Principles of AI Planning

16. Complexity of nondeterministic planning with full observability

Albert-Ludwigs-Universität Freiburg







Review

Complexity results

Summary

Motivation



Review

Complexity results

Summary

- Similar to the earlier analysis of deterministic planning, we will now study the computational complexity of nondeterministic planning with full observability.
- We consider the case of strong planning.
- The results for strong cyclic planning are identical.

As usual, the main motivation for such a study is to determine the limit of what is possible algorithmically: Should we try to develop a polynomial algorithm?



Review

Complexity results

- The basic proof idea is very similar to the PSPACE-completeness proof for deterministic planning.
- The main difference is that we consider alternating Turing Machines (ATMs) instead of deterministic Turing Machines (DTMs) in the reduction.
- Due to the similarity to the earlier proof, we first review some of the concepts introduced in the earlier lecture.



Review

ATMs Complexity classes

Complexity results

Summary

Review



Definition: Alternating Turing Machine

- Alternating Turing Machine (ATM) $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$:
 - 1 input alphabet Σ and blank symbol $\Box \notin \Sigma$
 - alphabets always non-empty and finite
 - tape alphabet $\Sigma_{\Box} = \Sigma \cup \{\Box\}$
 - 2 finite set Q of internal states with initial state $q_0 \in Q$
 - 3 state labeling $I: Q \to \{Y, N, \exists, \forall\}$
 - accepting, rejecting, existential, universal states $Q_{Y}, Q_{N}, Q_{\exists}, Q_{\forall}$
 - terminal states $Q_{\star} = Q_{Y} \cup Q_{N}$
 - nonterminal states $Q' = Q_\exists \cup Q_\forall$
 - 4 transition relation $\delta \subseteq (Q' \times \Sigma_{\Box}) \times (Q \times \Sigma_{\Box} \times \{-1, +1\})$

Motivation

Review

ATMs Complexity classes

Complexity results

Let $M = \langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ be an ATM.

Definition: Configuration

A configuration of *M* is a triple $(w,q,x) \in \Sigma_{\Box}^* \times Q \times \Sigma_{\Box}^+$.

- w: tape contents before tape head
- q: current state
- x: tape contents after and including tape head

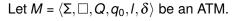


Review

ATMs Complexity classes

Complexity results





Definition: Yields relation

A configuration *c* of *M* yields a configuration *c'* of *M*, in symbols $c \vdash c'$, as defined by the following rules, where $a, a', b \in \Sigma_{\Box}, w, x \in \Sigma_{\Box}^*, q, q' \in Q$ and $((q, a), (q', a', \Delta)) \in \delta$:

$(w,q,ax) \vdash (wa',q',x)$	if Δ = +1, $ x \ge 1$
$(w,q,a) \vdash (wa',q',\Box)$	if $\Delta = +1$
$(wb,q,ax) \vdash (w,q',ba'x)$	if $\Delta = -1$
$(\varepsilon,q,ax) \vdash (\varepsilon,q',\Box a'x)$	if $\Delta = -1$



Motivation

Review

ATMs Complexity classes

Complexity results

Let $M = \langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ be an ATM.

Definition: Acceptance (space)

Let c = (w, q, x) be a configuration of M.

- *M* accepts c = (w, q, x) with $q \in Q_Y$ in space *n* iff $|w| + |x| \le n$.
- *M* accepts c = (w, q, x) with $q \in Q_{\exists}$ in space *n* iff *M* accepts some c' with $c \vdash c'$ in space *n*.
- *M* accepts c = (w, q, x) with $q \in Q_{\forall}$ in space *n* iff *M* accepts all c' with $c \vdash c'$ in space *n*.



Motivation

Review

ATMs Complexity classes

Complexity results

Let $M = \langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ be an ATM.

Definition: Accepting words

M accepts the word $w \in \Sigma^*$ in space $n \in \mathbb{N}_0$ iff *M* accepts (ε , q_0 , w) in space *n*.

Special case: *M* accepts *ε* in time (space) *n* ∈ ℕ₀ iff *M* accepts (*ε*, *q*₀, □) in time (space) *n*.

Definition: Accepting languages

Let $f : \mathbb{N}_0 \to \mathbb{N}_0$.

M accepts the language $L \subseteq \Sigma^*$ in space *f*

iff *M* accepts each word $w \in L$ in space f(|w|), and *M* does not accept any word $w \notin L$.



Motivation

Review

ATMs Complexity classes

Complexity results



Review

ATMs

Complexity classes

Complexity results

Summary

Definition: ASPACE, APSPACE

Let $f : \mathbb{N}_0 \to \mathbb{N}_0$.

Complexity class ASPACE(f) contains all languages accepted in space *f* by some ATM.

Let \mathscr{P} be the set of polynomials $p : \mathbb{N}_0 \to \mathbb{N}_0$.



Review

ATMs

Complexity classes

Complexity results

Summary

Theorem

 $\begin{array}{cccc} \mathsf{P} \subseteq & \mathsf{NP} & \subseteq \mathsf{AP} \\ \mathsf{PSPACE} \subseteq & \mathsf{NPSPACE} & \subseteq \mathsf{APSPACE} \\ \mathsf{EXP} \subseteq & \mathsf{NEXP} & \subseteq \mathsf{AEXP} \\ \mathsf{EXPSPACE} \subseteq \mathsf{NEXPSPACE} \subseteq \mathsf{AEXPSPACE} \\ \mathsf{2}\text{-}\mathsf{EXP} \subseteq & \dots \end{array}$



Review

ATMs

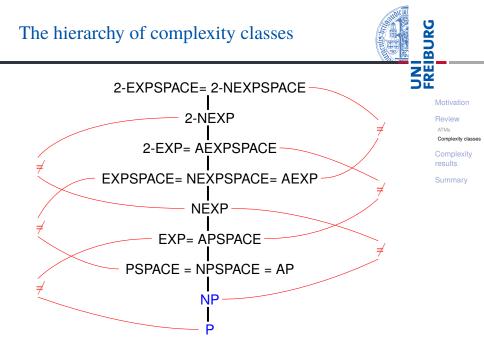
Complexity classes

Complexity results

Summary

Theorem (Chandra et al. 1981)

AP	=	PSPACE
APSPACE	=	EXP
AEXP	=	EXPSPACE
AEXPSPACE	=	2-EXP





Review

Complexity results

The problem The reduction The proof

Summary

Complexity results



Review

Complexity results

The problem The reduction The proof

Summary

STRONGPLANEX (strong plan existence)

- GIVEN: nondeterministic planning task $\langle A, I, O, G, V \rangle$ with full observability (A = V)
- QUESTION: Is there a strong plan for the task?
 - We do not consider a nondeterministic analog of the bounded plan existence problem (PLANLEN).



Review

Complexity results

The problem The reduction The proof

- We will prove that STRONGPLANEx is EXP-complete.
- We already know that the problem belongs to EXP, because we have presented a dynamic programming algorithm that generates strong plans in exponential time.
- We prove hardness for EXP by providing a generic reduction for alternating Turing Machines with polynomial space and use Chandra et al.'s theorem showing APSPACE = EXP.

- For a fixed polynomial p, given ATM M and input w, generate planning task which is solvable by a strong plan iff M accepts w in space p(|w|).
- For simplicity, restrict to ATMs which never move to the left of the initial head position (no loss of generality).
- Existential states of the ATM are modeled by states of the planning task where there are several applicable operators to choose from.
- Universal states of the ATM are modeled by states of the planning task where there is a single applicable operator with a nondeterministic effect.

Review

Complexity results

The reduction The proof

Summary

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

State variables

- state $_q$ for all $q \in Q$
- head_i for all $i \in X \cup \{0, p(n) + 1\}$
- content_{*i*,*a*} for all $i \in X$, $a \in \Sigma_{\Box}$



Motivation

Review

Complexity results

The problem

The reduction The proof

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Initial state formula

Specify a unique initial state.

Initially true:

- state_{q0}
- head₁
- content_{*i*,*w*_{*i*} for all $i \in \{1, ..., n\}$}
- content_{*i*, \Box} for all *i* \in *X* \ {1,...,*n*}

Initially false:

all others

January 16, 2019





Motivation

Review

Complexity results

The problem

The reduction The proof

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Goal

 $\bigvee_{q \in Q_{\mathsf{Y}}} \mathsf{state}_q$

- Without loss of generality, we can assume that Q_Y is a singleton set so that we do not need a disjunctive goal.
- This way, the hardness result also holds for a restricted class of planning tasks ("nondeterministic STRIPS").



Motivation

Review

Complexity results

The reduction The proof

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators

For $q,q' \in Q, \, a,a' \in \Sigma_{\Box}, \, \Delta \in \{-1,+1\}, \, i \in X,$ define



Motivation

Review

Complexity results

The reduction

The proof

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators (ctd.)

For existential states $q \in Q_{\exists}$, $a \in \Sigma_{\Box}$, $i \in X$: Let $(q'_j, a'_j, \Delta_j)_{j \in \{1, ..., k\}}$ be those triples with $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$.

For each $j \in \{1, \ldots, k\}$, introduce one operator:

precondition: pre_{q,a,i}

• effect: $eff_{q,a,q'_j,a'_j,\Delta_j,i}$



Motivation

Review

Complexity results

The problem

The reduction The proof

Given ATM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators (ctd.)

For universal states $q \in Q_{\forall}$, $a \in \Sigma_{\Box}$, $i \in X$: Let $(q'_j, a'_j, \Delta_j)_{j \in \{1, ..., k\}}$ be those triples with $((q, a), (q'_i, a'_j, \Delta_j)) \in \delta$.

Introduce only one operator:

precondition: pre_{q,a,i}

• effect: $eff_{q,a,q'_1,a'_1,\Delta_1,i}|\dots|eff_{q,a,q'_k,a'_k,\Delta_k,i}|$



Motivation

Review

Complexity results

The problem

The reduction The proof

EXP-completeness of strong planning with full observability

Theorem (Rintanen)

STRONGPLANEX is EXP-complete.

This is true even if we only allow operators in unary nondeterminism normal form where all deterministic sub-effects and the goal satisfy the STRIPS restriction and if we require a deterministic initial state.

Proof.

Membership in EXP has been shown by providing exponential-time algorithms that generate strong plans (and decide if one exists as a side effect).

Hardness follows from the previous generic reduction for ATMs with polynomial space bound and Chandra et al.'s theorem.



Motivation

Review

Complexit results The problem

The reducti



Review

Complexity results

Summary

Summary

problem.

Summary

B. Nebel, R. Mattmüller – Al Planning

27/27

The introduction of nondeterministic effects corresponds to the introduction of alternation in Turing Machines.

Later, we will see that restricted observability has an even more dramatic effect on the complexity of the planning

- goals satisfy some fairly strong syntactic restrictions and there is a unique initial state.
- deterministic planning. The hardness result already holds if the operators and
- planning. In particular, it is EXP-complete in the fully observable case, compared to the PSPACE-completeness of

Nondeterministic planning is harder than deterministic



results

Summarv