

# Principles of AI Planning

## 14. Planning as search: Partial-Order Reduction

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# Motivation

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- **Worst case:** Heuristic search may explore **exponentially** more states than necessary, even if heuristic is **almost perfect** (Helmert and Röger, 2008).
- **Example:** A\* search in GRIPPER domain explores all permutations of ball transportations if heuristic is off only by a small constant.
- **Idea:** Complement heuristic search with **orthogonal technique(s)** to reduce size of explored state space.
- **Desired properties of this technique:** preservation of **completeness** and, if possible, **optimality**.

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## Idea:

- Enforce particular ordering among operators.
- Ignore all other orderings.

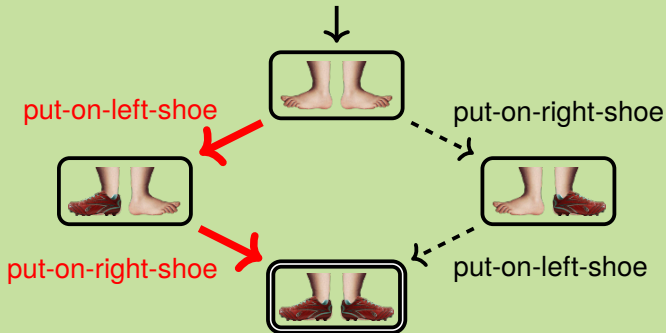
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## Example





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**Assumption:** For the rest of the chapter, we assume that all planning tasks are SAS<sup>+</sup> planning tasks  $\Pi = (V, I, O, \gamma)$ .

For convenience, we assume that operators have the form  $o = \langle pre(o), eff(o) \rangle$ , where  $pre(o)$  and  $eff(o)$  are both **partial states** over  $V$ , i.e., partial functions mapping variables  $v$  to values in  $\mathcal{D}_v$ . Similarly, we assume that  $\gamma$  is a partial state describing the goal.

## Example

Operator  $o = \langle pre(o), eff(o) \rangle$  with

- $pre(o) = \{v_1 \mapsto d_1, v_5 \mapsto d_5\}$  and
- $eff(o) = \{v_2 \mapsto d_2, v_3 \mapsto d_3\}$

corresponds to  $o = \langle \chi, e \rangle$  with

$\chi = (v_1 = d_1 \wedge v_5 = d_5)$  and  $e = (v_2 := d_2 \wedge v_3 := d_3)$ .

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## Definition (Operators)

Let  $\Pi = (V, I, O, \gamma)$  be a SAS<sup>+</sup> planning task and  $o \in O$  an operator. Then

- $prevars(o) := vars(pre(o))$  are the variables that occur in the precondition of  $o$ .
- $effvars(o) := vars(eff(o))$  are the variables that occur in the effect of  $o$ .
- $o$  **reads**  $v \in V$  iff  $v \in prevars(o)$ .
- $o$  **modifies**  $v \in V$  iff  $v \in effvars(o)$ .

Variable  $v \in V$  is **goal-related** iff  $v \in vars(\gamma)$ .

**Assumption:**  $effvars(o) \neq \emptyset$  for all  $o \in O$ .

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## Definition (Operator dependencies)

Let  $\Pi = \langle V, O, I, \gamma \rangle$  be a planning task and  $o, o' \in O$ .

- 1  $o$  **disables**  $o'$  iff there exists  $v \in \text{effvars}(o) \cap \text{prevars}(o')$  such that  $\text{eff}(o)(v) \neq \text{pre}(o')(v)$ .
- 2  $o$  **enables**  $o'$  iff there exists  $v \in \text{effvars}(o) \cap \text{prevars}(o')$  such that  $\text{eff}(o)(v) = \text{pre}(o')(v)$ .
- 3  $o$  and  $o'$  **conflict** iff there is  $v \in \text{effvars}(o) \cap \text{effvars}(o')$  such that  $\text{eff}(o)(v) \neq \text{eff}(o')(v)$ .
- 4  $o$  and  $o'$  **interfere** iff  $o$  disables  $o'$ , or  $o'$  disables  $o$ , or  $o$  and  $o'$  conflict.
- 5  $o$  and  $o'$  are **commutative** iff  $o$  and  $o'$  do not interfere, and neither  $o$  enables  $o'$ , nor  $o'$  enables  $o$ .

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$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$

$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$

$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$

$\text{go-to-gym} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{gym} \rangle$

Then:

- $\text{go-to-uni}$  and  $\text{go-to-gym}$  disable  $\text{put-on-left}$  and  $\text{put-on-right}$ .
- $\text{put-on-left}$  and  $\text{put-on-right}$  enable  $\text{go-to-uni}$  and  $\text{go-to-gym}$ .
- $\text{go-to-uni}$  and  $\text{go-to-gym}$  conflict.
- $\text{put-on-left}$  and  $\text{put-on-right}$  are commutative.

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## Definition (Necessary enabling set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task,  $s$  a state, and  $o \in O$  an operator that is not applicable in  $s$ . A set  $N$  of operators is a **necessary enabling set** (NES) for  $o$  in  $s$  if all operator sequences that lead from  $s$  to a goal state and include  $o$  contain an operator in  $N$  before the first occurrence of  $o$ .

**Note:** NESs not uniquely determined for given  $o$  and  $s$ .  
(E.g., supersets of NESs are still NESs.)

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# Necessary Enabling Sets and Disjunctive Action Landmarks



## Definition (Disjunctive action landmark)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A **disjunctive action landmark** (DAL)  $L$  in  $s$  is a set of operators such that all operator sequences that lead from  $s$  to a goal state contain some operator in  $L$ .

**Note:** This is really the same definition as in the chapter on LM-cut. Only the abbreviation “DAL” occurs here for the first time.

## Observation

For state  $s$  and operator  $o$  that is not applicable in  $s$ , disjunctive action landmarks for task  $\langle V, I, O, pre(o) \rangle$  are necessary enabling sets for  $o$  in  $s$ .

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# Necessary Enabling Sets and Disjunctive Action Landmarks



## Proof

Let  $L$  be such a disjunctive action landmark.

Then each operator sequence that leads from  $s$  to a state satisfying  $pre(o)$  contains some operator in  $L$ .

Thus, each operator sequence that leads from  $s$  to a goal state and includes  $o$  contains an operator in  $L$  before the first occurrence of  $o$ .

Therefore,  $L$  is an NES for  $o$  in  $s$ .

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## Back to the motivation:

If, in state  $s$ , some set of operators can be **applied in any order** and the order does not matter, we want to **commit to one such order** and **ignore all other orders**.

## Idea:

Identify operators that can be “postponed” since they are independent of all operators that are not “postponed”.

E.g., put-on-right could be postponed, since it is independent of put-on-left (that is not postponed).

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**Idea (more precisely):** Identify operators that **should not** be postponed, and postpone the rest.

**Question:** When should we consider an operator  $o$  **too important to be tentatively ignored?**

**Answer:**

- 1 Base case:** If  $o$  may be immediately relevant to reaching (part of) the goal, or
- 2 Inductive case I:** If  $o$  may be immediately relevant to contributing to making an operator applicable that we consider too important to be tentatively ignored, or
- 3 Inductive case II:** If  $o$  might not be applicable any more if we postponed it, or if its effect might conflict with the effect of an operator that we considered too important to be tentatively ignored ( $\approx o$  interferes with such an operator).

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Let's formalize the above answer:

## Definition (Strong stubborn set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_s \subseteq O$  is a **strong stubborn set in  $s$**  if

- 1  $T_s$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_s$  that are not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $o$  and  $s$ , and
- 3 for all  $o \in T_s$  that are applicable in  $s$ ,  $T_s$  contains all operators that interfere with  $o$ .

Instead of applying all applicable operators in  $s$  only apply those that are applicable and contained in  $T_s$ .

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## Example

$$I = \{\text{pos} \mapsto \text{home}, \text{left} \mapsto \text{f}, \text{right} \mapsto \text{f}\}, \quad \gamma = \{\text{pos} \mapsto \text{uni}\}$$
$$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$$
$$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$$
$$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$$

- Step 1: DAL in  $I$  is  $\{\text{go-to-uni}\} \rightsquigarrow T_S := \{\text{go-to-uni}\}$ .
- Step 2:  $\text{go-to-uni}$  not applicable in  $I$ . One possible NES for  $\text{go-to-uni}$  in  $I$  is  $\{\text{put-on-left}\} \rightsquigarrow T_S := T_S \cup \{\text{put-on-left}\}$ .
- Step 3:  $\text{put-on-left}$  is applicable in  $I$ . The only operator that interferes with it,  $\text{go-to-uni}$ , is already in  $T_S$ .
- Hence,  $T_S = \{\text{go-to-uni}, \text{put-on-left}\}$ , and  $T_S$  restricted to the applicable operators is  $\{\text{put-on-left}\}$ . During search, only apply  $\text{put-on-left}$  (not  $\text{put-on-right}$ ).

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## Example

Let  $V = \{u_1, u_2, v, w\}$ ,  $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$ ,  
 $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$ , and  $O = \{o_1, o_2, o_3\}$ , where:

- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$ ,
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$ ,
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$ .

Strong stubborn set:

- Step 1: Include  $o_1$  (or  $o_2$ ) in  $T_s$  as DAL.
- Step 2: Include  $o_3$  in  $T_s$  since it interferes with  $o_1$  (or  $o_2$ ).
- Step 3: Include  $o_2$  (or  $o_1$ ) in  $T_s$  since it interferes with  $o_3$ .

$\rightsquigarrow$  all applicable operators included in  $T_s$ , no pruning.

**Question:** Can we do better than that in this example?

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## Definition (Domain transition graph)

Let  $\Pi = (V, I, O, \gamma)$  be a SAS<sup>+</sup> planning task and  $v \in V$ . The

**domain transition graph** for  $v$  is the directed graph

$DTG(v) = \langle \mathcal{D}_v, E \rangle$  where  $(d, d') \in E$  iff there is an operator

$o \in O$  with

- $eff(o)(v) = d'$ , and
- $v \notin prevars(o)$  or  $pre(o)(v) = d$ .

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## Example

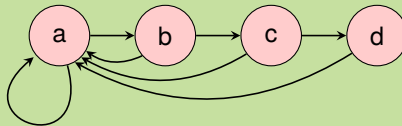
move-a-b =  $\langle \text{pos} = \text{a}, \text{pos} := \text{b} \rangle$

move-b-c =  $\langle \text{pos} = \text{b}, \text{pos} := \text{c} \rangle$

move-c-d =  $\langle \text{pos} = \text{c}, \text{pos} := \text{d} \rangle$

reset =  $\langle \top, \text{pos} := \text{a} \wedge \text{othersvar} := \text{othersvar} \rangle$

Then  $DTG(\text{pos})$ :



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## Definition (Active operators)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and let  $s$  be a state. The set of **active operators**  $Act(s) \subseteq O$  in  $s$  is defined as the set of operators such that for all  $o \in Act(s)$ :

- For every variable  $v \in prevars(o)$ , there is a path in  $DTG(v)$  from  $s(v)$  to  $pre(o)(v)$ . If  $v$  is goal-related, then there is also a path from  $pre(o)(v)$  to the goal value  $\gamma(v)$ .
- For every goal-related variable  $v \in effvars(o)$ , there is a path in  $DTG(v)$  from  $eff(o)(v)$  to the goal value  $\gamma(v)$ .

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## Proposition

- 1  $Act(s)$  can be identified efficiently for a given state  $s$  by considering paths in the projection of  $\Pi$  onto  $v$ .
- 2 Operators not in  $Act(s)$  can be treated as nonexistent when reasoning about  $s$  because they are not applicable in all states reachable from  $s$ , or they lead to a dead-end from  $s$ .

## Proof

- 1 Homework: Specify efficient algorithm for identification of  $Act(s)$ .
- 2 Obvious. □

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**Remark 1:** Even when excluding **inactive** operators, this preserves completeness and even optimality of a search algorithm (see proof below).

**Remark 2:** Excluding **inactive** operators can “cascade” in the sense that additional **active** operators need not be considered.

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## Definition (Strong stubborn set with active operator pruning)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_s \subseteq O$  is a **strong stubborn set in  $s$**  if

- 1  $T_s$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_s$  that are not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $o$  and  $s$ , and
- 3 for all  $o \in T_s$  that are applicable in  $s$ ,  $T_s$  contains all operators that **are active in  $s$  and** interfere with  $o$ .

Instead of applying all applicable operators in  $s$  only apply those that are applicable and contained in  $T_s$ .

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Recall the previous example where strong stubborn sets without active operator pruning were useless.

### Example

- $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$ ,  
 $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$
- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$

Now, **with** active operator pruning:

- Step 1: Include  $o_1$  (or  $o_2$ ) in  $T_s$  as DAL.
- Step 2: Operator  $o_3$  is not active in any reachable state.  
 $\rightsquigarrow o_3$  not in  $T_s$ , although it interferes with  $o_1$  (or  $o_2$ ).

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### Example (Example, ctd.)

Now, **with** active operator pruning:

- Step 1: Include  $o_1$  (or  $o_2$ ) in  $T_s$  as DAL.
- Step 2: Operator  $o_3$  is not active in any reachable state.  
 $\rightsquigarrow o_3$  not in  $T_s$ , although it interferes with  $o_1$  (or  $o_2$ ).
- Hence, e. g.,  $T_s = \{o_1\}$  strong stubborn set (with active operator pruning) in  $I$ .
- Even **active** operator  $o_2$  is not included in  $T_s = \{o_1\}$ .

$\rightsquigarrow$  some pruning occurs.

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With **weak** stubborn sets, some operators that disable an operator in  $T_s$  need not be included in  $T_s$ .

Therefore, weak stubborn sets potentially allow more pruning than strong stubborn sets.

## Definition (Weak stubborn set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_s \subseteq O$  is a **weak stubborn set in  $s$**  if

- 1  $T_s$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_s$  that are not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $o$  and  $s$ , and
- 3 for all  $o \in T_s$  that are applicable in  $s$ ,  $T_s$  contains the active operators in  $s$  that have conflicting effects with  $o$  or that are disabled by  $o$ .

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For weak stubborn sets, it suffices to include active operators  $o'$  that **are disabled** or **conflict** with applicable operators  $o \in T_S$ . However,  $o'$  **does not need to be included** if  $o'$  **disables** an applicable operator  $o \in T_S$ .

**No computational overhead** of computing weak stubborn sets over computing strong stubborn sets.

## Theorem

In the best case, weak stubborn sets admit **exponentially more pruning** than strong stubborn sets.

## Proof

Homework. □

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**compute-DAL:** Compute a disjunctive action landmark.

## Procedure compute-DAL

```
def compute-DAL( $\gamma$ ):  
  select  $v \in vars(\gamma)$  with  $s(v) \neq \gamma(v)$   
   $L \leftarrow \{o' \in Act(s) \mid eff(o')(v) = \gamma(v)\}$   
  return  $L$ 
```

Selection of  $v \in vars(\gamma)$  arbitrary. Any variable will do.  
Selection heuristics?

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**compute-NES:** Compute a necessary enabling set.

## Procedure compute-NES

**def** compute-NES( $o, s$ ):

  select  $v \in \text{prevars}(o)$  with  $s(v) \neq \text{pre}(o)(v)$

$N \leftarrow \{o' \in \text{Act}(s) \mid \text{eff}(o')(v) = \text{pre}(o)(v)\}$

**return**  $N$

Selection of  $v \in \text{prevars}(o)$  arbitrary. Any variable will do.

Selection heuristics?

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**compute-interfering-operators:** Compute interfering operators.

## Precedure compute-interfering-operators (for strong SS)

```
def compute-interfering-operators(o):  
  disablers ← {o' ∈ O | o' disables o}  
  disablees ← {o' ∈ O | o disables o'}  
  conflicting ← {o' ∈ O | o and o' conflict}  
  return disablers ∪ disablees ∪ conflicting
```

## Precedure compute-interfering-operators (for weak SS)

```
def compute-interfering-operators(o):  
  disablees ← {o' ∈ O | o disables o'}  
  conflicting ← {o' ∈ O | o and o' conflict}  
  return disablees ∪ conflicting
```

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Computing (strong and weak) stubborn sets for planning can be achieved with a **fixpoint iteration** until the constraints of  $T_s$  are satisfied:

**compute-stubborn-set**: Compute (strong or weak) stubborn set.

## Precedure compute-stubborn-set

**def** compute-stubborn-set( $s$ ):

$T_s \leftarrow \text{compute-DAL}(\gamma)$

**while** no fixed-point of  $T_s$  reached **do**

for  $o \in T_s$  applicable in  $s$ :

$T_s \leftarrow T_s \cup \text{compute-interfering-operators}(o)$

for  $o \in T_s$  not applicable in  $s$ :

$T_s \leftarrow T_s \cup \text{compute-NES}(o, s)$

**end while**

**return**  $T_s$

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**Observation:** stubborn sets are state-dependent, but not path-dependent.

This allows filtering the applicable operators in  $s$  in graph search algorithms like A\* that perform duplicate detection, too.

Instead of applying all applicable operators  $app(s)$  in  $s$ , only apply operators in  $T_{app(s)} := T_s \cap app(s)$ .

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## Theorem

*Weak stubborn sets are completeness and optimality preserving.*

## Proof

Let  $T_{app(s)} := T_s \cap app(s)$  for a weak stubborn set  $T_s$ .

We show that for all states  $s$  from which an optimal plan consisting of  $n > 0$  operators exists,  $T_{app(s)}$  contains an operator that starts such a plan.

We show by induction that  $A^*$  restricting successor generation to  $T_{app(s)}$  is optimal.

Let  $T_s$  be a weak stubborn set and  $\pi = o_1, \dots, o_n$  be an optimal plan that starts in  $s$ .

...

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## Proof (ctd.)

As  $T_S$  contains a disjunctive action landmark,  $\pi$  must contain an operator from  $T_S$ .

Let  $o_k$  be the operator with smallest index in  $\pi$  that is also contained in  $T_S$ , i.e.,  $o_k \in T_S$  and  $\{o_1, \dots, o_{k-1}\} \cap T_S = \emptyset$ .

We observe:

1.  $o_k \in \text{app}(s)$ : otherwise by definition of weak stubborn sets, a necessary enabling set  $N$  for  $o_k$  in  $s$  would have to be contained in  $T_S$ , and at least one operator from  $N$  would have to occur before  $o_k$  in  $\pi$  to enable  $o_k$ , contradicting that  $o_k$  was chosen with smallest index.
2. ...

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## Proof (ctd.)

1. ...
2.  $o_k$  does not disable any of the operators  $o_1, \dots, o_{k-1}$ , and all these operators have non-conflicting effects with  $o_k$ : otherwise, as  $o_k \in \text{app}(s)$ , and by definition of weak stubborn sets, at least one of  $o_1, \dots, o_{k-1}$  would have to be contained in  $T_s$ , again contradicting the assumption.

Hence, we can move  $o_k$  to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$  is also a plan for  $\Pi$ .

It has the same cost as  $\pi$  and is hence optimal.

Thus, we have found an optimal plan of length  $n$  started by an operator  $o_k \in T_{\text{app}(s)}$ , completing the proof.  $\square$

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**Remark:** The argument to move  $o_k$  to the front also holds for strong stubborn sets: in this case,  $o_k$  is not even disabled by any of  $o_1, \dots, o_{k-1}$  (and hence,  $o_k$  is independent of  $o_1, \dots, o_{k-1}$ ), which is a stronger property than needed in the proof.

## Corollary

*Strong stubborn sets are completeness and optimality preserving.*



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# Some Experiments: Overview

Optimal Planning, A\* with LM-cut Heuristic, Selected Domains



Domain (problems)	Coverage		Nodes generated	
	A*	+SSS	A*	+SSS
PARCPRINTER-08 (30)	18	+12	2455181	<1%
PARCPRINTER-OPT11 (20)	13	+7	2454533	<1%
WOODWORKING-OPT08 (30)	17	+10	26796212	<1%
WOODWORKING-OPT11 (20)	12	+7	26795517	<1%
SATELLITE (36)	7	+5	5116312	2%
ROVERS (40)	7	+2	1900691	22%
AIRPORT (50)	28	±0	545072	93%
OPENSTACKS-OPT08 (30)	19	+2	56584063	51%
OPENSTACKS-OPT11 (20)	14	+2	56456969	51%
DRIVERLOG (20)	13	+1	3679376	82%
SCANALYZER-08 (30)	15	-3	14203012	100%
SCANALYZER-OPT11 (20)	12	-3	14202884	100%
PARKING-OPT11 (20)	3	-1	560914	100%
SOKOBAN-OPT08 (30)	30	-1	20519270	100%
VISITALL-OPT11 (20)	11	-1	1991169	100%
REMAINING DOMAINS (980)	544	±0	436017004	93%
SUM (1396)	763	+39	670278179	77%

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## Weak compared to strong stubborn sets



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Domain (problems)	Coverage		Nodes generated		# problems w. diff. gen.
	SSS	WSS	SSS	WSS	
OPENSTACKS-OPT08 (30)	<b>21</b>	$\pm 0$	152711917	<b>99.936%</b>	18
OPENSTACKS-OPT11 (20)	<b>16</b>	$\pm 0$	152642101	<b>99.936%</b>	16
PATHWAYS-NONEG (30)	<b>5</b>	$\pm 0$	162347	<b>99.702%</b>	2
PSR-SMALL (50)	<b>49</b>	$\pm 0$	18119489	<b>99.998%</b>	6
SATELLITE (36)	<b>12</b>	$\pm 0$	70299721	<b>92.804%</b>	12

⇒ In practice (or, at least, in the standard benchmark problems) there is no significant difference between weak and strong stubborn sets.



# Conclusion

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- Need for **techniques orthogonal to heuristic search**, complementing heuristics.
- One idea: **Commit to one order of operators** if they are independent. Prune other orders.
- Class of such techniques: **partial-order reduction** (POR)
- One such technique: **strong/weak stubborn sets**
- Can lead to **substantial pruning** compared to plain  $A^*$ .
- Many other POR techniques exist.
- Other pruning techniques exist as well, e.g., symmetry reduction.

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