Dynamic Epistemic Logic

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Exercise Sheet 6

Due: December 8th, 2016, 10:00

Exercise 6.1 (Muddy Children Puzzle, 3+1 points)

Consider the muddy children puzzle as described in the lecture (and in Example 4.10 of the book *Dynamic Epistemic Logic*).

(a) Show that for the general case with n children of which m are muddy, it takes m requests of the father for the muddy children to finally step forward. (Exercise 4.61)

(b) Suppose that in the beginning of the game, instead of announcing that at least one agent is muddy, the father announces that some specific agent (e.g., Alice) is muddy. What happens now when the father makes his subsequent requests? (Exercise 4.62)

Exercise 6.2 (Gossip Problem, 2+2+2+2 points)

Consider a simplified version of the gossip problem: There are three agents a, b, and c who each have their own secret (the truth value of proposition p_i for agent $i \in \{a, b, c\}$). A call can be made by an arbitrary agent to any other agent, in which the secrets of both agents are exchanged. In our simplified setting (and in contrast to the version that can be found, e.g., in the book *One Hundred Prisoners and a Light Bulb*), agents do not exchange secrets they learned from previous calls.

(a) Specify the initial situation where every agent knows only its own secret (and this is common knowledge between the agents) as epistemic state s_0. Assume that in the actual world p_i is true for all $i \in \{a, b, c\}$.

(b) Specify the epistemic actions $call_{ij}$ in which agents i and j exchange their secrets. Note that we allow the third agent to be aware that the call takes place and that agents i and j are exchanging secrets.

(c) Specify the updated epistemic state $s_1 = s_0 \otimes call_{ab}$. Confirm that $s_1 \models C_{ab}(p_a \land p_b) \land C_{abc}((K_a p_b \lor K_a \neg p_b) \land (K_b p_a \lor K_b \neg p_a))$

(d) Specify a sequence of call actions $a_1, \ldots, a_n$ and the updated epistemic state $s_n = s_0 \otimes a_1 \otimes \ldots \otimes a_n$ such that $s_n \models C_{abc}(p_a \land p_b \land p_c)$.