Principles of AI Planning

6. Planning as search: search algorithms

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Introduction to search algorithms for planning

Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   ⇝ previous chapter
2. search space representation: states/sets of states
   ⇝ previous chapter
3. search algorithm: uninformed/heuristic; systematic/local
   ⇝ this chapter
4. search control: heuristics, pruning techniques
   ⇝ next chapters

Search

- Search algorithms are used to find solutions (plans) for transition systems in general, not just for planning tasks.
- Planning is one application of search among many.
- In this chapter, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.
- Most of this is review of material that should be known (details: Russell and Norvig's textbook).

Search states vs. search nodes

In search, one distinguishes:

- search states s ⇝ states (vertices) of the transition system
- search nodes σ ⇝ search states plus information on where/when/how they are encountered during search

What is in a search node?

Different search algorithms store different information in a search node σ, but typical information includes:

- $\mathit{state}(\sigma)$: associated search state
- $\mathit{parent}(\sigma)$: pointer to search node from which σ is reached
- $\mathit{action}(\sigma)$: action leading from $\mathit{state}(\mathit{parent}(\sigma))$ to $\mathit{state}(\sigma)$
- $g(\sigma)$: cost of σ (length of path from the root node)

For the root node, $\mathit{parent}(\sigma)$ and $\mathit{action}(\sigma)$ are undefined.

Search states vs. planning states

Search states $\neq$ (planning) states:

- Search states don't have to correspond to states in the planning sense.
    - progression: search states $\approx$ (planning) states
    - regression: search states $\approx$ sets of states (formulae)
- Search algorithms for planning where search states are planning states are called state-space search algorithms.
- Strictly speaking, regression is not an example of state-space search, although the term is often used loosely.
- However, we will put the emphasis on progression, which is almost always state-space search.

Required ingredients for search

A general search algorithm can be applied to any transition system for which we can define the following three operations:

- init(): generate the initial state
- is-goal(s): test if a given state is a goal state
- succ(s): generate the set of successor states of state s, along with the operators through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a search space (a very similar notion to a transition system).

Search for planning: progression

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for progression search

states: all states of Π (assignments to A)

- init() = I
- is-goal(s) = $\begin{cases} \textbf{true} & \text{if } s \models \gamma \\ \textbf{false} & \text{otherwise} \end{cases}$
- succ(s) = $\{\langle o, s' \rangle \mid \text{applicable } o \in O, s' = app_o(s)\}$

Search for planning: regression

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for regression search

states: all formulae over A (how many?)

- init() = γ
- is-goal(φ) = $\begin{cases} \textbf{true} & \text{if } I \models \varphi \\ \textbf{false} & \text{otherwise} \end{cases}$
- succ(φ) = $\{\langle o, \varphi' \rangle \mid o \in O, \varphi' = regr_o(\varphi), \varphi' \text{ is satisfiable}\}$ (modified if splitting is used)

Classification of search algorithms

uninformed search vs. heuristic search:

- uninformed search algorithms only use the basic ingredients for general search algorithms
- heuristic search algorithms additionally use heuristic functions which estimate how close a node is to the goal

systematic search vs. local search:

- systematic algorithms consider a large number of search nodes simultaneously
- local search algorithms work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are crossbreeds (e. g., enforced hill-climbing)

Classification: what works where in planning?

uninformed vs. heuristic search:

- For satisficing planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For optimal planning, the difference is less pronounced.

systematic search vs. local search:

- For satisficing planning, the most successful algorithms are somewhere between the two extremes.
- For optimal planning, systematic algorithms are required.

Common procedures for search algorithms

Before we describe the different search algorithms, we introduce three procedures used by all of them:

- make-root-node: Create a search node without parent.
- make-node: Create a search node for a state generated as the successor of another state.
- extract-solution: Extract a solution from a search node representing a goal state.

Procedure make-root-node

make-root-node: Create a search node without parent.

Procedure make-root-node

```
def make-root-node($s$):
    $\sigma$ := new node
    $state(\sigma)$ := $s$
    $parent(\sigma)$ := undefined
    $action(\sigma)$ := undefined
    $g(\sigma)$ := 0
    return $\sigma$
```

Procedure make-node

make-node: Create a search node for a state generated as the successor of another state.

Procedure make-node

```
def make-node($\sigma$, $o$, $s$):
    $\sigma'$ := new node
    $state(\sigma') := s$
    $parent(\sigma') := \sigma$
    $action(\sigma') := o$
    $g(\sigma') := g(\sigma) + 1$
    return $\sigma'$
```

Procedure extract-solution

extract-solution: Extract a solution from a search node representing a goal state.

Procedure extract-solution

```
def extract-solution($\sigma$):
    solution := new list
    while parent($\sigma$) is defined:
        solution.push-front(action($\sigma$))
        $\sigma$ := parent($\sigma$)
    return solution
```

Uninformed search algorithms

Uninformed search algorithms

- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

Popular uninformed systematic search algorithms:

- breadth-first search
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:

- random walk

Breadth-first search without duplicate detection

Breadth-first search

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
while not queue.empty():
    $\sigma$ = queue.pop-front()
    if is-goal(state($\sigma$)):
        return extract-solution($\sigma$)
    for each $\langle o, s \rangle \in$ succ(state($\sigma$)):
        $\sigma'$ := make-node($\sigma, o, s$)
        queue.push-back($\sigma'$)
return unsolvable
```

- Possible improvement: duplicate detection (see next slide).
- Another possible improvement: test if σ' is a goal node; if so, terminate immediately. (We don't do this because it obscures the similarity to some of the later algorithms.)

Breadth-first search with duplicate detection

Breadth-first search with duplicate detection

```
$queue$ := new fifo-queue
$queue$.push-back(make-root-node(init()))
$closed := \emptyset$
while not $queue$.empty():
    $\sigma$ = $queue$.pop-front()
    if $state(\sigma) \notin closed$:
        $closed := closed \cup \{state(\sigma)\}$
        if is-goal(state($\sigma$)):
            return extract-solution($\sigma$)
        for each $\langle o, s \rangle \in$ succ($state(\sigma)$):
            $\sigma'$ := make-node($\sigma, o, s$)
            $queue$.push-back($\sigma'$)
return unsolvable
```

Breadth-first search with duplicate detection

Breadth-first search with duplicate detection

```
$queue$ := **new** fifo-queue
$queue$.push-back(make-root-node(init()))
$closed$ := $\emptyset$
**while not** $queue$.empty():
    $\sigma$ = $queue$.pop-front()
    **if** $state(\sigma) \notin closed$:
        $closed := closed \cup \{state(\sigma)\}$
        **if** is-goal(state($\sigma$)):
            **return** extract-solution($\sigma$)
        **for each** $\langle o, s \rangle \in$ succ($state(\sigma)$):
            $\sigma'$ := make-node($\sigma, o, s$)
            $queue$.push-back($\sigma'$)
**return** unsolvable
```

Random walk

Random walk

```
$\sigma$ := make-root-node(init())
forever:
    if is-goal(state($\sigma$)):
        return extract-solution($\sigma$)
    Choose a random element $\langle o, s \rangle$ from succ(state($\sigma$)).
    $\sigma$ := make-node($\sigma, o, s$)
```

- The algorithm usually does not find any solutions, unless almost every sequence of actions is a plan.
- Often, it runs indefinitely without making progress.
- It can also fail by reaching a dead end, a state with no successors. This is a weakness of many local search approaches.

Heuristic search algorithms

Heuristic search algorithms: systematic

- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular systematic heuristic search algorithms:

- greedy best-first search
- A*
- weighted A*
- IDA*
- depth-first branch-and-bound search
- …

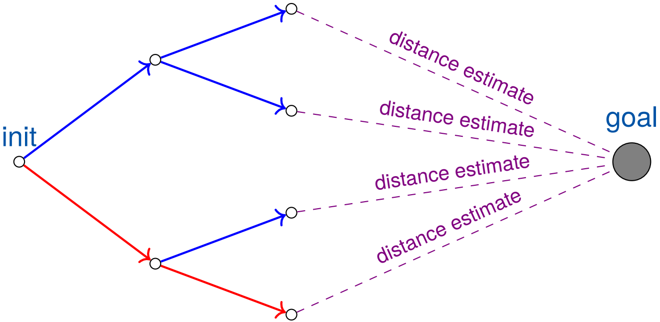
Heuristic search algorithms: local

- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic local search algorithms:

- hill-climbing
- enforced hill-climbing
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- . . .

Heuristic search: idea



Required ingredients for heuristic search

A heuristic search algorithm requires one more operation in addition to the definition of a search space.

Definition (heuristic function)

Let Σ be the set of nodes of a given search space.
A heuristic function or heuristic (for that search space) is a function $h : \Sigma \to \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the heuristic estimate or heuristic value of heuristic h for node σ. It is supposed to estimate the distance from σ to the nearest goal node.

What exactly is a heuristic estimate?

What does it mean that *h* “estimates the goal distance”?

- For most heuristic search algorithms, *h* does not need to have any strong properties for the algorithm to work (= be correct and complete).
- However, the efficiency of the algorithm closely relates to how accurately *h* reflects the actual goal distance.
- For some algorithms, like A*, we can prove strong formal relationships between properties of *h* and properties of the algorithm (optimality, dominance, run-time for bounded error, . . .)
- For other search algorithms, “it works well in practice” is often as good an analysis as one gets.

Heuristics applied to nodes or states?

- Most texts apply heuristic functions to states, not nodes.
- This is slightly less general than our definition:
  - Given a state heuristic h, we can define an equivalent node heuristic as $h'(\sigma) := h(state(\sigma))$.
  - The opposite is not possible. (Why not?)
- There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on how we ended up in a given state s?
- We call heuristics which don't just depend on $state(\sigma)$ pseudo-heuristics.
- In practice there are sometimes good reasons to have the heuristic value depend on the generating path of σ (e. g., landmark pseudo-heuristic, Richter et al. 2008).

Perfect heuristic

Let Σ be the set of nodes of a given search space.

Definition (optimal/perfect heuristic)

The optimal or perfect heuristic of a search space is the heuristic h^* which maps each search node σ to the length of a shortest path from $\mathit{state}(\sigma)$ to any goal state.

Note: $h^*(\sigma) = \infty$ iff no goal state is reachable from σ.

Properties of heuristics

A heuristic h is called

- safe if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- goal-aware if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- admissible if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- consistent if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that σ' is a successor of σ.[1]

Relationships?

---

[1] or: $h(\sigma) \leq h(\sigma') + cost(\sigma, \sigma')$ for non-unit costs, where $cost(\sigma, \sigma')$ is the cost of the transition from σ to σ'.

Greedy best-first search

Greedy best-first search (with duplicate detection)

```
$open$ := new min-heap ordered by $(\sigma \mapsto h(\sigma))$
$open$.insert(make-root-node(init()))
$closed := \emptyset$
while not $open$.empty():
    $\sigma$ = $open$.pop-min()
    if $state(\sigma) \notin closed$:
        $closed := closed \cup \{state(\sigma)\}$
        if is-goal(state($\sigma$)):
            return extract-solution($\sigma$)
        for each $\langle o, s \rangle \in$ succ($state(\sigma)$):
            $\sigma'$ := make-node($\sigma, o, s$)
            if $h(\sigma') < \infty$:
                $open$.insert($\sigma'$)
return unsolvable
```

Properties of greedy best-first search

- one of the three most commonly used algorithms for satisficing planning
- complete for safe heuristics (due to duplicate detection)
- suboptimal unless h satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of h (e. g., scaling with a positive constant or adding a constant)

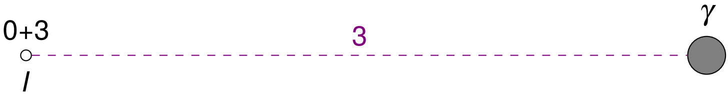
A^*

A^* (with duplicate detection and reopening)

```
$open$ := **new** min-heap ordered by $(\sigma \mapsto g(\sigma) + h(\sigma))$
$open$.insert(make-root-node(init()))
$closed := \emptyset$
$distance := \emptyset$
while not $open$.empty():
    $\sigma$ = $open$.pop-min()
    if $state(\sigma) \notin closed$ or $g(\sigma) < distance(state(\sigma))$:
        $closed := closed \cup \{state(\sigma)\}$
        $distance(state(\sigma)) := g(\sigma)$
        if is-goal(state($\sigma$)):
            return extract-solution($\sigma$)
        for each $\langle o, s\rangle \in$ succ($state(\sigma)$):
            $\sigma'$ := make-node($\sigma, o, s$)
            if $h(\sigma') < \infty$: $open$.insert($\sigma'$)
return unsolvable
```

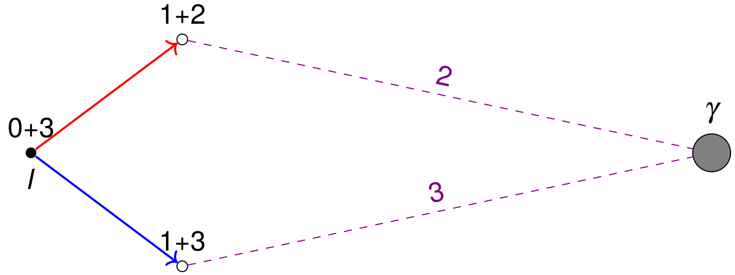
A* example

Example



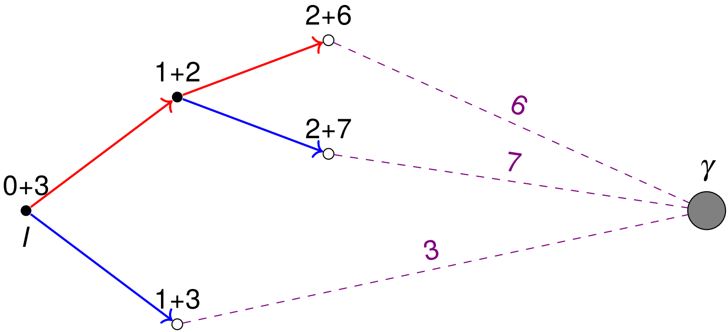
A* example

Example



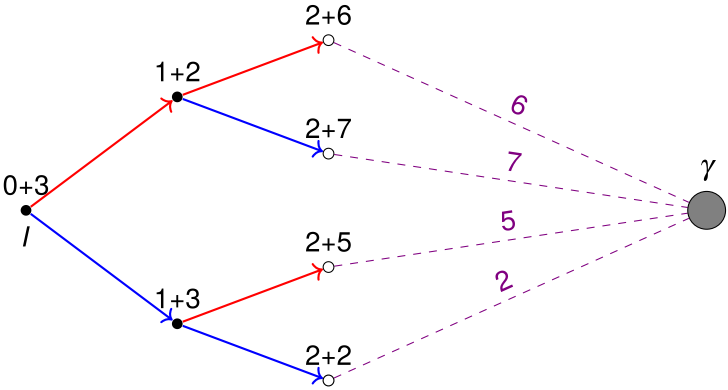
A* example

Example



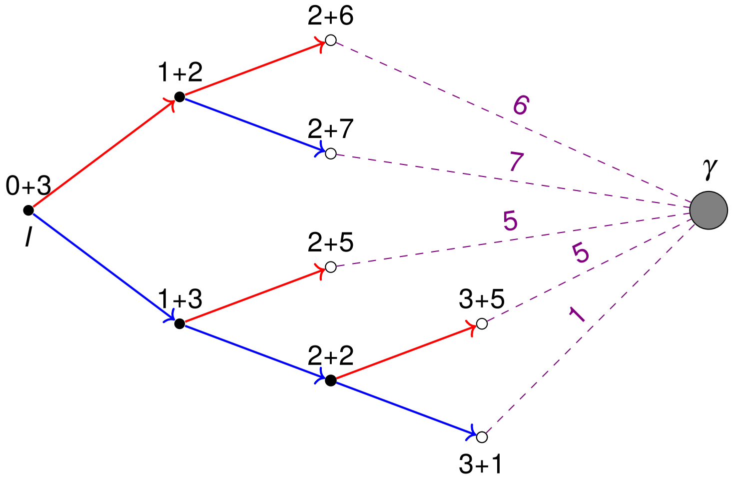
A^* example

Example



A* example

Example



Terminology for A*

- f value of a node: defined by $f(\sigma) := g(\sigma) + h(\sigma)$
- generated nodes: nodes inserted into *open* at some point
- expanded nodes: nodes σ popped from *open* for which the test against *closed* and *distance* succeeds
- reexpanded nodes: expanded nodes for which $\mathit{state}(\sigma) \in \mathit{closed}$ upon expansion (also called reopened nodes)

Properties of A*

- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- complete for safe heuristics (even without duplicate detection)
- optimal if *h* is admissible (even without duplicate detection)
- never reopens nodes if *h* is consistent

Implementation notes:

- in the heap-ordering procedure, it is considered a good idea to break ties in favour of lower *h* values
- can simplify algorithm if we know that we only have to deal with consistent heuristics
- common, hard to spot bug: test membership in *closed* at the wrong time

Weighted A*

Weighted A* (with duplicate detection and reopening)

```
$open$ := new min-heap ordered by $(\sigma \mapsto g(\sigma) + W \cdot h(\sigma))$
$open$.insert(make-root-node(init()))
$closed := \emptyset$
$distance := \emptyset$
while not $open$.empty():
    $\sigma$ = $open$.pop-min()
    if $state(\sigma) \notin closed$ or $g(\sigma) < distance(state(\sigma))$:
        $closed := closed \cup \{state(\sigma)\}$
        $distance(\sigma) := g(\sigma)$
        if is-goal(state($\sigma$)):
            return extract-solution($\sigma$)
        for each $\langle o, s \rangle \in$ succ$(state(\sigma))$:
            $\sigma'$ := make-node$(\sigma, o, s)$
            if $h(\sigma') < \infty$: $open$.insert$(\sigma')$
return unsolvable
```

Properties of weighted A^*

The weight $W \in \mathbb{R}_0^+$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like A^*
- for $W \to \infty$, behaves like greedy best-first search

Properties:

- one of the most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to A^*, replacing optimal with bounded suboptimal: generated solutions are at most a factor W as long as optimal ones

Hill-climbing

Hill-climbing

```
$\sigma$ := make-root-node(init())
forever:
    if is-goal(state($\sigma$)):
        return extract-solution($\sigma$)
    $\Sigma'$ := { make-node($\sigma, o, s$) | $\langle o, s \rangle \in$ succ(state($\sigma$)) }
    $\sigma$ := an element of $\Sigma'$ minimizing $h$ (random tie breaking)
```

- can easily get stuck in local minima where immediate improvements of $h(\sigma)$ are not possible
- many variations: tie-breaking strategies, restarts

Enforced hill-climbing

Enforced hill-climbing: procedure improve

```
def $improve(\sigma_0)$:
    $queue$ := new fifo-queue
    $queue$.push-back($\sigma_0$)
    $closed := \emptyset$
    while not $queue$.empty():
        $\sigma$ = $queue$.pop-front()
        if $state(\sigma) \notin closed$:
            $closed := closed \cup \{state(\sigma)\}$
            if $h(\sigma) < h(\sigma_0)$:
                return $\sigma$
            for each $\langle o, s \rangle \in$ succ($state(\sigma)$):
                $\sigma'$ := make-node($\sigma, o, s$)
                $queue$.push-back($\sigma'$)
    fail
```

⇝ breadth-first search for more promising node than σ_0

Enforced hill-climbing (ctd.)

Enforced hill-climbing

```
$\sigma$ := make-root-node(init())
while not is-goal(state($\sigma$)):
    $\sigma$ := improve($\sigma$)
return extract-solution($\sigma$)
```

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from σ_0)
- complete for undirected search spaces (where the successor relation is symmetric) if $h(\sigma) = 0$ for all goal nodes and only for goal nodes

Summary

- distinguish: planning states, search states, search nodes
    - planning state: situation in the world modelled by the task
    - search state: subproblem remaining to be solved
        - In state-space search (usually progression search), planning states and search states are identical.
        - In regression search, search states usually describe sets of states ("subgoals").
    - search node: search state + info on "how we got there"
- search algorithms mainly differ in order of node expansion
    - uninformed vs. informed (heuristic) search
    - local vs. systematic search

Summary (ctd.)

- heuristics: estimators for “distance to goal node”
  - usually: the more accurate, the better performance
  - desiderata: safe, goal-aware, admissible, consistent
  - the ideal: perfect heuristic h^*
- most common algorithms for satisficing planning:
  - greedy best-first search
  - weighted A^*
  - enforced hill-climbing
- most common algorithm for optimal planning:
  - A^*