Principles of AI Planning

15. Strong nondeterministic planning

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Strong planning

In this chapter, we will consider the simplest case of nondeterministic planning by restricting attention to strong plans.

Concepts

Recall the definition of strong plans:

Definition (strong plan)

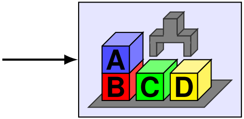
Let S be the set of states of a planning task Π. Then a strong plan for Π is a function $\pi : S_\pi \to O$ for some subset $S_\pi \subseteq S$ such that

- $\pi(s)$ is applicable in s for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi \cup S_\star$ (π is closed),
- $S_\pi(s') \cap S_\star \neq \emptyset$ for all $s' \in S_\pi(s_0)$ (π is proper), and
- there is no state $s' \in S_\pi(s_0)$ such that s' is reachable from s' following π in a strictly positive number of steps (π is acyclic).

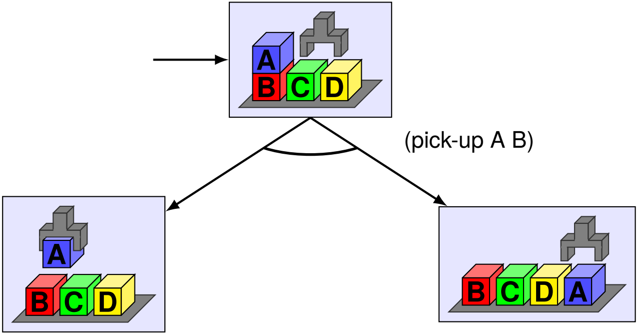
Execution of a strong plan

1. Determine the current state s.
2. If s is a goal state then terminate.
3. Execute action $\pi(s)$.
4. Repeat from first step.

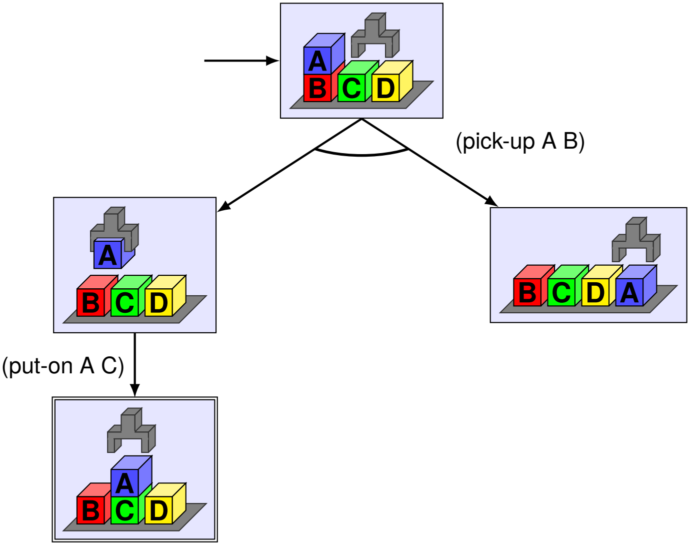
Strong plans



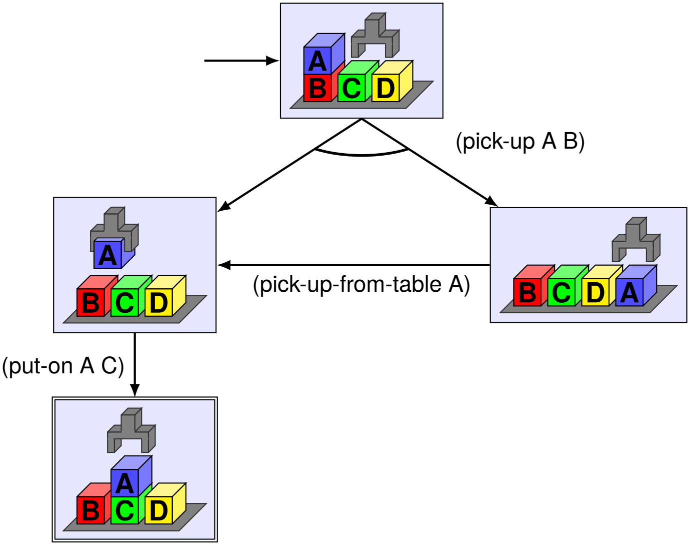
Strong plans



Strong plans



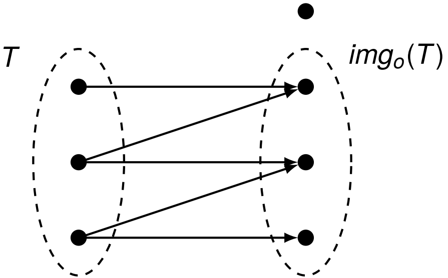
Strong plans



Images

Image

The image of a set T of states with respect to an operator o is the set of those states that can be reached by executing o in a state in T.



Definition (image of a state)

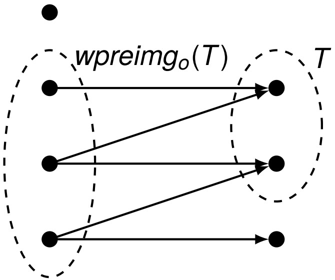
$img_o(s) = \{s' \in S | s \xrightarrow{o} s'\} = app_o(s)$

Definition (image of a set of states)

$img_o(T) = \bigcup_{s \in T} img_o(s)$

Weak preimage

The weak preimage of a set T of states with respect to an operator o is the set of those states from which a state in T can be reached by executing o.



Weak preimages

Definition (weak preimage of a state)

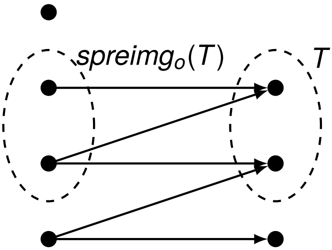
$wpreimg_o(s') = \{s \in S | s \xrightarrow{o} s'\}$

Definition (weak preimage of a set of states)

$wpreimg_o(T) = \bigcup_{s \in T} wpreimg_o(s)$.

Strong preimage

The strong preimage of a set T of states with respect to an operator o is the set of those states from which a state in T is always reached when executing o.



Strong preimages

Definition (strong preimage of a set of states)

$$spreimg_o(T) = \{s \in S \mid \exists s' \in T : s \xrightarrow{o} s' \wedge img_o(s) \subseteq T\}$$

Algorithms

Algorithms for strong planning

1. Dynamic programming (backward)
   Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.
   1. Zero actions needed for goal states.
   2. If states with i actions to goals are known, states with $\leq i+1$ actions to goals can be easily identified.

   Automatic reuse of plan suffixes already found.
2. Heuristic search (forward)
   Strong planning can be viewed as AND/OR graph search.

   OR nodes: Choice between operators
   AND nodes: Choice between effects

   Heuristic AND/OR search algorithms:
   AO*, Proof Number Search, …

Dynamic programming

Planning by dynamic programming

If for all successors of state s with respect to operator o a plan exists, assign operator o to s.

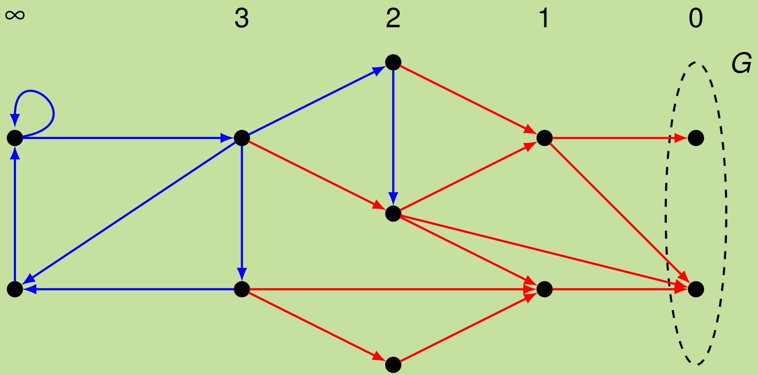
- Base case $i = 0$: In goal states there is nothing to do.
- Inductive case $i \geq 1$: If $\pi(s)$ is still undefined and there is $o \in O$ such that for all $s' \in img_o(s)$, the state s' is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

Backward distances

If s is assigned a value on iteration $i \geq 1$, then the backward distance of s is i. The dynamic programming algorithm essentially computes the backward distances of states.

Example

distance to G



Definition (backward distance sets)

Let G be a set of states and O a set of operators.
The backward distance sets D_i^{bwd} for G and O consist of those states for which there is a guarantee of reaching a state in G with at most i operator applications using operators in O:

$$D_0^{bwd} := G$$
$$D_i^{bwd} := D_{i-1}^{bwd} \cup \bigcup_{o \in O} spreimg_o(D_{i-1}^{bwd}) \text{ for all } i \geq 1$$

Backward distances

Definition (backward distance)

Let G be a set of states and O a set of operators, and let $D_0^{bwd}, D_1^{bwd}, \ldots$ be the backward distance sets for G and O. Then the **backward distance** of a state s for G and O is

$$\delta_G^{bwd}(s) = \min\{i \in \mathbb{N} \mid s \in D_i^{bwd}\}$$

(where $\min \emptyset = \infty$).

Strong plans based on distances

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states $S_\star$.

Extraction of a strong plan from distance sets

1. Let $S' \subseteq S$ be those states having a finite backward distance for $G = S_\star$ and O.
2. Let $s \in S'$ be a state with distance $i = \delta_G^{bwd}(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $img_o(s) \subseteq D_{i-1}^{bwd}$. Hence o decreases the backward distance by at least one.

Then π is a strong plan for $\mathscr{T}$ iff $I \in S'$.

Question: What is the worst-case runtime of the algorithm?

Summary

Summary

- We have considered the special case of nondeterministic planning where
  - planning tasks are fully observable and
  - we are interested in strong plans.
- We have introduced important concepts also relevant to other variants of nondeterministic planning such as
  - images and
  - weak and strong preimages.
- We have discussed one basic classes of algorithms: backward induction by dynamic programming.