Dynamic Epistemic Logic

Chapter 3 - Public announcements

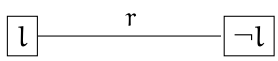
3.1 Introduction

Announcement := Public and truthful announcement

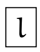
Example 1 *I say "The sun is shining".*
It is public knowledge afterwards

Example 2 $\neg K_r l \wedge l$
After the announcements: $\neg(\neg K_r l \wedge l)$ *is true*

<u>Intuition:</u>
How should epistemic models look like before and after?
Before:



After:Only those states survive where the announced formula is true.



Example 3 *Anne, Bill and Cath have drawn one card from a stack of three cards 0,1,2. Anne has drawn a 0, Bill has drawn a 1 and Cath 2. And now Anne says:" I do not have card 1"*

<u>Notation:</u> We write 0_a for the fact that Anne has card 0. In order to describe states we write three digits, e.g. 012 to describe the distribution.
The epistemic state resulting from the situation (Hexa, 012):
Hexa,012 $\models K_a \neg(K_b 0_a \vee K_b 1_a \vee K_b 2_a)$
Hexa,012 $\models 1_b \wedge \hat{K}_a 2_b$
If Anne says: "$\neg 1_a$", we get:

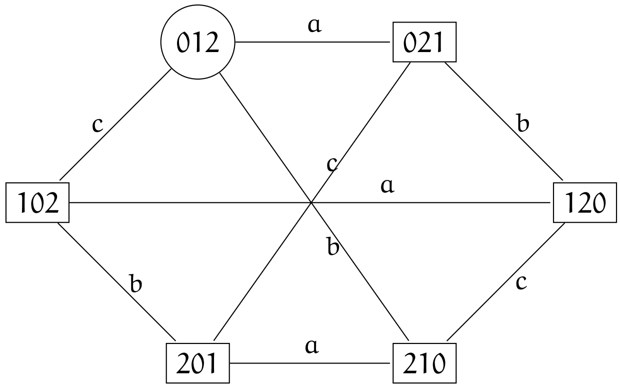


Figure 1: Hexa

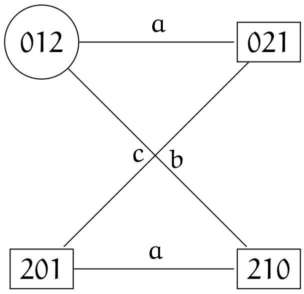


Figure 2: Hexa'

Now we have:
Hexa',012 $\models K_c 0_a \wedge \neg K_a K_c 0_a$, but
Hexa',012 $\models \neg(K_b 0_a \vee K_b 1_a \vee K_b 2_a)$
Now Bill states: $\neg(K_b 0_a \vee K_b 1_a \vee K_b 2_a)$.
Now assume Anne says: "I know Bill's card". $K_a 0_b \vee K_a 1_b \vee K_a 2_b$. Nothing changes in

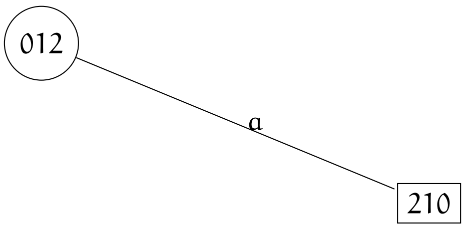


Figure 3: Hexa"

the model. Finally Anne says: $0_a \wedge 1_b \wedge 2_c$. Only one state remains.

3.2 Syntax of Announcements

Definition 1 *(Languages $\mathcal{L}_{K[]}$ and $\mathcal{L}_{KC[]}$)*
Given a finite set of agent names A and a countable set of atomic propositions p, the language $\mathcal{L}_{KC[]}$ is defined by:
$\phi ::= p|\neg\phi|\phi \vee \phi|K_a\phi|C_B\phi|[\phi]\phi$

The language $\mathcal{L}_{K[]}$ is the same without the C_B clause.
$[\phi]\psi$ reads " after a truthful announcement of ϕ it holds that ψ. $\langle\phi\rangle\psi$ is the dual to $[\phi]$ "after some announcement..."

Example 4 *After Anne announces $\neg 1_a$, Cath knows that 1_b.* $[\neg 1_a]K_c 1_b$

Example 5 *After the announcement of Bill that he does not know Anne's card, Anne knows Bill's card.* $[\neg 1_a][\neg(K_b(0_a \vee 1_a \vee 2_a))]K_a 1_b$

3.3 Semantics

Notation:
Let M=$\langle M, \sim, V\rangle$ be a model then $M|_\phi = \langle S', \sim', V'\rangle$ with
$s' = [\![\phi]\!]$
$\sim' = \sim \cap([\![\phi]\!] \times [\![\phi]\!])$
$V'(p) = V(p) \cap [\![\phi]\!]$

Definition 2 *(Semantics of announcement logic)*
Given an epistemic model for agents A and atoms P:
M,s $\models$ *p iff* $p \in V(p)$
.
.
.
$M, s \models [\phi]\psi$ *iff (*$M, s \models \phi$ *implies* $M|_\phi, s \models \psi$*)*

Note: $[\phi]\psi$ is true in a state if ϕ is not satisfied in this state!
The dual $\langle\phi\rangle\psi = \neg[\phi]\neg\psi$ has a truth condition $M, s \models \phi$ and $M|_\phi, s \models \psi$
The set of all valid announcement principles in $\mathcal{L}_{K[]}$ is denoted by PA. The set for $\mathcal{L}_{KC[]}$ is denoted by PAC.

3.4 Announcements and Revelations

z An announcement by an agent implies that the agent knows the fact. Actually one can use $[K_a\phi]$ instead of $[\phi]$. This can make a difference.
Revelation: Some outside agent (which can distinguish between every state) makes the statement.