Dynamic Epistemic Logic

Chapter 3 - Public announcements

3.8 Axiomatisation

3.8.1 PA (Without Common Knowledge)

(cont.)

Notes:
The axioms involving announcements were shown to be sound.
We cannot substitute freely propositional symbols in the atomic permanence axiom.

A potential inference rule could be "neccessitation of announcement"

Example 1 *Infer* $[\phi]\psi$.

This rule is sound in our system.
The rule is derivable with what we have.
The completeness prrof will be done by reduction EL.

3.8.2 Public Announcements with Common Knowledge (PAC)

Additional axioms ($B \subseteq A$):
$C_B(\phi \to \psi) \to (C_B\phi \to C_B\psi)$ (distribution of C_B over $\to$)
$C_B\phi \to (\phi \wedge E_B C_B\phi)$ (mix)
$C_B(\phi \to E_B\phi) \to (\phi \to C_B\phi)$ (induction of common knowledge)
Additional inference rules:
From ϕ infer $[\psi]\phi$ (neccessitation of announcements)
From ϕ infer $C_B\phi$ (nec. of common knowledge)
From $\chi \to [\phi]\psi$ and $\chi \wedge \phi \to E_B\chi$ infer $\chi \to [\phi]C_B\psi$ (mix of ann. and common knowledge)

Notes:
The additional rules and axioms were shown to be sound.
The induction axiom is derivable in PAC-induction axioms.

Example 2 $\vdash C_A\neg p$

1. $\neg p \to \neg(\neg p \to p)$ *taut.*
2. $\neg[\neg p]p \leftrightarrow \neg(\neg p \to p)$ *atomic permanence*
3. $\neg p \to \neg[\neg p]p$ *1,2,prop.*
4. $[\neg p]\neg p \leftrightarrow (\neg p \to \neg[\neg p]p)$ *ann.+neg.*
5. $[\neg p]\neg p$
6. $\top \to [\neg p]\neg p$ *5,prop.*
7. $\top$ *taut.*
8. $K_a\top$ *7,nec.*
9. $\top \wedge \neg p \to K_a\top$ *8,prop.*
10.$\top \wedge \neg p \to E_A\top$ *9,*$\forall a \in A$
11.$\top \to [\neg p]C_A\neg p$ *10,6,ann.+common knowledge*
12.$[\neg p]C_A\neg p$ *11,prop.*

Example 3 *(Muddy children)*
n children, some have a muddy forehead.
They only see whether the other children are muddy.
Perfect reasoners/logicians.
Father says:"At least one of you is muddy. Those of you who know whether they know they are muddy please raise your hand".

<u>Announcements:</u> Raising the hands or not.

For three children (a, b, c) a and b muddy, c not.
$m_a \wedge m_b \wedge \neg m_c$
$muddy = m_a \vee m_b \vee m_c$

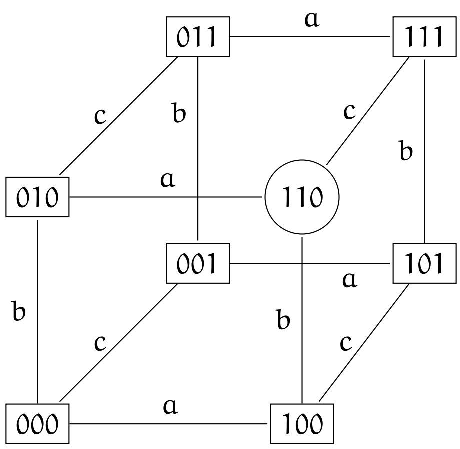

Figure 1: Cube

$\mathsf{Cube}, 100 \models E_{abc}\mathsf{muddy}$
$\mathsf{Cube}, 100 \not\models C_{abc}\mathsf{muddy}$ because $110 \sim_a 010 \sim_b 000$ and $\mathsf{Cube}, 000 \not\models \mathsf{muddy}$
Father's public announcement that muddy is true leads to a new model.

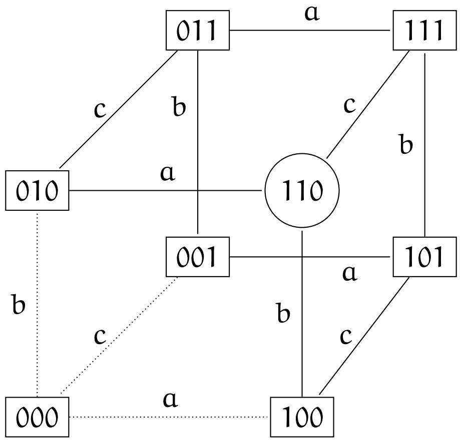

Figure 2: $\mathsf{Cube}|_{\mathsf{muddy}}$

$\mathsf{knowmuddy} = (K_a m_a \vee K_a\neg m_a) \vee (K_b m_b \vee K_b\neg m_b) \vee (K_c \vee K_c\neg m_c)$.
Nobody raises their hand which corresponds in a public announcement of $\neg\mathsf{knowmuddy}$.
$C|_{\mathsf{muddy}}, 110 \models \langle\mathsf{knowmuddy}\rangle\mathsf{knowmuddy}$ (This is a unsuccessful update).

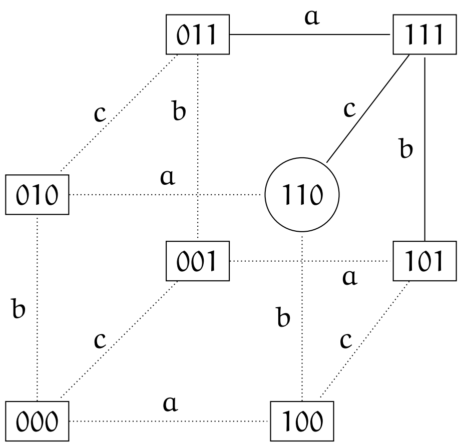

Figure 3: $\mathsf{Cube}|_{\mathsf{muddy}}|_{\neg\mathsf{knowmuddy}}$

Now after the father's next quesiton, a and b will raise their hands. This is a public announcement of the formula $\mathsf{abknowmuddy} = (K_a m_a \vee K_a\neg m_a) \wedge (K_b m_b \vee K_b\neg m_b)$ and leads to model $\mathsf{Cube}|_{\mathsf{muddy}}|_{\neg\mathsf{knowmuddy}}|_{\mathsf{abknowmuddy}} = \mathsf{Cube}'''$
$\mathsf{Cube}''', 110 \models \mathsf{knowmuddy}$