## Dynamic Epistemic Logic

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# Exercise Sheet 5

**Due: December 1st, 2016, 10:00**

**Exercise 5.1** (Consecutive Numbers Game, 2+2 points)

Consider another variant[1] of the consecutive numbers puzzle from Exercise 4.1. Again, both agents receive a number that can be seen only by the other agent. And again, it is common knowledge that the numbers are consecutive integers (this time between 1 and 10). Both agents now independently have to choose whether they want to take a risk or not. Depending on their choice, they will be rewarded or punished as follows: If both of the agents take the risk and both have a number less than 10, they both get one million euros. If both take the risk and one of the agents has the number 10, they both die. If only one of the agents takes the risk, he also dies (regardless of the agents' numbers). An agent that does not take the risk remains alive, but is not rewarded either. Of course, the agents are not allowed to coordinate after having seen the other agents' number.

(a) Assume you are one of the agents. You see that the other agent has the number 1. Would you take the risk or not? Explain your choice.

(b) Now assume that the agents are allowed to agree on a protocol before the game starts (before the numbers are randomly assigned to the agents), containing binding instructions for each agent in which cases to take and in which cases not to take the risk (depending on the agent's own knowledge). Show whether or not there is a protocol that guarantees the agents to survive while allowing them to collect the reward in at least some cases.

**Exercise 5.2** (Hexa, 2+2 points)

In $(Hexa, 012)$, Anne says to Bill: "(I hold card 0 and) You don't know that I hold card 0".

(a) Show that this is an unsuccessful update.

(b) In the resulting epistemic state Bill says to Anne: "But (I hold card 1 and) you don't know that I hold card 1". Show that this is also an unsuccessful update.

**Exercise 5.3** (Substitution in **PA**, 4 points)

(a) Show the *substitution of equals* property of **PA**:

$$\text{If } \vdash \psi \leftrightarrow \chi, \text{ then } \vdash \varphi(p/\psi) \leftrightarrow \varphi(p/\chi).$$

Use induction on the formula $\varphi$.

[1] Thanks go to Thomas Bolander for the idea for this game.