# Principles of AI Planning

## 7. Planning as search: relaxed planning tasks

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# How to obtain a heuristic

# A simple heuristic for deterministic planning

STRIPS (Fikes & Nilsson, 1971) used the number of state variables that differ in current state $s$ and a STRIPS goal $a_1 \wedge \cdots \wedge a_n$:

$$h(s) := \left|\{i \in \{1, \ldots, n\} \mid s \not\models a_i\}\right|.$$

Intuition: more true goal literals $\rightsquigarrow$ closer to the goal

$\rightsquigarrow$ STRIPS heuristic (a.k.a. goal-count heuristic) (properties?)

Note: From now on, for convenience we usually write heuristics as functions of states (as above), not nodes. Node heuristic $h'$ is defined from state heuristic $h$ as $h'(\sigma) := h(state(\sigma))$.

# Criticism of the STRIPS heuristic



What is wrong with the STRIPS heuristic?

- quite uninformative:
  the range of heuristic values in a given task is small;
  typically, most successors have the same estimate
- very sensitive to reformulation:
  can easily transform any planning task into an equivalent one where $h(s) = 1$ for all non-goal states (how?)
- ignores almost all problem structure:
  heuristic value does not depend on the set of operators!

$\rightsquigarrow$ need a better, principled way of coming up with heuristics

# Coming up with heuristics in a principled way



**General procedure for obtaining a heuristic**

Solve an easier version of the problem.

Two common methods:

- relaxation: consider less constrained version of the problem
- abstraction: consider smaller version of real problem

Both have been very successfully applied in planning.
We consider both in this course, beginning with relaxation.

# Relaxing a problem



How do we relax a problem?

**Example (Route planning for a road network)**

The road network is formalized as a weighted graph over points in the Euclidean plane. The weight of an edge is the road distance between two locations.

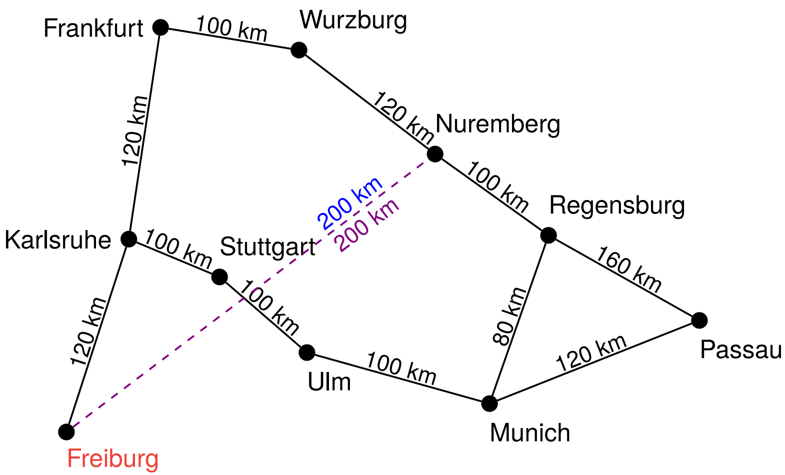
A relaxation drops constraints of the original problem.

**Example (Relaxation for route planning)**

Use the Euclidean distance $\sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}$ as a heuristic for the road distance between $\langle x_1, y_1 \rangle$ and $\langle x_2, y_2 \rangle$ This is a lower bound on the road distance ($\rightsquigarrow$ admissible).

$\rightsquigarrow$ We drop the constraint of having to travel on roads.

# A* using the Euclidean distance heuristic



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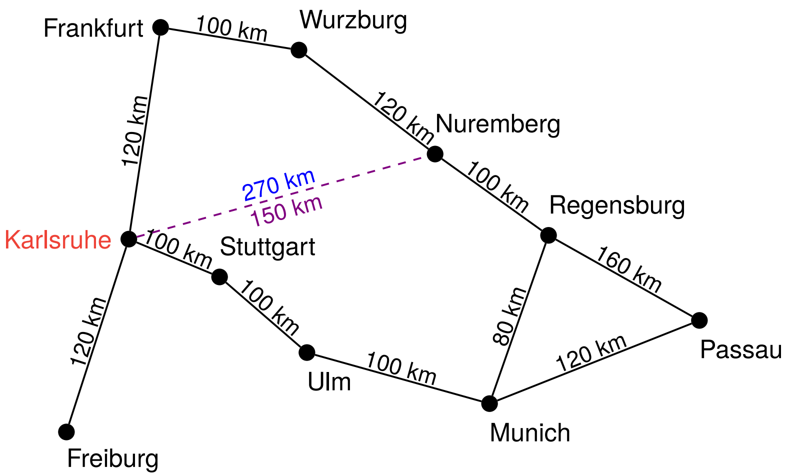

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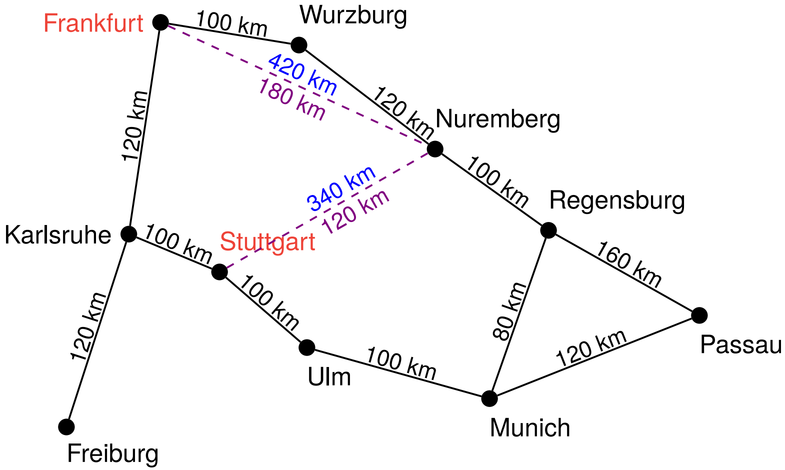

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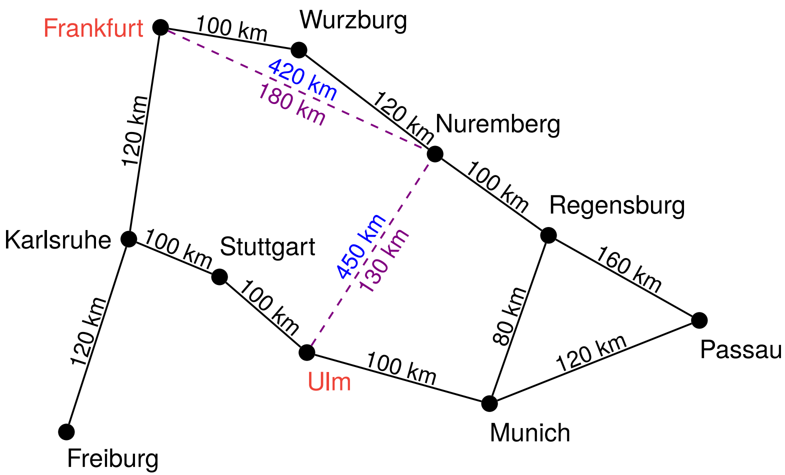

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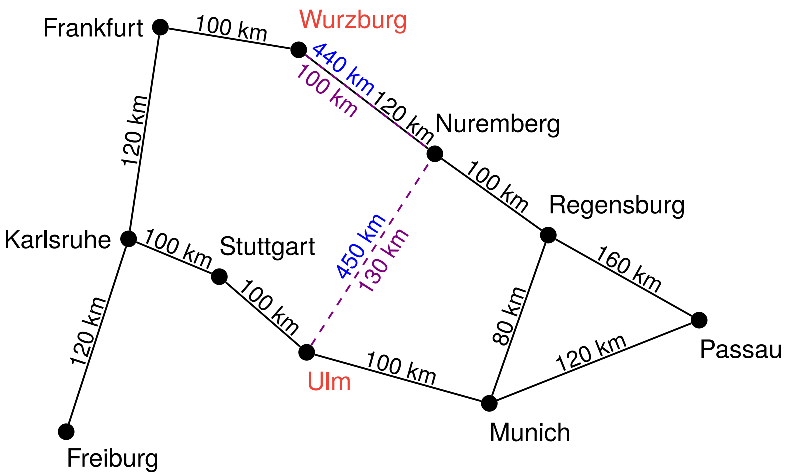

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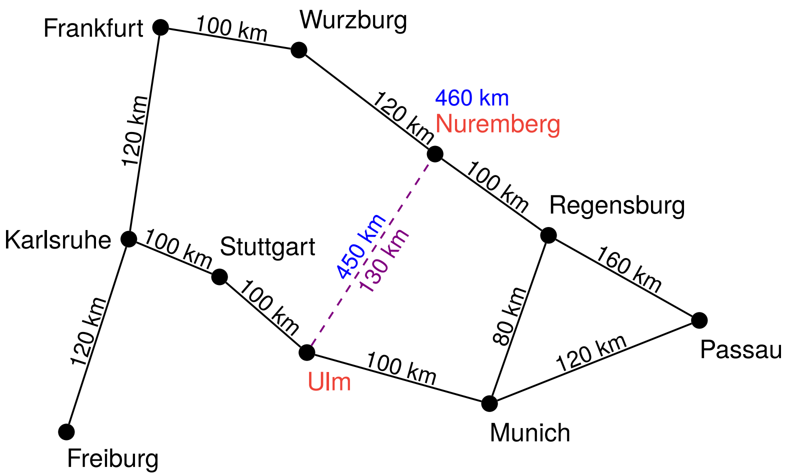

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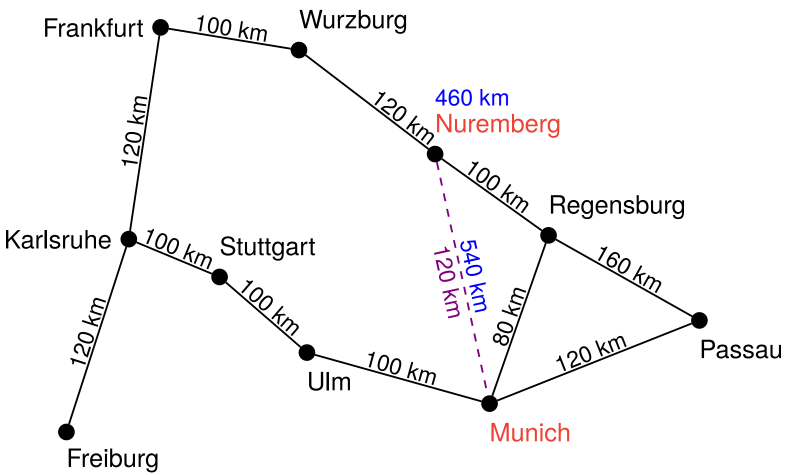
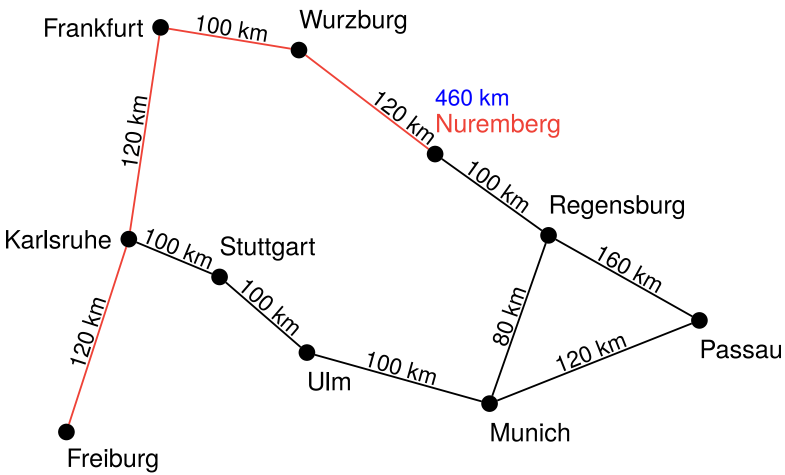


# $A^*$ using the Euclidean distance heuristic



# Relaxed planning tasks

# Relaxed planning tasks: idea



In positive normal form (remember?), good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

Idea for the heuristic: Ignore all delete effects.

# Relaxed planning tasks

## Definition (relaxation of operators)

The relaxation $o^+$ of an operator $o = \langle \chi, e \rangle$ in positive normal form is the operator which is obtained by replacing all negative effects $\neg a$ within $e$ by the do-nothing effect $\top$.

## Definition (relaxation of planning tasks)

The relaxation $\Pi^+$ of a planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form is the planning task $\Pi^+ := \langle A, I, \{o^+ \mid o \in O\}, \gamma \rangle$.

## Definition (relaxation of operator sequences)

The relaxation of an operator sequence $\pi = o_1 \dots o_n$ is the operator sequence $\pi^+ := {o_1}^+ \dots {o_n}^+$.

# Relaxed planning tasks: terminology

- Planning tasks in positive normal form without delete effects are called relaxed planning tasks.
- Plans for relaxed planning tasks are called relaxed plans.
- If $\Pi$ is a planning task in positive normal form and $\pi^+$ is a plan for $\Pi^+$, then $\pi^+$ is called a relaxed plan for $\Pi$.

# Dominating states

The on-set $on(s)$ of a state $s$ is the set of true state variables in $s$, i.e. $on(s) = \{a \in A \mid s(a) = 1\}$.
A state $s'$ dominates another state $s$ iff $on(s) \subseteq on(s')$.

## Lemma (domination)

*Let $s$ and $s'$ be valuations of a set of propositional variables $A$ and let $\chi$ be a propositional formula over $A$ which does not contain negation symbols.*
*If $s \models \chi$ and $s'$ dominates $s$, then $s' \models \chi$.*

## Proof.

Proof by induction over the structure of $\chi$.

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \bot$: then $s \not\models \bot$.

# Dominating states (ctd.)

## Proof (ctd.)

- Base case $\chi = a \in A$: assume $s \models a$ and $on(s) \subseteq on(s')$. With $a \in on(s)$ we get $a \in on(s')$, hence $s' \models a$.
- Inductive case $\chi = \chi_1 \land \chi_2$: by induction hypothesis, our claim holds for the proper subformulas $\chi_1$ and $\chi_2$ of $\chi$.

$$
\begin{aligned}
s \models \chi \quad & \Longleftrightarrow \quad s \models \chi_1 \land \chi_2 \\
& \Longleftrightarrow \quad s \models \chi_1 \text{ and } s \models \chi_2 \\
& \overset{\text{I.H. (twice)}}{\Longrightarrow} \quad s' \models \chi_1 \text{ and } s' \models \chi_2 \\
& \Longleftrightarrow \quad s' \models \chi_1 \land \chi_2 \\
& \Longleftrightarrow \quad s' \models \chi.
\end{aligned}
$$

- Inductive case $\chi = \chi_1 \lor \chi_2$: Analogous.

□

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# The relaxation lemma

For the rest of this chapter, we assume that all planning tasks are in positive normal form.

## Lemma (relaxation)

*Let $s$ be a state, let $s'$ be a state that dominates $s$, and let $\pi$ be an operator sequence which is applicable in $s$. Then $\pi^+$ is applicable in $s'$ and $app_{\pi^+}(s')$ dominates $app_\pi(s)$. Moreover, if $\pi$ leads to a goal state from $s$, then $\pi^+$ leads to a goal state from $s'$.*

## Proof.

The "moreover" part follows from the rest by the domination lemma. Prove the rest by induction over the length of $\pi$.

Base case: $\pi = \varepsilon$

$app_{\pi^+}(s') = s'$. Dominates $app_\pi(s) = s$ by assumption.

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# The relaxation lemma (ctd.)

## Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$.

By the induction hypothesis, $o_1{}^+ \dots o_n{}^+$ is applicable in $s'$, and $t' = app_{o_1{}^+ \dots o_n{}^+}(s')$ dominates $t = app_{o_1 \dots o_n}(s)$.

Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, $o$ is applicable in $t$, and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence $o^+$ is applicable in $t'$. Therefore, $\pi^+$ is applicable in $s'$.

Because $o$ is in positive normal form, all effect conditions satisfied by $t$ are also satisfied by $t'$ (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where $A$ is the set of state variables, or positive literals). We get $on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. □

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Because $o$ is in positive normal form, all effect conditions satisfied by $t$ are also satisfied by $t'$ (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where $A$ is the set of state variables, or positive literals). We get
$on(app_\pi(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$,
and thus $app_{\pi^+}(s')$ dominates $app_\pi(s)$. □

# Consequences of the relaxation lemma

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Obtaining heuristics

Relaxed planning tasks

Definition

The relaxation lemma

Greedy algorithm

Optimality

Discussion

Summary

**Corollary (relaxation leads to dominance and preserves plans)**

*Let $\pi$ be an operator sequence that is applicable in state $s$. Then $\pi^+$ is applicable in $s$ and $app_{\pi^+}(s)$ dominates $app_{\pi}(s)$. If $\pi$ is a plan for $\Pi$, then $\pi^+$ is a plan for $\Pi^+$.*

**Proof.**

Apply relaxation lemma with $s' = s$.

- ⇝ Relaxations of plans are relaxed plans.
- ⇝ Relaxations are no harder to solve than original task.
- ⇝ Optimal relaxed plans are never longer than optimal plans for original tasks.

# Consequences of the relaxation lemma (ctd.)



## Corollary (relaxation preserves dominance)

*Let $s$ be a state, let $s'$ be a state that dominates $s$, and let $\pi^+$ be a relaxed operator sequence applicable in $s$. Then $\pi^+$ is applicable in $s'$ and $app_{\pi^+}(s')$ dominates $app_{\pi^+}(s)$.*

## Proof.

Apply relaxation lemma with $\pi^+$ for $\pi$, noting that $(\pi^+)^+ = \pi^+$.

- ⇝ If there is a relaxed plan starting from state $s$, the same plan can be used starting from a dominating state $s'$.
- ⇝ Making a transition to a dominating state never hurts in relaxed planning tasks.

# Monotonicity of relaxed planning tasks

We need one final property before we can provide an algorithm for solving relaxed planning tasks.

**Lemma (monotonicity)**

*Let $o^+ = \langle \chi, e^+ \rangle$ be a relaxed operator and let $s$ be a state in which $o^+$ is applicable.*
*Then $app_{o^+}(s)$ dominates $s$.*

**Proof.**

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup [e^+]_s = on(app_{o^+}(s))$.

⇝ Together with our previous results, this means that making a transition in a relaxed planning task never hurts.

# Greedy algorithm for relaxed planning tasks



The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

## Greedy planning algorithm for $\langle A, I, O^+, \gamma \rangle$

$s := I$
$\pi^+ := \varepsilon$
**forever**:
    **if** $s \models \gamma$:
        **return** $\pi^+$
    **else if** there is an operator $o^+ \in O^+$ applicable in $s$
        with $app_{o^+}(s) \neq s$:
        Append such an operator $o^+$ to $\pi^+$.
        $s := app_{o^+}(s)$
    **else**:
        **return** unsolvable

# Correctness of the greedy algorithm



The algorithm is sound:

- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable
  - Upon termination, there clearly is no relaxed plan from $s$.
  - By iterated application of the monotonicity lemma, $s$ dominates $I$.
  - By the relaxation lemma, there is no solution from $I$.

What about completeness (termination) and runtime?

- Each iteration of the loop adds at least one atom to $on(s)$.
- This guarantees termination after at most $|A|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
  - A good implementation runs in $O(\|\Pi\|)$.

# Using the greedy algorithm as a heuristic



We can apply the greedy algorithm within heuristic search:

- In a search node $\sigma$, solve the relaxation of the planning task with $state(\sigma)$ as the initial state.
- Set $h(\sigma)$ to the length of the generated relaxed plan.

Is this an admissible heuristic?

- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because our greedy planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

# The set cover problem

To obtain an admissible heuristic, we need to generate optimal relaxed plans. Can we do this efficiently?
This question is related to the following problem:

## Problem (set cover)

*Given:* a finite set $U$, a collection of subsets $C = \{C_1, \ldots, C_n\}$ with $C_i \subseteq U$ for all $i \in \{1, \ldots, n\}$, and a natural number $K$.
*Question:* Does there exist a set cover of size at most $K$, i. e., a subcollection $S = \{S_1, \ldots, S_m\} \subseteq C$ with $S_1 \cup \cdots \cup S_m = U$ and $m \leq K$?

The following is a classical result from complexity theory:

## Theorem (Karp 1972)

*The set cover problem is NP-complete.*

# Hardness of optimal relaxed planning





## Theorem (optimal relaxed planning is hard)

*The problem of deciding whether a given relaxed planning task has a plan of length at most $K$ is NP-complete.*

## Proof.

For membership in NP, guess a plan and verify. It is sufficient to check plans of length at most $|A|$, so this can be done in nondeterministic polynomial time.

For hardness, we reduce from the set cover problem.

# Hardness of optimal relaxed planning (ctd.)

## Proof (ctd.)

Given a set cover instance $\langle U, C, K \rangle$, we generate the following relaxed planning task $\Pi^+ = \langle A, I, O^+, \gamma \rangle$:

- $A = U$
- $I = \{a \mapsto 0 \mid a \in A\}$
- $O^+ = \{\langle \top, \bigwedge_{a \in C_i} a \rangle \mid C_i \in C\}$
- $\gamma = \bigwedge_{a \in U} a$

If $S$ is a set cover, the corresponding operators form a plan. Conversely, each plan induces a set cover by taking the subsets corresponding to the operators. There exists a plan of length at most $K$ iff there exists a set cover of size $K$.

Moreover, $\Pi^+$ can be generated from the set cover instance in polynomial time, so this is a polynomial reduction. □

# Using relaxations in practice

How can we use relaxations for heuristic planning in practice?

Different possibilities:

- Implement an optimal planner for relaxed planning tasks and use its solution lengths as an estimate, even though it is NP-hard.
  $\rightsquigarrow$ $h^+$ heuristic
- Do not actually solve the relaxed planning task, but compute an estimate of its difficulty in a different way.
  $\rightsquigarrow$ $h_{\text{max}}$ heuristic, $h_{\text{add}}$ heuristic, $h_{\text{LM-cut}}$ heuristic
- Compute a solution for relaxed planning tasks which is not necessarily optimal, but "reasonable".
  $\rightsquigarrow$ $h_{\text{FF}}$ heuristic

# Summary



- Two general methods for coming up with heuristics:
  - relaxation: solve a less constrained problem
  - abstraction: solve a small problem
- Here, we consider the delete relaxation, which requires tasks in positive normal form and ignores delete effects.
- Delete-relaxed tasks have a domination property: it is always beneficial to make more fluents true.
- They also have a monotonicity property: applying operators leads to dominating states.
- Because of these two properties, finding some relaxed plan greedily is easy (polynomial).
- For an informative heuristic, we would ideally want to find optimal relaxed plans. This is NP-complete.