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Ranks: Ties



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In case of ties, the average rank is assigned to the whole group of scores that constitutes the tie.

Example

- Data: 1, 6, 4, 4, 2, 2, 2
- Rank: 1, 7, 5.5, 5.5, 3, 3, 3

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Wilcoxon Signed-Rank Test: Example Continued

Example

Five participants are asked to rate their belief in the possibility that humans will one day be the slaves of robots before and after they have watched a Sci-Fi movie. As a measurement instrument, a 3-Point Likert-Scale "never ever!" (1), "maybe" (2), "yes, sure!" (3) was used.

- Before: 1, 2, 2, 3, 3; After: 2, 3, 3, 3, 1
- Difference: -1, -1, -1, 0, +2; Without 0: -1, -1, -1, +2
- Ranks: 2, 2, 2, 4
- Let $V = \sum_{i=1}^{n} Z_i R_i$ be the sum of the positive ranks ($Z_i = 1$ if difference *i* is positive, and $Z_i = 0$ else).
- In the example V = 4. Well, so what?

Wilcoxon Signed-Rank Test: Motivation



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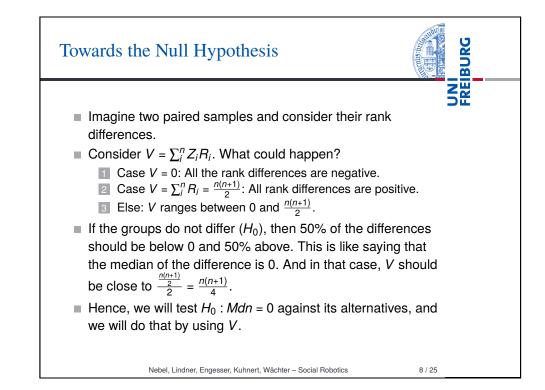
- Likert scales are a popular means of measurement.
- Likert scales in most cases have no interval-scale reading.

Example

Five participants are asked to rate their belief in the possibility that humans will one day be the slaves of robots before and after they have watched a Sci-Fi movie. As a measurement instrument, a 3-Point Likert-Scale "never ever!" (1), "maybe" (2), "yes, sure!" (3) was used.

- Before: 1, 2, 2, 3, 3; After: 2, 3, 3, 3, 1
- Difference: -1, -1, -1, 0, +2
- Differences without 0: -1, -1, -1, +2
- Ranks: 2, 2, 2, 4

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Wilcoxon Signed-Rank Test

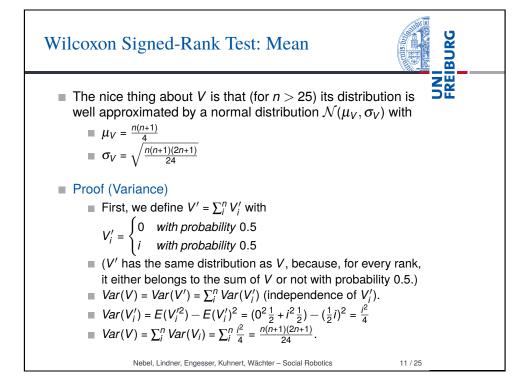
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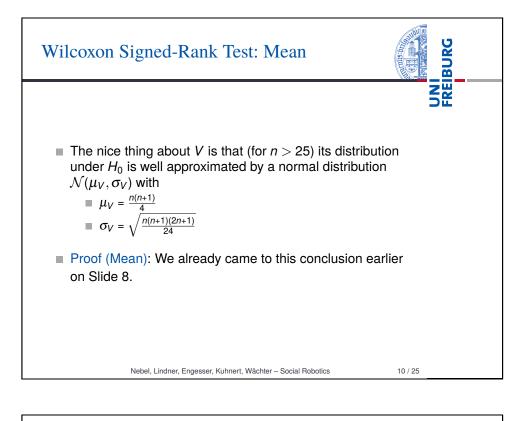
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■ The nice thing about *V* is that (for *n* > 25) its distribution is well approximated by a normal distribution *N*(µ_V, σ_V) with

$$\mu_V = \frac{n(n+1)}{4}$$
$$\sigma_V = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

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Wilcoxon Signed-Rank Test: Example Again



Example

Five participants are asked to rate their belief in the possibility that humans will one day be the slaves of robots before and after they have watched a Sci-Fi movie. As a measurement instrument, a 3-Point Likert-Scale "never ever!" (1), "maybe" (2), "yes, sure!" (3) was used. Before: 1, 2, 2, 3, 3; After: 2, 3, 3, 3, 1

- Difference: -1, -1, -1, 0, +2; Without 0: -1, -1, -1, +2
- Ranks: 2, 2, 2, 4

 $V = 4, \ \mu_V = 4(4+1)/4 = 5, \ \sigma_V = \sqrt{4(4+1)(2 \times 4+1)/24}$ $z = \frac{V - \mu_V}{\sigma_V} = (4-5)/2.74 = -0.365$ $p = P(z \le -0.365) + 1 - P(z \le 0.365) = 0.715$

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Comparison to Paired t-Test



Example: t-Test

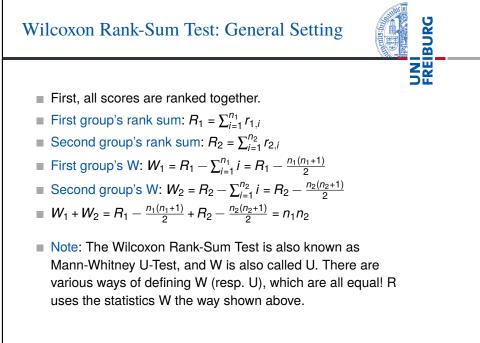
Five participants are asked to rate their belief in the possibility that humans will one day be the slaves of robots before and after they have watched a Sci-Fi movie. As a measurement instrument, a 3-Point Likert-Scale "never ever!" (1), "maybe" (2), "yes, sure!" (3) was used.

- Before: 1, 2, 2, 3, 3; After: 2, 3, 3, 3, 1
- Difference: -1, -1, -1, 0, +2
- \overline{D} = 0.20, s_D = 1.30, n = 5

■ $t = \sqrt{5} \times 0.20 / 1.30 = 0.344$, df = 4

■ *p* = 0.748

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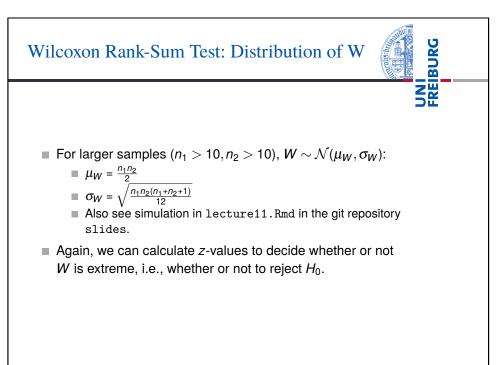
Example

Five participants are asked to rate their belief in the possibility that humans will one day be the slaves of robots after they have watched the Sci-Fi movie M1, and five participants rate their belief after watching Sci-Fi movie M2. As a measurement instrument, a 3-Point Likert-Scale "never ever!" (1), "maybe" (2), "yes, sure!" (3) was used.

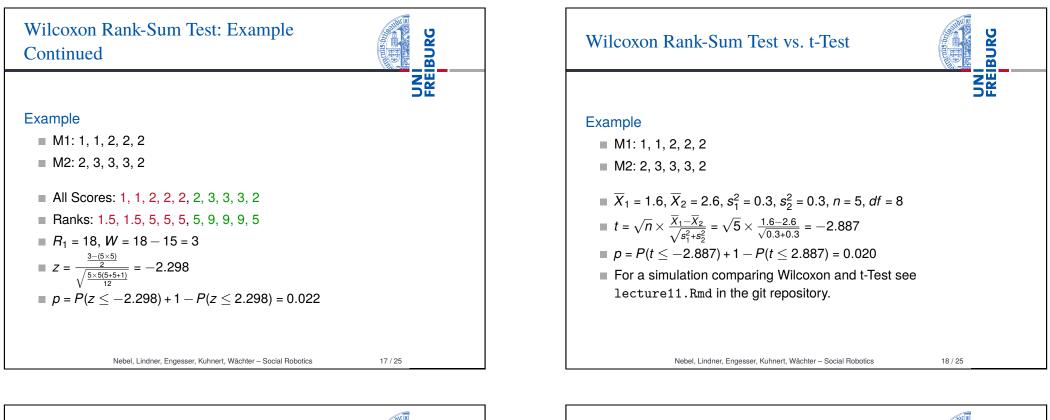
- M1: 1, 1, 2, 2, 2; M2: 2, 3, 3, 3, 2
- \blacksquare H_0 : The two groups are equal.
- Reject H_0 or not?

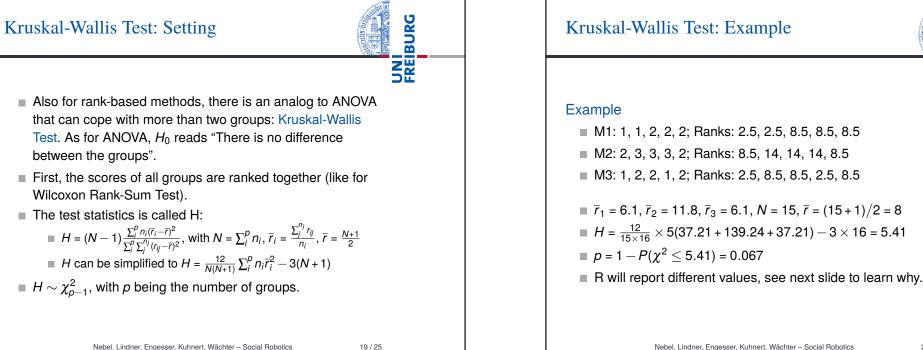
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Ties call for Corrections



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If there are long ties (i.e., a lot of scores getting the same rank), the variance of the statistics become smaller and thus some corrections have to be applied.

The V-statistics's standard deviation becomes:

$$\sigma_V = \sqrt{\frac{n(n+1)(2n+1)}{24}} - \sum_i^k \frac{t_i^3 - t_i}{48}$$
 (cf., slide 9)

The W-statistics's standard deviation becomes:

$$\sigma_W = \sqrt{\frac{n_1 n_2}{12} \left((n_1 + n_2 + 1) - \sum_i^k \frac{t_i^3 - t_i}{(n_1 + n_2)(n_1 + n_2 - 1)} \right)$$
 (cf., slide 16)

And the H-statistics can be corrected by dividing H by the term *corr* = $1 - \frac{\sum_{i}^{k} (t_{i}^{3} - t_{i})}{N^{3} - N}$

In the example:
$$corr = 1 - \frac{(4^3-4)+(8^3-8)+(3^3-3)}{(15^3-15)}$$

- The corrected *H* value then is $H_{corr} = 6.56$
- Because all this is rather tedious, you are allowed to skip these corrections in your assignments (also in the exam).
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