Dynamic Epistemic Logic

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Exercise Sheet 5

Due: November 24th, 2016, 10:00

Exercise 5.1 (Announcement Logic, 2+1+1+1+3 points)

We want to model and solve a logics puzzle using Public Announcement Logic (PAL).[1] Consider the following situation with two agents. Both have a number written on their forehead, which can only be seen by the other agent. It is also common knowledge between the agents that both numbers have to be consecutive integers between 0 and 9. Furthermore, let us assume that Agent 1 sees the number 4 on Agent 2's forehead. Now consider the following sequence of truthful announcements:

Agent 1: *"I don't know my number!"*
Agent 2: *"I don't know my number!"*
Agent 1: *"I don't know my number!"*
Agent 2: *"I don't know my number!"*
Agent 1: *"I know my number!"*

(a) Model the initial situation (prior to the announcements) as epistemic model. Identify the worlds that are candidates for being the actual world, following from the problem description (but not from the following dialogue).

(b) Model the announcements from the dialogue as $\mathcal{L}_K$ formulas.

(c) What is Agent 1's number? Formulate your hypothesis as $\mathcal{L}_K$ formula.

(d) Will Agent 2 know its own number after witnessing all the announcements? Formulate your hypothesis as $\mathcal{L}_{K[]}$ formula.

(e) Verify your hypotheses by first identifying the actual world (for which the successive announcements have to be truthful) and then checking whether the hypotheses hold in this world.

Exercise 5.2 (**S5C**: Deriving theorems, 1+1 points)

Derive the following **S5C** theorems, where $a \in B \subseteq A$ and A is the set of agents.

(a) $C_B p \leftrightarrow C_B C_B p$

(b) $\neg C_B p \leftrightarrow K_a \neg C_B p$

Exercise 5.3 (Equivalences in $\mathcal{L}_{K[]}$, 1+1 points)

Prove the following equivalences.

(a) φ and $\neg[\varphi]\bot$

(b) $\langle\varphi\rangle\,\psi$ and $\varphi \wedge [\varphi]\psi$

[1] Different versions of this puzzle and their origins are discussed in the book *One Hundred Prisoners and a Light Bulb* by Hans van Ditmarsch and Barteld Kooi, Springer, 2015.