

# Principles of AI Planning

## 19. Complexity of nondeterministic planning with full observability

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- Similar to the earlier analysis of deterministic planning, we will now study the computational complexity of nondeterministic planning with full observability.
- We consider the case of **strong planning**.
- The results for **strong cyclic planning** are identical.

As usual, the main **motivation** for such a study is to determine the **limit** of what is possible algorithmically: Should we try to develop a polynomial algorithm?



- The basic proof idea is very similar to the PSPACE-completeness proof for deterministic planning.
- The main difference is that we consider **alternating** Turing Machines (ATMs) instead of deterministic Turing Machines (DTMs) in the reduction.
- Due to the similarity to the earlier proof, we first review some of the concepts introduced in the earlier lecture.

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## Definition: Alternating Turing Machine

**Alternating Turing Machine (ATM)**  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$ :

- input alphabet**  $\Sigma$  and **blank symbol**  $\square \notin \Sigma$ 
  - alphabets always non-empty and finite
  - tape alphabet**  $\Sigma_{\square} = \Sigma \cup \{\square\}$
- finite set  $Q$  of **internal states** with **initial state**  $q_0 \in Q$
- state labeling  $l : Q \rightarrow \{Y, N, \exists, \forall\}$ 
  - accepting, rejecting, existential, universal** states  
 $Q_Y, Q_N, Q_{\exists}, Q_{\forall}$
  - terminal** states  $Q_{*} = Q_Y \cup Q_N$
  - nonterminal** states  $Q' = Q_{\exists} \cup Q_{\forall}$
- transition relation**  $\delta \subseteq (Q' \times \Sigma_{\square}) \times (Q \times \Sigma_{\square} \times \{-1, +1\})$

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Let  $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$  be an ATM.

## Definition: Configuration

A **configuration** of  $M$  is a triple  $(w, q, x) \in \Sigma_{\square}^* \times Q \times \Sigma_{\square}^+$ .

- $w$ : tape contents before tape head
- $q$ : current state
- $x$ : tape contents after and including tape head

Let  $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$  be an ATM.

## Definition: Yields relation

A configuration  $c$  of  $M$  **yields** a configuration  $c'$  of  $M$ , in symbols  $c \vdash c'$ , as defined by the following rules, where  $a, a', b \in \Sigma_{\square}$ ,  $w, x \in \Sigma_{\square}^*$ ,  $q, q' \in Q$  and  $((q, a), (q', a', \Delta)) \in \delta$ :

$$\begin{array}{ll} (w, q, ax) \vdash (wa', q', x) & \text{if } \Delta = +1, |x| \geq 1 \\ (w, q, a) \vdash (wa', q', \square) & \text{if } \Delta = +1 \\ (wb, q, ax) \vdash (w, q', ba'x) & \text{if } \Delta = -1 \\ (\varepsilon, q, ax) \vdash (\varepsilon, q', \square a'x) & \text{if } \Delta = -1 \end{array}$$

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Let  $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$  be an ATM.

## Definition: Acceptance (space)

Let  $c = (w, q, x)$  be a configuration of  $M$ .

- $M$  **accepts**  $c = (w, q, x)$  with  $q \in Q_Y$  **in space**  $n$  iff  $|w| + |x| \leq n$ .
- $M$  **accepts**  $c = (w, q, x)$  with  $q \in Q_{\exists}$  **in space**  $n$  iff  $M$  accepts some  $c'$  with  $c \vdash c'$  in space  $n$ .
- $M$  **accepts**  $c = (w, q, x)$  with  $q \in Q_{\forall}$  **in space**  $n$  iff  $M$  accepts all  $c'$  with  $c \vdash c'$  in space  $n$ .

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Let  $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$  be an ATM.

## Definition: Accepting words

$M$  **accepts the word**  $w \in \Sigma^*$  in space  $n \in \mathbb{N}_0$

iff  $M$  accepts  $(\varepsilon, q_0, w)$  in space  $n$ .

- Special case:  $M$  accepts  $\varepsilon$  in time (space)  $n \in \mathbb{N}_0$   
iff  $M$  accepts  $(\varepsilon, q_0, \square)$  in time (space)  $n$ .

## Definition: Accepting languages

Let  $f : \mathbb{N}_0 \rightarrow \mathbb{N}_0$ .

$M$  **accepts the language**  $L \subseteq \Sigma^*$  in space  $f$

iff  $M$  accepts each word  $w \in L$  in space  $f(|w|)$ ,  
and  $M$  does not accept any word  $w \notin L$ .

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## Definition: ASPACE, APSPACE

Let  $f : \mathbb{N}_0 \rightarrow \mathbb{N}_0$ .

Complexity class **ASPACE**( $f$ ) contains all languages accepted in space  $f$  by some ATM.

Let  $\mathcal{P}$  be the set of polynomials  $p : \mathbb{N}_0 \rightarrow \mathbb{N}_0$ .

$$\mathbf{APSPACE} := \bigcup_{p \in \mathcal{P}} \mathbf{ASPACE}(p)$$

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# Standard complexity classes relationships



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## Theorem

$$\begin{array}{l} P \subseteq NP \subseteq AP \\ PSPACE \subseteq NPSPACE \subseteq APSPACE \\ EXP \subseteq NEXP \subseteq AEXP \\ EXPSPACE \subseteq NEXPSPACE \subseteq AEXPSPACE \\ 2\text{-EXP} \subseteq \dots \end{array}$$



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## Theorem (Chandra et al. 1981)

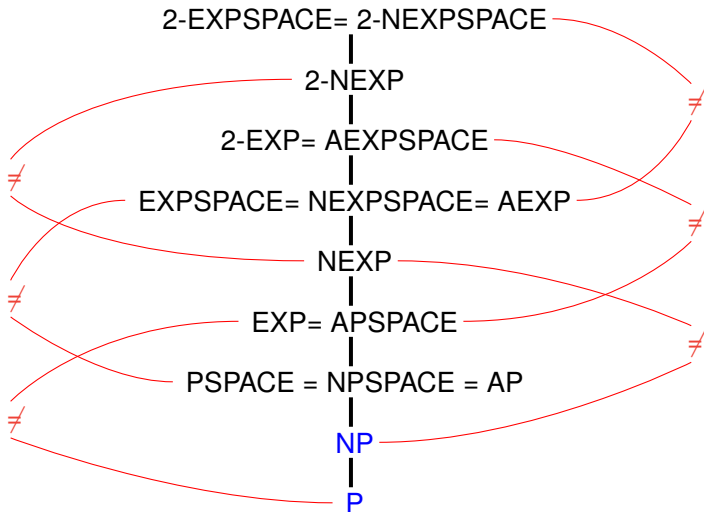
$$AP = PSPACE$$

$$APSPACE = EXP$$

$$AEXP = EXPSPACE$$

$$AEXPSPACE = 2-EXP$$

# The hierarchy of complexity classes



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# Complexity results

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## STRONGPLANEx (strong plan existence)

GIVEN: nondeterministic planning task  $\langle A, I, O, G, V \rangle$   
with full observability ( $A = V$ )

QUESTION: Is there a **strong plan** for the task?

- We do **not** consider a nondeterministic analog of the bounded plan existence problem (PLANLEN).





- We will prove that `STRONGPLANEx` is EXP-complete.
- We already know that the problem belongs to EXP, because we have presented a dynamic programming algorithm that generates strong plans in exponential time.
- We prove **hardness** for EXP by providing a **generic reduction** for **alternating Turing Machines with polynomial space** and use Chandra et al.'s theorem showing  $\text{APSPACE} = \text{EXP}$ .

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- For a fixed polynomial  $p$ , given ATM  $M$  and input  $w$ , generate planning task which is solvable by a strong plan iff  $M$  accepts  $w$  in space  $p(|w|)$ .
- For simplicity, restrict to ATMs which never move to the left of the initial head position (no loss of generality).
- **Existential states** of the ATM are modeled by states of the planning task where there are **several applicable operators** to choose from.
- **Universal states** of the ATM are modeled by states of the planning task where there is **a single applicable operator with a nondeterministic effect**.

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# Reduction: state variables



Let  $p$  be the space-bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ ,  
define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## State variables

- $\text{state}_q$  for all  $q \in Q$
- $\text{head}_i$  for all  $i \in X \cup \{0, p(n) + 1\}$
- $\text{content}_{i,a}$  for all  $i \in X, a \in \Sigma \cup \square$

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# Reduction: initial state



Let  $p$  be the space bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ ,  
define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## Initial state formula

Specify a **unique initial state**.

Initially true:

- state $_{q_0}$
- head $_1$
- content $_{i,w_i}$  for all  $i \in \{1, \dots, n\}$
- content $_{i,\square}$  for all  $i \in X \setminus \{1, \dots, n\}$

Initially false:

- all others

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# Reduction: goal



Let  $p$  be the space bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ ,  
define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## Goal

$\bigvee_{q \in Q_Y} \text{state}_q$

- Without loss of generality, we can assume that  $Q_Y$  is a singleton set so that we do not need a disjunctive goal.
- This way, the hardness result also holds for a restricted class of planning tasks (“nondeterministic STRIPS”).

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Let  $p$  be the space bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ , define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## Operators

For  $q, q' \in Q$ ,  $a, a' \in \Sigma_{\square}$ ,  $\Delta \in \{-1, +1\}$ ,  $i \in X$ , define

- $\text{pre}_{q,a,i} = \text{state}_q \wedge \text{head}_i \wedge \text{content}_{i,a}$
- $\text{eff}_{q,a,q',a',\Delta,i} = \neg \text{state}_q \wedge \neg \text{head}_i \wedge \neg \text{content}_{i,a} \wedge \text{state}_{q'} \wedge \text{head}_{i+\Delta} \wedge \text{content}_{i,a'}$ 
  - If  $q = q'$ , omit the effects  $\neg \text{state}_q$  and  $\text{state}_{q'}$ .
  - If  $a = a'$ , omit the effects  $\neg \text{content}_{i,a}$  and  $\text{content}_{i,a'}$ .

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# Reduction: operators (continued)



Let  $p$  be the space bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ ,  
define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## Operators (ctd.)

For **existential** states  $q \in Q_{\exists}$ ,  $a \in \Sigma_{\square}$ ,  $i \in X$ :

Let  $(q'_j, a'_j, \Delta_j)_{j \in \{1, \dots, k\}}$  be those triples with  
 $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$ .

For each  $j \in \{1, \dots, k\}$ , introduce one operator:

- precondition:  $\text{pre}_{q,a,i}$
- effect:  $\text{eff}_{q,a,q'_j,a'_j,\Delta_j,i}$

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# Reduction: operators (continued)



Let  $p$  be the space bound polynomial.

Given ATM  $\langle \Sigma, \square, Q, q_0, l, \delta \rangle$  and input  $w_1 \dots w_n$ ,  
define **relevant tape positions**  $X = \{1, \dots, p(n)\}$ .

## Operators (ctd.)

For **universal** states  $q \in Q_{\forall}$ ,  $a \in \Sigma_{\square}$ ,  $i \in X$ :

Let  $(q'_j, a'_j, \Delta_j)_{j \in \{1, \dots, k\}}$  be those triples with  
 $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$ .

Introduce only one operator:

- precondition:  $\text{pre}_{q,a,i}$
- effect:  $\text{eff}_{q,a,q'_1,a'_1,\Delta_1,i} \mid \dots \mid \text{eff}_{q,a,q'_k,a'_k,\Delta_k,i}$

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# EXP-completeness of strong planning with full observability



## Theorem (Rintanen)

**STRONGPLANEx** is EXP-complete.

*This is true even if we only allow operators in unary nondeterminism normal form where all deterministic sub-effects and the goal satisfy the STRIPS restriction and if we require a deterministic initial state.*

## Proof.

Membership in EXP has been shown by providing exponential-time algorithms that generate strong plans (and decide if one exists as a side effect).

Hardness follows from the previous generic reduction for ATMs with polynomial space bound and Chandra et al.'s theorem. □

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# EXP-completeness of strong planning with full observability



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- Nondeterministic planning is harder than deterministic planning.
- In particular, it is **EXP-complete** in the fully observable case, compared to the PSPACE-completeness of deterministic planning.
- The hardness result already holds if the operators and goals satisfy some fairly strong syntactic restrictions and there is a unique initial state.
- The introduction of nondeterministic effects corresponds to the introduction of **alternation** in Turing Machines.
- Later, we will see that **restricted observability** has an even more dramatic effect on the complexity of the planning problem.

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