and go-to-gym disable put-on-left and put-on-right .
- put-on-left and put-on-right enable go-to-uni and go-to-gym .
- go-to-uni and go-to-gym conflict.
- put-on-left and put-on-right are commutative.

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Definition (Necessary enabling set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task, s a state, and $o \in O$ an operator that is not applicable in s . A set N of operators is a **necessary enabling set** (NES) for o in s if all operator sequences that lead from s to a goal state and include o contain an operator in N before the first occurrence of o .

Note: NESs not uniquely determined for given o and s .
(E.g., supersets of NESs are still NESs.)

Definition (Disjunctive action landmark)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A **disjunctive action landmark** (DAL) L in s is a set of operators such that all operator sequences that lead from s to a goal state contain some operator in L .

Observation

For state s and operator o that is not applicable in s , disjunctive action landmarks for task $\langle V, I, O, pre(o) \rangle$ are necessary enabling sets for o in s .

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Proof

Let L be such a disjunctive action landmark.

Then each operator sequence that leads from s to a state satisfying $pre(o)$ contains some operator in L .

Thus, each operator sequence that leads from s to a goal state and includes o contains an operator in L before the first occurrence of o .

Therefore, L is an NES for o in s .

Necessary Enabling Sets and Disjunctive Action Landmarks



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Necessary Enabling Sets and Disjunctive Action Landmarks



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Back to the motivation:

If, in state s , some set of operators can be **applied in any order** and the order does not matter, we want to **commit to one such order** and **ignore all other orders**.

Idea:

Identify operators that can be postponed since they are independent of all operators that are not postponed.

E.g., put-on-right could be postponed, since it is independent of put-on-left (that is not postponed).

Idea (more precisely): Identify operators that **should not** be postponed, and postpone the rest.

Question: When should an operator o **not be postponed**?

Answer:

- 1 Base case:** If o may be immediately relevant to reaching (part of) the goal, or
- 2 Inductive case I:** If o may be immediately relevant to contributing to making another operator applicable that should not be postponed, or
- 3 Inductive case II:** If o might not be applicable any more if we postponed it, or if its effect might conflict with the effect of another operator that should not be postponed ($\approx o$ interferes with such an operator).

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Let's formalize the above answer:

Definition (Strong stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a **strong stubborn set in s** if

- 1 T_s contains a disjunctive action landmark in s , and
- 2 for all $o \in T_s$ that are not applicable in s , T_s contains a necessary enabling set for o and s , and
- 3 for all $o \in T_s$ that are applicable in s , T_s contains all operators that interfere with o .

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s .

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Example

$I = \{\text{pos} \mapsto \text{home}, \text{left} \mapsto \text{f}, \text{right} \mapsto \text{f}\}, \quad \gamma = \{\text{pos} \mapsto \text{uni}\}$

$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$

$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$

$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$

- Step 1: DAL in I is $\{\text{go-to-uni}\} \rightsquigarrow T_s := \{\text{go-to-uni}\}$.
- Step 2: go-to-uni not applicable in I . One possible NES for go-to-uni in I is $\{\text{put-on-left}\} \rightsquigarrow T_s := T_s \cup \{\text{put-on-left}\}$.
- Step 3: put-on-left is applicable in I . The only operator that interferes with it, go-to-uni , is already in T_s .
- Hence, $T_s = \{\text{go-to-uni}, \text{put-on-left}\}$, and T_s restricted to the applicable operators is $\{\text{put-on-left}\}$. During search, only apply put-on-left (not put-on-right).

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Example

Let $V = \{u_1, u_2, v, w\}$, $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$,
 $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$, and $O = \{o_1, o_2, o_3\}$, where:

- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$,
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$,
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$.

Strong stubborn set:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Include o_3 in T_s since it interferes with o_1 (or o_2).
- Step 3: Include o_2 (or o_1) in T_s since it interferes with o_3 .

\rightsquigarrow all applicable operators included in T_s , no pruning.

Question: Can we do better than that in this example?

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Definition (Domain transition graph)

Let $\Pi = (V, I, O, \gamma)$ be a SAS⁺ planning task and $v \in V$. The **domain transition graph** for v is the directed graph

$DTG(v) = \langle \mathcal{D}_v, E \rangle$ where $(d, d') \in E$ iff there is an operator $o \in O$ with

- $eff(o)(v) = d'$, and
- $v \notin prevars(o)$ or $pre(o)(v) = d$.

Example

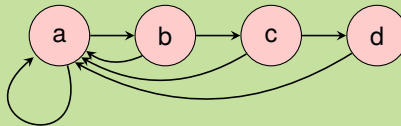
$\text{move-a-b} = \langle \text{pos} = \text{a}, \text{pos} := \text{b} \rangle$

$\text{move-b-c} = \langle \text{pos} = \text{b}, \text{pos} := \text{c} \rangle$

$\text{move-c-d} = \langle \text{pos} = \text{c}, \text{pos} := \text{d} \rangle$

$\text{reset} = \langle \top, \text{pos} := \text{a} \wedge \text{othersvar} := \text{othersvar} \rangle$

Then $DTG(\text{pos})$:



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Definition (Active operators)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and let s be a state. The set of **active operators** $Act(s) \subseteq O$ in s is defined as the set of operators such that for all $o \in Act(s)$:

- For every variable $v \in prevars(o)$, there is a path in $DTG(v)$ from $s(v)$ to $pre(o)(v)$. If v is goal-related, then there is also a path from $pre(o)(v)$ to the goal value $\gamma(v)$.
- For every goal-related variable $v \in effvars(o)$, there is a path in $DTG(v)$ from $eff(o)(v)$ to the goal value $\gamma(v)$.

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Proposition

- 1 $Act(s)$ can be identified efficiently for a given state s by considering paths in the projection of Π onto v .
- 2 Operators not in $Act(s)$ can be treated as nonexistent when reasoning about s because they are not applicable in all states reachable from s , or they lead to a dead-end from s .

Proof

- 1 Homework: Specify efficient algorithm for identification of $Act(s)$.
- 2 Obvious. □

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Remark 1: Even when excluding **inactive** operators, this preserves completeness and even optimality of a search algorithm (see proof below).

Remark 2: Excluding **inactive** operators can “cascade” in the sense that additional **active** operators need not be considered.

Definition (Strong stubborn set with active operator pruning)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a **strong stubborn set in s** if

- 1 T_s contains a disjunctive action landmark in s , and
- 2 for all $o \in T_s$ that are not applicable in s , T_s contains a necessary enabling set for o and s , and
- 3 for all $o \in T_s$ that are applicable in s , T_s contains all operators that **are active in s and** interfere with o .

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s .

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Strong Stubborn Sets

Why operator activity matters



Recall the previous example where strong stubborn sets without active operator pruning were useless.

Example

- $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\},$
 $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$
- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$

Now, **with** active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state.
 $\rightsquigarrow o_3$ not in T_s , although it interferes with o_1 (or o_2).

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Example (Example, ctd.)

Now, **with** active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state.
 $\rightsquigarrow o_3$ not in T_s , although it interferes with o_1 (or o_2).
- Hence, e. g., $T_s = \{o_1\}$ strong stubborn set (with active operator pruning) in I .
- Even **active** operator o_2 is not included in $T_s = \{o_1\}$.

\rightsquigarrow some pruning occurs.

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With **weak** stubborn sets, some operators that disable an operator in T_s need not be included in T_s .

Therefore, weak stubborn sets potentially allow more pruning than strong stubborn sets.

Definition (Weak stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a **weak stubborn set in s** if

- 1 T_s contains a disjunctive action landmark in s , and
- 2 for all $o \in T_s$ that are not applicable in s , T_s contains a necessary enabling set for o and s , and
- 3 for all $o \in T_s$ that are applicable in s , T_s contains the active operators in s that have conflicting effects with o or that are disabled by o .

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For weak stubborn sets, it suffices to include active operators o' that **are disabled** or **conflict** with applicable operators $o \in T_s$. However, o' **does not need to be included** if o' **disables** an applicable operator $o \in T_s$.

No computational overhead of computing weak stubborn sets over computing strong stubborn sets.

Theorem

In the best case, weak stubborn sets admit **exponentially more pruning** than strong stubborn sets.

Proof

Homework. □

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compute-DAL: Compute a disjunctive action landmark.

Procedure compute-DAL

```
def compute-DAL( $\gamma$ ):  
    select  $v \in vars(\gamma)$  with  $s(v) \neq \gamma(v)$   
     $L \leftarrow \{o' \in Act(s) \mid eff(o')(v) = \gamma(v)\}$   
    return  $L$ 
```

Selection of $v \in vars(\gamma)$ arbitrary. Any variable will do.
Selection heuristics?

compute-NES: Compute a necessary enabling set.

Procedure compute-NES

```
def compute-NES( $o, s$ ):  
    select  $v \in \text{prevars}(o)$  with  $s(v) \neq \text{pre}(o)(v)$   
     $N \leftarrow \{o' \in \text{Act}(s) \mid \text{eff}(o')(v) = \text{pre}(o)(v)\}$   
    return  $N$ 
```

Selection of $v \in \text{prevars}(o)$ arbitrary. Any variable will do.
Selection heuristics?

compute-interfering-operators: Compute interfering operators.

Procedure compute-interfering-operators (for strong SS)

```
def compute-interfering-operators( $o$ ):  
     $\text{disablers} \leftarrow \{o' \in O \mid o' \text{ disables } o\}$   
     $\text{disablees} \leftarrow \{o' \in O \mid o \text{ disables } o'\}$   
     $\text{conflicting} \leftarrow \{o' \in O \mid o \text{ and } o' \text{ conflict}\}$   
    return  $\text{disablers} \cup \text{disablees} \cup \text{conflicting}$ 
```

Procedure compute-interfering-operators (for weak SS)

```
def compute-interfering-operators( $o$ ):  
     $\text{disablees} \leftarrow \{o' \in O \mid o \text{ disables } o'\}$   
     $\text{conflicting} \leftarrow \{o' \in O \mid o \text{ and } o' \text{ conflict}\}$   
    return  $\text{disablees} \cup \text{conflicting}$ 
```

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Computing (strong and weak) stubborn sets for planning can be achieved with a **fixpoint iteration** until the constraints of T_s are satisfied:

compute-stubborn-set: Compute (strong or weak) stubborn set.

Precedure compute-stubborn-set

```
def compute-stubborn-set(s):  
     $T_s \leftarrow \text{compute-DAL}(\gamma)$   
    while no fixed-point of  $T_s$  reached do  
        for  $o \in T_s$  applicable in s:  
             $T_s \leftarrow T_s \cup \text{compute-interfering-operators}(o)$   
        for  $o \in T_s$  not applicable in s:  
             $T_s \leftarrow T_s \cup \text{compute-NES}(o, s)$   
    end while  
    return  $T_s$ 
```

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Observation: stubborn sets are state-dependent, but not path-dependent.

This allows filtering the applicable operators in s in graph search algorithms like A* that perform duplicate detection, too.

Instead of applying all applicable operators $app(s)$ in s , only apply operators in $T_{app(s)} := T_s \cap app(s)$.

Theorem

Weak stubborn sets are completeness and optimality preserving.

Proof

Let $T_{app(s)} := T_s \cap app(s)$ for a weak stubborn set T_s .

We show that for all states s from which an optimal plan consisting of $n > 0$ operators exists, $T_{app(s)}$ contains an operator that starts such a plan.

We show by induction that A^* restricting successor generation to $T_{app(s)}$ is optimal.

Let T_s be a weak stubborn set and $\pi = o_1, \dots, o_n$ be an optimal plan that starts in s .

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Let $T_{app(s)} := T_s \cap app(s)$ for a weak stubborn set T_s .

We show that for all states s from which an optimal plan consisting of $n > 0$ operators exists, $T_{app(s)}$ contains an operator that starts such a plan.

We show by induction that A^* restricting successor generation to $T_{app(s)}$ is optimal.

Let T_s be a weak stubborn set and $\pi = o_1, \dots, o_n$ be an optimal plan that starts in s .

...

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Proof (ctd.)

As T_s contains a disjunctive action landmark, π must contain an operator from T_s .

Let o_k be the operator with smallest index in π that is also contained in T_s , i.e., $o_k \in T_s$ and $\{o_1, \dots, o_{k-1}\} \cap T_s = \emptyset$.

We observe:

1. $o_k \in \text{app}(s)$: otherwise by definition of weak stubborn sets, a necessary enabling set N for o_k in s would have to be contained in T_s , and at least one operator from N would have to occur before o_k in π to enable o_k , contradicting that o_k was chosen with smallest index.
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1. ...
2. o_k does not disable any of the operators o_1, \dots, o_{k-1} , and all these operators have non-conflicting effects with o_k : otherwise, as $o_k \in \text{app}(s)$, and by definition of weak stubborn sets, at least one of o_1, \dots, o_{k-1} would have to be contained in T_s , again contradicting the assumption.

Hence, we can move o_k to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$ is also a plan for Π .

It has the same cost as π and is hence optimal.

Thus, we have found an optimal plan of length n started by an operator $o_k \in T_{\text{app}(s)}$, completing the proof. \square

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Remark: The argument to move o_k to the front also holds for strong stubborn sets: in this case, o_k is not even disabled by any of o_1, \dots, o_{k-1} (and hence, o_k is independent of o_1, \dots, o_{k-1}), which is a stronger property than needed in the proof.

Corollary

Strong stubborn sets are completeness and optimality preserving.



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Some Experiments: Overview

Optimal Planning, A* with LM-cut Heuristic, Selected Domains



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Domain (problems)	Coverage		Nodes generated	
	A*	+SSS	A*	+SSS
PARCPRINTER-08 (30)	18	+12	2455181	<1%
PARCPRINTER-OPT11 (20)	13	+7	2454533	<1%
WOODWORKING-OPT08 (30)	17	+10	26796212	<1%
WOODWORKING-OPT11 (20)	12	+7	26795517	<1%
SATELLITE (36)	7	+5	5116312	2%
ROVERS (40)	7	+2	1900691	22%
AIRPORT (50)	28	±0	545072	93%
OPENSTACKS-OPT08 (30)	19	+2	56584063	51%
OPENSTACKS-OPT11 (20)	14	+2	56456969	51%
DRIVERLOG (20)	13	+1	3679376	82%
SCANALYZER-08 (30)	15	-3	14203012	100%
SCANALYZER-OPT11 (20)	12	-3	14202884	100%
PARKING-OPT11 (20)	3	-1	560914	100%
SOKOBAN-OPT08 (30)	30	-1	20519270	100%
VISITALL-OPT11 (20)	11	-1	1991169	100%
REMAINING DOMAINS (980)	544	±0	436017004	93%
SUM (1396)	763	+39	670278179	77%

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Weak compared to strong stubborn sets



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Domain (problems)	Coverage		Nodes generated		# problems w. diff. gen.
	SSS	WSS	SSS	WSS	
OPENSTACKS-OPT08 (30)	21	± 0	152711917	99.936%	18
OPENSTACKS-OPT11 (20)	16	± 0	152642101	99.936%	16
PATHWAYS-NONEG (30)	5	± 0	162347	99.702%	2
PSR-SMALL (50)	49	± 0	18119489	99.998%	6
SATELLITE (36)	12	± 0	70299721	92.804%	12

⇒ In practice (or, at least, in the standard benchmark problems) there is no significant difference between weak and strong stubborn sets.



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- Need for **techniques orthogonal to heuristic search**, complementing heuristics.
- One idea: **Commit to one order of operators** if they are independent. Prune other orders.
- Class of such techniques: **partial-order reduction** (POR)
- One such technique: **strong/weak stubborn sets**
- Can lead to **substantial pruning** compared to plain A^* .
- Many other POR techniques exist.
- Other pruning techniques exist as well, e.g., symmetry reduction.

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