

# Principles of AI Planning

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## Exercise Sheet 5

**Due: Friday, November 24th, 2017**

**Exercise 5.1** (Delete relaxation, 2+2 points)

Consider the planning task  $\Pi = \langle A, I, O, \gamma \rangle$  in positive normal form with

$$\begin{aligned} A &= \{haveCake, eatenCake, haveNoCake\}, \\ I &= \{have-cake \mapsto 0, eatenCake \mapsto 0, haveNoCake \mapsto 1\} \\ O &= \{eatCake, bakeCake\}, \\ eatCake &= \langle haveCake, \neg haveCake \wedge haveNoCake \wedge eatenCake \rangle, \\ bakeCake &= \langle haveNoCake, haveCake \wedge \neg haveNoCake \rangle \text{ und} \\ \gamma &= haveCake \wedge eatenCake. \end{aligned}$$

- Give the relaxation  $\Pi^+$  of  $\Pi$ .
- Give a sequence  $\pi$  of operators (as short as possible) from  $O$  such that  $\pi$  is *not* a plan of  $\Pi$ , but  $\pi^+$  is a plan of  $\Pi^+$ .

**Exercise 5.2** ( $h^+$  heuristic, 2+2 points)

A 15-puzzle planning task  $\Pi = \langle A, I, O, \gamma \rangle$  is given as

$$\begin{aligned} A &= \{empty(p_{i,j}) \mid 0 \leq i, j \leq 3\} \cup \{at(t_k, p_{i,j}) \mid 0 \leq i, j \leq 3, 0 \leq k \leq 14\}, \\ O &= \{move(t_m, p_{i,j}, p_{k,l}) \mid 0 \leq i, j, k, l \leq 3, 0 \leq m \leq 14, \\ &\quad (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1)\}, \\ \gamma &= \bigwedge_{0 \leq m \leq 14} at(t_m, p_{\lfloor m/4 \rfloor, m \% 4}) \end{aligned}$$

Action  $move(t_m, p_{i,j}, p_{k,l})$  moves tile  $t_m$  from position  $p_{i,j}$  to position  $p_{k,l}$ :

$$\begin{aligned} move(t_m, p_{i,j}, p_{k,l}) &= \langle at(t_m, p_{i,j}) \wedge empty(p_{k,l}), \\ &\quad at(t_m, p_{k,l}) \wedge empty(p_{i,j}) \wedge \neg at(t_m, p_{i,j}) \wedge \neg empty(p_{k,l}) \rangle \end{aligned}$$

A syntactically possible state is *legal* if each tile  $t_m$  is at some position  $p_{i,j}$ , if no two tiles are at the same position and if the remaining position is the only one that is *empty*. The initial state is an arbitrary state that is legal.

One possible heuristic for the 15-puzzle is the Manhattan-distance heuristic  $h^{Manhattan}$ : It sums the Manhattan distances of all tiles from their current positions to their target positions, where the Manhattan distance between position  $p_{i,j}$  and  $p_{k,l}$  is given as  $|i - k| + |j - l|$ .

The  $h^+$  heuristic estimates the distance of state  $s$  to the closest goal state as the length of the optimal plan in the relaxed planning task (with initial state  $s$ ).

- Show that  $h^+(s) \geq h^{Manhattan}(s)$  for each legal state  $s$  of a 15-puzzle planning task.
- Show that  $h^+(s) > h^{Manhattan}(s)$  for at least one state  $s$  of a 15-puzzle planning task.

**Exercise 5.3** (Inaccuracy of  $h_{\max}$ , 2 points)

Prove that the heuristic  $h_{\max}$  is arbitrarily inaccurate, i.e., for all  $c \in \mathbb{R}^+$  there exists a relaxed planning task  $\Pi = \langle A, I, O^+, \gamma \rangle$  such that  $c \cdot h_{\max}(I) \leq h^+(I)$ .

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.