Dynamic Epistemic Logic

B. Nebel, R. Mattmüller, T. Engesser — University of Freiburg
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Exercise Sheet 3

Due: November 10th, 2016, 10:00

Exercise 3.1 (**S5**: Axioms and frame properties I, 6 points)

A Kripke frame $\mathcal{F} = \langle S, R \rangle$ is defined exactly like a Kripke model $\langle S, R, V \rangle$, but without the valuation V. The set of all models over $\langle S, R \rangle$ is the set of all models $\langle S, R, V \rangle$ where V is any propositional valuation. A formula is valid in a frame $\mathcal{F}$, if it is valid in all models over $\mathcal{F}$. It is valid in a class of frames, if it is valid in each frame in that class. We say that an axiom defines a class of frames if the axiom is valid exactly in this class of frames. Show that

(a) the axiom **T** defines the class of *reflexive* frames,

(b) the axiom **4** defines the class of *transitive* frames,

(c) the axiom **5** defines the class of *Euclidean* frames.

Note: You might be able to re-use parts of your solutions for Exercise 2.2.

Exercise 3.2 (**S5**: Axioms and frame properties II, 3 points)

Show that the class of frames that is defined by the axioms **K**, **T** and **5** is the same as the class of frames that is defined by the axioms **K**, **T**, **4** and **5**. You can use the correspondences of frame properties to axioms from the previous exercise.

Exercise 3.3 (**S5**: Deriving theorems, 1+1+1 points)

Derive the following **S5** theorems. Recall that a derivation is a finite sequence of formulas, such that each formula is either an instance of one of the axioms, an instance of a propositional tautology, or the result of the application of one of the rules (necessitation, modus ponens) on previous formulas.

(a) $K_a(p \to p)$

(b) $K_a p \to \hat{K}_a p$

(c) $K_a K_b p \to K_a p$