Dynamic Epistemic Logic

Chapter 4 - Action models

4.1 Motivation

So far. Only public announcements.
How can we model "private announcements", etc...?

Example 1 *a and b both don't know the value of proposition p. This is common knowledge among them. In fact p is true. Then a receives a letter containing the value of p and reads it. Agent b observes a reading the letter and knows that it is about p, but b does not learn the value of p.*

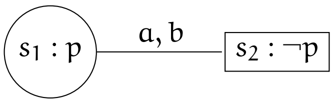

Figure 1: Before

Question: How to get from Before to After?
Answer: Action models.

$$After, s_1' | = K_a p \wedge (\neg K_b p \wedge \neg K_b \neg p) \wedge K_b(K_a p \vee K_a \neg p) \wedge K_a(\neg K_b p \wedge \neg K_b \neg p$$

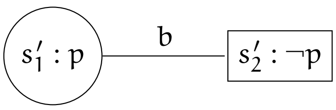

Figure 2: After

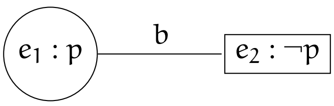

Figure 3: Action model Read

Then, $After = Before \bigotimes Read$
For an appropriate definition of $\bigotimes$:

Definition 1 $\otimes$ *is a restricted modal update with component worlds (s,e) only present if* $(M, s) \models \text{pre}(e)$

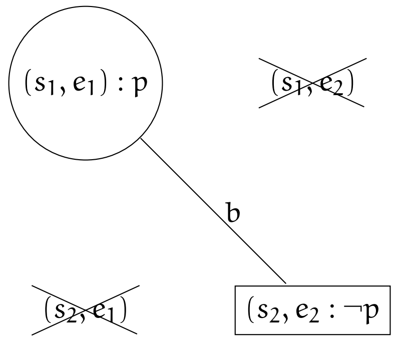

Figure 4: Before $\otimes$ Read

$(s_1, e_1) \sim_b (s_2, e_2)$ because $s_1 \sim_b s_2$ and $e_1 \sim_b e_2$.
(s_1, e_2) and (s_2, e_1) were eliminated because e_2 cannot be applies in s_1 and e_1 cannot be applied in s_2.

4.2 Action models

Definition 2 *Let $\mathcal{L}$ be any logical language for a set of agents A and a set of atoms P. Then an S5 action model m is a structure* $\langle S, \sim, \text{pre} \rangle$ *s.t.:*
S is the domain of action events,
$\forall a \in A$, *$\sim_a$ is an equivalence relation on S,*
pre: $S \to \mathcal{L}$ *is the precondition function that assigns a precondition* $\text{pre}(s) \in \mathcal{L}$ $\forall s \in S$

A pointed action model is such a structure (M,s) with $s \in S$

4.3 Syntax of action model logic

Definition 3 *Given A and P, the language of action model logic $\mathcal{L}_{KC\otimes}$ is the union of the formulas* $\phi \in \mathcal{L}^{stat}_{KC\otimes}(A, P)$ *and the actions* $\alpha \in \mathcal{L}^{act}_{KC\otimes}(A, P)$ *defined by:*
$\phi ::= p | \neg p | (\phi \wedge \phi) | K_a \phi | C_B \phi | [\alpha] \phi$
$\alpha ::= (M, s) | \alpha \vee \alpha$
where $p \in P$, $a \in A$, $B \subseteq A$ *and (M,s) is a pointed action model with:*
(1) finite domain S,
(2) such that $\forall t \in S$, *the precondition pre(t) is a* $\mathcal{L}^s tat_{KC\otimes}(A, P)$ *formula that has already been constructed in a previous step of the induction*

Example 2 *(Public announcements)*
Public announcements can be viewed as action models.

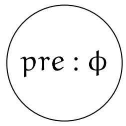

Figure 5: Action model for the public announcement of ϕ

$\langle\alpha\rangle := \neg[\alpha]\neg\phi$
$M = \bigcup_{s\in S}(M, s)$

Definition 4 *(Composition)*
Let $m = \langle S, \sim, pre\rangle$ *and* $m' = \langle S', \sim', pre'\rangle$ *be action models in* $\mathcal{L}^a ct_{KC\bigotimes}(A, P)$*. Then their* *composition* *(m;m') is the action model* $\langle S'', \sim'', pre''\rangle$ *s.t. :*
$S = S \times S'$
$(s, s') \sim''_a (t, t')$ *iff* $s \sim_a t$ *and* $s' \sim'_a t'$
$pre((s, s')) = \langle M, s\rangle pre'(s')$

For pointed action models: $((m, t); (m', t')) = ((m; m'), (t; t'))$

4.4 Semantics of action model logic

Definition 5 *Let* $m = \langle S, \sim, V\rangle$ *be an epistemic models and let* $m' = \langle S, \sim, pre\rangle$ *be an action model. Then the product update of* $m \bigotimes m'$ *is the model* $m'' = \langle S'', \sim'', V''\rangle$*, where:*
$S'' = \{(s, s') \in S \times S' | m, s \models pre(s')\}$
$(s, s') \sim''_a (t, t')$ *iff* $s \sim_a t$ *and* $s' \sim'_a t'$ *for* $a \in A$
$(s, s') \in V_p$ *iff* $s \in V_p$

Example 3 $(Before, S_1) \bigotimes (Read, e_1) = (After, (s_1, e_1))$

Definition 6 *(Semantics of formulas and actions)*
Let (m,s) be an epistemic state and $m' = \langle S', \sim', pre\rangle$ *an action model,* $\phi \in \mathcal{L}^{stat}_{KC\bigotimes}(A, P)$ *and* $\alpha \in \mathcal{L}^{act}_{KC\bigotimes}(A, P)$*. Then:*
$m, s \models \phi | \neg\phi | \phi \wedge \phi | K_a\phi | C_B\phi$ *are as usual*
$m, s \models [\alpha]\phi$ *iff* $\forall m'', s''$ *:* $(m, s)[\![\alpha]\!](m'', s'')$ *implies* $m'', s'' \models \phi$ *where*
$(m, s)[\![(m', s')]\!](m'', s'')$ *iff* $(m, s) \models pre(s')$ *and* $(m'', s'') = (m \bigotimes, m', (s, s'))$ *and* $[\![\alpha \cup \alpha']\!] = [\![\alpha]\!] \cup [\![\alpha']\!]$.
For $\alpha = (m', s')$ *:* $m, s \models [m', s']\phi$ *iff* $(m, s) \models pre(s')$ *implies* $(m \bigotimes m', (s, s')) \models \phi$

Remark: Compare to semantics of $[\phi]\psi$ and $\langle\phi\rangle\psi$ in PA.

Proposition: Let $(m, s), (m', s') \in \mathcal{L}^{act}_{KC\bigotimes}(A, P)$ and $\phi \in \mathcal{L}^{stat}_{KC\bigotimes}(A, P)$. Then: $[(m, s); (m', s')]\phi$ is equivalent to $[(m, s)][(m', s')]\phi$