Principles of AI Planning

7. Planning as search: relaxed planning tasks

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel and Robert Mattmüller

November 13th, 2018

How to obtain a heuristic

A simple heuristic for deterministic planning

STRIPS (Fikes & Nilsson, 1971) used the number of state variables that differ in current state s and a STRIPS goal $a_1 \wedge \cdots \wedge a_n$:

$$h(s) := |\{i \in \{1, \ldots, n\} \mid s \not\models a_i\}|.$$

Intuition: more true goal literals $\rightsquigarrow$ closer to the goal

$\rightsquigarrow$ STRIPS heuristic (a.k.a. goal-count heuristic) (properties?)

Note: From now on, for convenience we usually write heuristics as functions of states (as above), not nodes. Node heuristic h' is defined from state heuristic h as $h'(\sigma) := h(\mathit{state}(\sigma))$.

Criticism of the STRIPS heuristic

What is wrong with the STRIPS heuristic?

- quite uninformative:
  the range of heuristic values in a given task is small;
  typically, most successors have the same estimate
- very sensitive to reformulation:
  can easily transform any planning task into an equivalent one where $h(s) = 1$ for all non-goal states (how?)
- ignores almost all problem structure:
  heuristic value does not depend on the set of operators!

$\rightsquigarrow$ need a better, principled way of coming up with heuristics

Coming up with heuristics in a principled way

General procedure for obtaining a heuristic

Solve an easier version of the problem.

Two common methods:

- relaxation: consider less constrained version of the problem
- abstraction: consider smaller version of real problem

Both have been very successfully applied in planning.
We consider both in this course, beginning with relaxation.

Relaxing a problem

How do we relax a problem?

Example (Route planning for a road network)

The road network is formalized as a weighted graph over points in the Euclidean plane. The weight of an edge is the road distance between two locations.

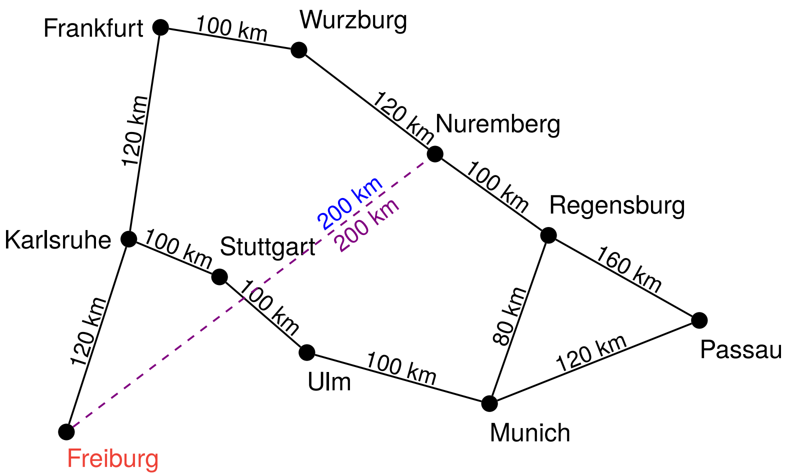
A relaxation drops constraints of the original problem.

Example (Relaxation for route planning)

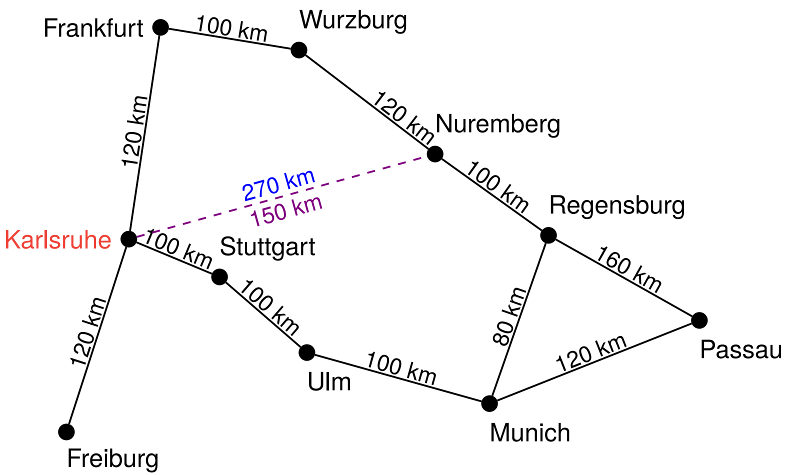
Use the Euclidean distance $\sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}$ as a heuristic for the road distance between $\langle x_1, y_1 \rangle$ and $\langle x_2, y_2 \rangle$
This is a lower bound on the road distance ($\rightsquigarrow$ admissible).

$\rightsquigarrow$ We drop the constraint of having to travel on roads.

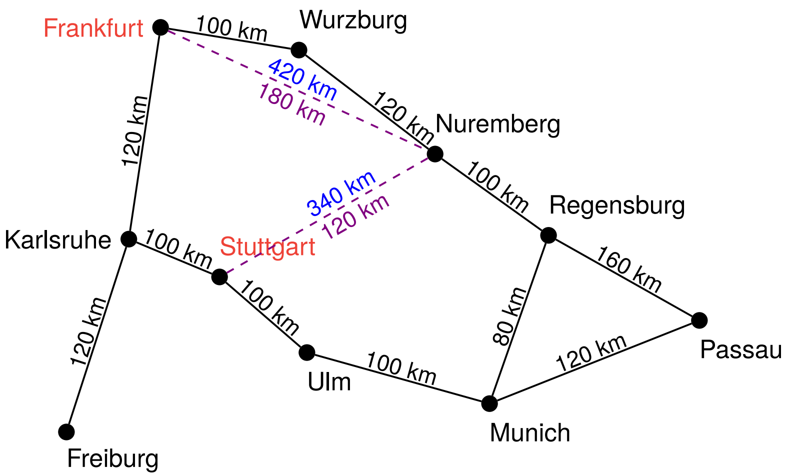
A* using the Euclidean distance heuristic



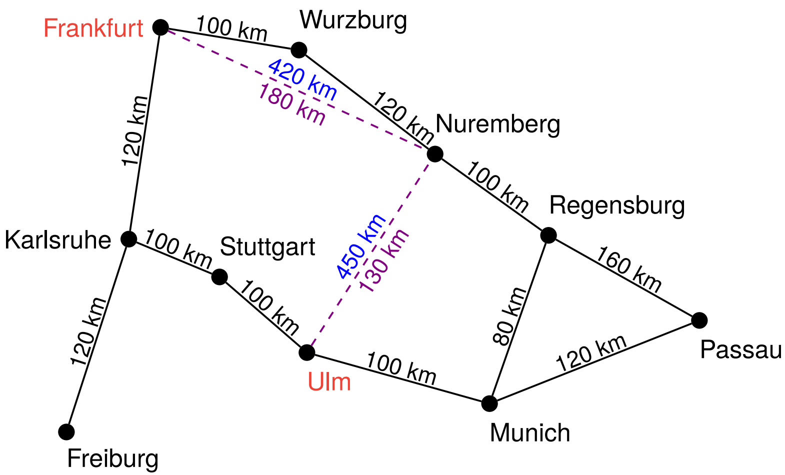
A* using the Euclidean distance heuristic



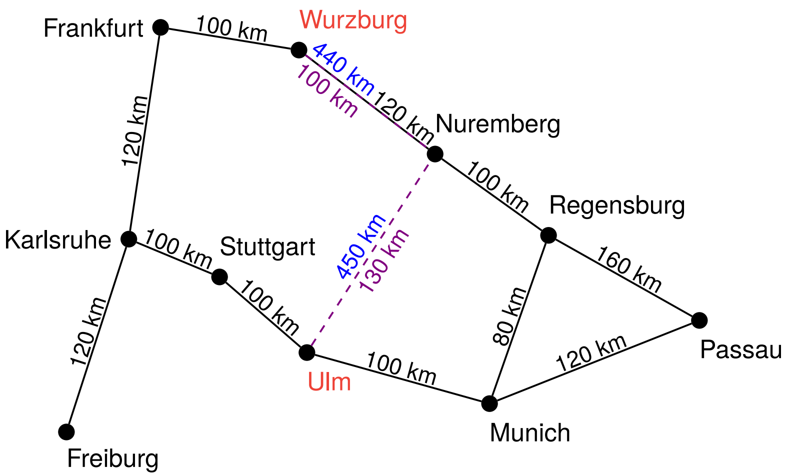
A* using the Euclidean distance heuristic



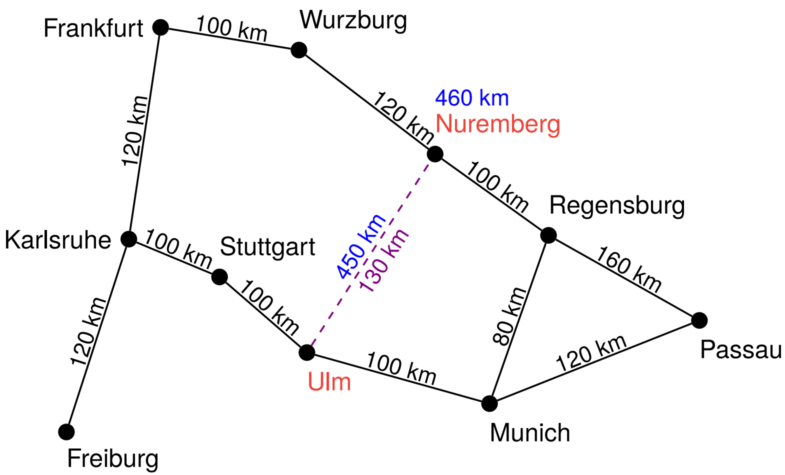
A* using the Euclidean distance heuristic



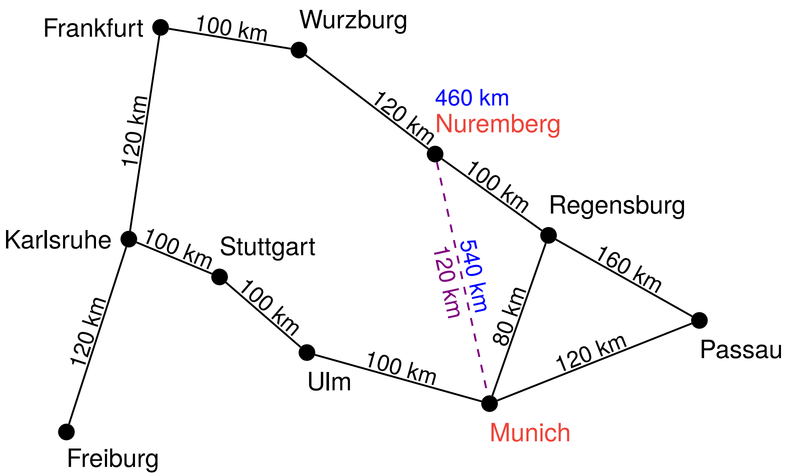
A* using the Euclidean distance heuristic



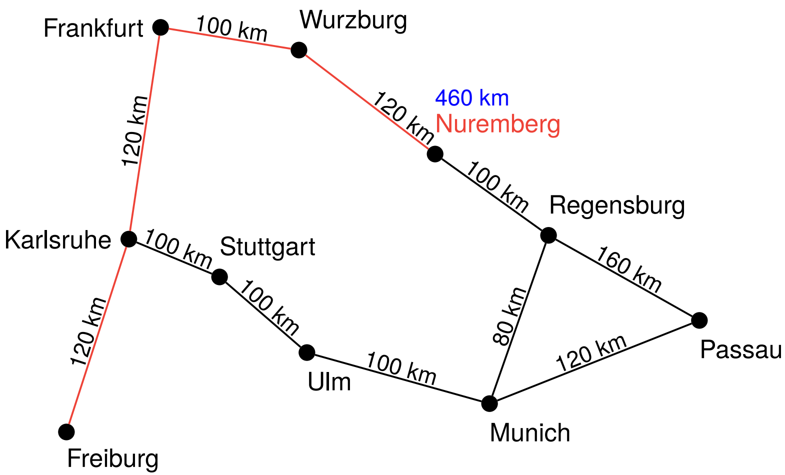
A* using the Euclidean distance heuristic



A^* using the Euclidean distance heuristic



A* using the Euclidean distance heuristic



Relaxed planning tasks

Relaxed planning tasks: idea

In positive normal form (remember?), good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

Idea for the heuristic: Ignore all delete effects.

Relaxed planning tasks

Definition (relaxation of operators)

The relaxation o^+ of an operator $o = \langle \chi, e \rangle$ in positive normal form is the operator which is obtained by replacing all negative effects $\neg a$ within e by the do-nothing effect $\top$.

Definition (relaxation of planning tasks)

The relaxation Π^+ of a planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form is the planning task $\Pi^+ := \langle A, I, \{o^+ \mid o \in O\}, \gamma \rangle$.

Definition (relaxation of operator sequences)

The relaxation of an operator sequence $\pi = o_1 \dots o_n$ is the operator sequence $\pi^+ := {o_1}^+ \dots {o_n}^+$.

Relaxed planning tasks: terminology

- Planning tasks in positive normal form without delete effects are called relaxed planning tasks.
- Plans for relaxed planning tasks are called relaxed plans.
- If Π is a planning task in positive normal form and π^+ is a plan for Π^+, then π^+ is called a relaxed plan for Π.

Dominating states

The on-set $on(s)$ of a state s is the set of true state variables in s, i.e. $on(s) = \{a \in A \mid s(a) = 1\}$.
A state s' dominates another state s iff $on(s) \subseteq on(s')$.

Lemma (domination)

Let s and s' be valuations of a set of propositional variables A and let χ be a propositional formula over A which does not contain negation symbols.
If $s \models \chi$ and s' dominates s, then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ.

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \bot$: then $s \not\models \bot$.

Dominating states (ctd.)

Proof (ctd.)

- Base case $\chi = a \in A$: assume $s \models a$ and $on(s) \subseteq on(s')$. With $a \in on(s)$ we get $a \in on(s')$, hence $s' \models a$.
- Inductive case $\chi = \chi_1 \wedge \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ.

$$
\begin{aligned}
s \models \chi \quad &\Longleftrightarrow \quad s \models \chi_1 \wedge \chi_2 \\
&\Longleftrightarrow \quad s \models \chi_1 \text{ and } s \models \chi_2 \\
&\overset{\text{I.H. (twice)}}{\Longrightarrow} \quad s' \models \chi_1 \text{ and } s' \models \chi_2 \\
&\Longleftrightarrow \quad s' \models \chi_1 \wedge \chi_2 \\
&\Longleftrightarrow \quad s' \models \chi.
\end{aligned}
$$

- Inductive case $\chi = \chi_1 \vee \chi_2$: Analogous.

□

The relaxation lemma

For the rest of this chapter, we assume that all planning tasks are in positive normal form.

Lemma (relaxation)

Let s be a state, let s' be a state that dominates s, and let π be an operator sequence which is applicable in s. Then π^+ is applicable in s' and $app_{\pi^+}(s')$ dominates $app_\pi(s)$. Moreover, if π leads to a goal state from s, then π^+ leads to a goal state from s'.

Proof.

The "moreover" part follows from the rest by the domination lemma. Prove the rest by induction over the length of π.

Base case: $\pi = \varepsilon$

$app_{\pi^+}(s') = s'$. Dominates $app_\pi(s) = s$ by assumption.

The relaxation lemma

For the rest of this chapter, we assume that all planning tasks are in positive normal form.

Lemma (relaxation)

Let s be a state, let s' be a state that dominates s, and let π be an operator sequence which is applicable in s. Then π^+ is applicable in s' and $app_{\pi^+}(s')$ dominates $app_\pi(s)$. Moreover, if π leads to a goal state from s, then π^+ leads to a goal state from s'.

Proof.

The "moreover" part follows from the rest by the domination lemma. Prove the rest by induction over the length of π.

Base case: $\pi = \varepsilon$

$app_{\pi^+}(s') = s'$. Dominates $app_\pi(s) = s$ by assumption.

The relaxation lemma (ctd.)

Proof (ctd.)

Inductive case: $\pi = o_1 \ldots o_{n+1}$.

By the induction hypothesis, $o_1{}^+ \ldots o_n{}^+$ is applicable in s', and $t' = app_{o_1{}^+ \ldots o_n{}^+}(s')$ dominates $t = app_{o_1 \ldots o_n}(s)$.

Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, o is applicable in t, and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence o^+ is applicable in t'. Therefore, π^+ is applicable in s'.

Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals). We get $on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. □

The relaxation lemma (ctd.)

Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$.

By the induction hypothesis, ${o_1}^+ \dots {o_n}^+$ is applicable in s', and $t' = app_{{o_1}^+ \dots {o_n}^+}(s')$ dominates $t = app_{o_1 \dots o_n}(s)$.

Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, o is applicable in t, and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence o^+ is applicable in t'. Therefore, π^+ is applicable in s'.

Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals). We get $on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. □

Obtaining heuristics

Relaxed planning tasks

Definition

The relaxation lemma

Greedy algorithm

Optimality

Discussion

Summary

The relaxation lemma (ctd.)

Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$.

By the induction hypothesis, ${o_1}^+ \dots {o_n}^+$ is applicable in s', and $t' = app_{{o_1}^+ \dots {o_n}^+}(s')$ dominates $t = app_{o_1 \dots o_n}(s)$.

Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, o is applicable in t, and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence o^+ is applicable in t'. Therefore, π^+ is applicable in s'.

Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals). We get $on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. □

The relaxation lemma (ctd.)

Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$.
By the induction hypothesis, ${o_1}^+ \dots {o_n}^+$ is applicable in s', and $t' = app_{{o_1}^+ \dots {o_n}^+}(s')$ dominates $t = app_{o_1 \dots o_n}(s)$.
Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, o is applicable in t, and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence o^+ is applicable in t'. Therefore, π^+ is applicable in s'.
Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals). We get
$on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. $\square$

Consequences of the relaxation lemma

Corollary (relaxation leads to dominance and preserves plans)

Let π be an operator sequence that is applicable in state s.
Then π^+ is applicable in s and $app_{\pi^+}(s)$ dominates $app_\pi(s)$.
If π is a plan for Π, then π^+ is a plan for Π^+.

Proof.

Apply relaxation lemma with $s' = s$.

- $\rightsquigarrow$ Relaxations of plans are relaxed plans.
- $\rightsquigarrow$ Relaxations are no harder to solve than original task.
- $\rightsquigarrow$ Optimal relaxed plans are never longer than optimal plans for original tasks.

Consequences of the relaxation lemma (ctd.)

Corollary (relaxation preserves dominance)

Let s be a state, let s' be a state that dominates s, and let π^+ be a relaxed operator sequence applicable in s. Then π^+ is applicable in s' and $app_{\pi^+}(s')$ dominates $app_{\pi^+}(s)$.

Proof.

Apply relaxation lemma with π^+ for π, noting that $(\pi^+)^+ = \pi^+$.

- $\rightsquigarrow$ If there is a relaxed plan starting from state s, the same plan can be used starting from a dominating state s'.
- $\rightsquigarrow$ Making a transition to a dominating state never hurts in relaxed planning tasks.

Monotonicity of relaxed planning tasks

We need one final property before we can provide an algorithm for solving relaxed planning tasks.

Lemma (monotonicity)

Let $o^+ = \langle \chi, e^+ \rangle$ be a relaxed operator and let s be a state in which o^+ is applicable.
Then $app_{o^+}(s)$ dominates s.

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup [e^+]_s = on(app_{o^+}(s))$.

$\rightsquigarrow$ Together with our previous results, this means that making a transition in a relaxed planning task never hurts.

Greedy algorithm for relaxed planning tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy planning algorithm for $\langle A, I, O^+, \gamma \rangle$

$s := I$
$\pi^+ := \varepsilon$
forever:
    if $s \models \gamma$:
        return π^+
    else if there is an operator $o^+ \in O^+$ applicable in s
            with $app_{o^+}(s) \neq s$:
        Append such an operator o^+ to π^+.
        $s := app_{o^+}(s)$
    else:
        return unsolvable

Correctness of the greedy algorithm

The algorithm is sound:

- If it returns a plan, this is indeed a correct solution.
- If it returns “unsolvable”, the task is indeed unsolvable
  - Upon termination, there clearly is no relaxed plan from s.
  - By iterated application of the monotonicity lemma, s dominates I.
  - By the relaxation lemma, there is no solution from I.

What about completeness (termination) and runtime?

- Each iteration of the loop adds at least one atom to $on(s)$.
- This guarantees termination after at most $|A|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
  - A good implementation runs in $O(\|\Pi\|)$.

Using the greedy algorithm as a heuristic

We can apply the greedy algorithm within heuristic search:

- In a search node σ, solve the relaxation of the planning task with *state*(σ) as the initial state.
- Set $h(\sigma)$ to the length of the generated relaxed plan.

Is this an admissible heuristic?

- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because our greedy planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

The set cover problem

To obtain an admissible heuristic, we need to generate optimal relaxed plans. Can we do this efficiently?

This question is related to the following problem:

Problem (set cover)

Given: a finite set U, a collection of subsets $C = \{C_1, \ldots, C_n\}$ with $C_i \subseteq U$ for all $i \in \{1, \ldots, n\}$, and a natural number K.

Question: Does there exist a set cover of size at most K, i. e., a subcollection $S = \{S_1, \ldots, S_m\} \subseteq C$ with $S_1 \cup \cdots \cup S_m = U$ and $m \leq K$?

The following is a classical result from complexity theory:

Theorem (Karp 1972)

The set cover problem is NP-complete.

Hardness of optimal relaxed planning

Theorem (optimal relaxed planning is hard)

The problem of deciding whether a given relaxed planning task has a plan of length at most K is NP-complete.

Proof.

For membership in NP, guess a plan and verify. It is sufficient to check plans of length at most $|A|$, so this can be done in nondeterministic polynomial time.

For hardness, we reduce from the set cover problem.

Hardness of optimal relaxed planning (ctd.)

Proof (ctd.)

Given a set cover instance $\langle U, C, K\rangle$, we generate the following relaxed planning task $\Pi^+ = \langle A, I, O^+, \gamma\rangle$:

- $A = U$
- $I = \{a \mapsto 0 \mid a \in A\}$
- $O^+ = \{\langle \top, \bigwedge_{a \in C_i} a\rangle \mid C_i \in C\}$
- $\gamma = \bigwedge_{a \in U} a$

If S is a set cover, the corresponding operators form a plan. Conversely, each plan induces a set cover by taking the subsets corresponding to the operators. There exists a plan of length at most K iff there exists a set cover of size K.

Moreover, Π^+ can be generated from the set cover instance in polynomial time, so this is a polynomial reduction. $\square$

Using relaxations in practice

How can we use relaxations for heuristic planning in practice?

Different possibilities:

- Implement an optimal planner for relaxed planning tasks and use its solution lengths as an estimate, even though it is NP-hard.
  $\rightsquigarrow$ h^+ heuristic
- Do not actually solve the relaxed planning task, but compute an estimate of its difficulty in a different way.
  $\rightsquigarrow$ h_{max} heuristic, h_{add} heuristic, $h_{\text{LM-cut}}$ heuristic
- Compute a solution for relaxed planning tasks which is not necessarily optimal, but “reasonable”.
  $\rightsquigarrow$ h_{FF} heuristic

Summary

- Two general methods for coming up with heuristics:
  - relaxation: solve a less constrained problem
  - abstraction: solve a small problem
- Here, we consider the delete relaxation, which requires tasks in positive normal form and ignores delete effects.
- Delete-relaxed tasks have a domination property: it is always beneficial to make more fluents true.
- They also have a monotonicity property: applying operators leads to dominating states.
- Because of these two properties, finding some relaxed plan greedily is easy (polynomial).
- For an informative heuristic, we would ideally want to find optimal relaxed plans. This is NP-complete.