Principles of AI Planning

5. Planning as search: progression and regression

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Planning as (classical) search

What do we mean by search?

- Search is a very generic term.

$\rightsquigarrow$ Every algorithm that tries out various alternatives can be said to “search” in some way.

- Here, we mean classical search algorithms.
  - Search nodes are expanded to generate successor nodes.
  - Examples: breadth-first search, A^*, hill-climbing, …
- To be brief, we just say search in the following (not “classical search”).

Do you know this stuff already?

- We assume prior knowledge of basic search algorithms:
  - uninformed vs. informed
  - systematic vs. local
- There will be a small refresher in the next chapter.
- Background: Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)

Search in planning

- search: one of the big success stories of AI
- many planning algorithms based on classical AI search (we'll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)

Satisficing or optimal planning?

Must carefully distinguish two different problems:

- satisficing planning: any solution is OK (although shorter solutions typically preferred)
- optimal planning: plans must have shortest possible length

Both are often solved by search, but:

- details are very different
- almost no overlap between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners

How to apply search to planning? $\rightsquigarrow$ many choices to make!

Choice 1: Search direction

- progression: forward from initial state to goal
- regression: backward from goal states to initial state
- bidirectional search

Planning by search

How to apply search to planning? ⇝ many choices to make!

Choice 2: Search space representation

- search nodes are associated with states
  (⇝ state-space search)
- search nodes are associated with sets of states

Planning by search

How to apply search to planning? $\rightsquigarrow$ many choices to make!

Choice 3: Search algorithm

- uninformed search:
  depth-first, breadth-first, iterative depth-first, ...
- heuristic search (systematic):
  greedy best-first, A^*, Weighted A^*, IDA^*, ...
- heuristic search (local):
  hill-climbing, simulated annealing, beam search, ...

Planning by search

How to apply search to planning? $\rightsquigarrow$ many choices to make!

Choice 4: Search control

- heuristics for informed search algorithms
- pruning techniques: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, ...

Search-based satisficing planners

FF (Hoffmann & Nebel, 2001)

- search direction: forward search
- search space representation: single states
- search algorithm: enforced hill-climbing (informed local)
- heuristic: FF heuristic (inadmissible)
- pruning technique: helpful actions (incomplete)

$\rightsquigarrow$ one of the best satisficing planners

Search-based optimal planners

Fast Downward Stone Soup (Helmert et al., 2011)

- search direction: forward search
- search space representation: single states
- search algorithm: A^* (informed systematic)
- heuristic: multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, . . .)
- pruning technique: none

$\rightsquigarrow$ one of the best optimal planners

Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   $\rightsquigarrow$ this chapter
2. search space representation: states/sets of states
   $\rightsquigarrow$ this chapter
3. search algorithm: uninformed/heuristic; systematic/local
   $\rightsquigarrow$ next chapter
4. search control: heuristics, pruning techniques
   $\rightsquigarrow$ following chapters

Progression

Planning by forward search: progression

Progression: Computing the successor state $app_o(s)$ of a state s with respect to an operator o.

Progression planners find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and progress it through an operator, generating a new state
- solution found when a goal state generated

pro: very easy and efficient to implement

Search space representation in progression planners

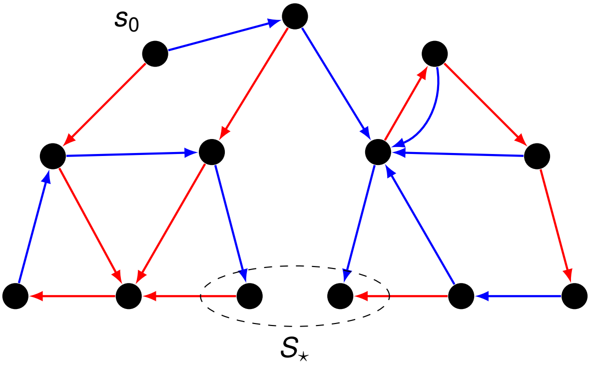
Two alternative search spaces for progression planners:

1. search nodes correspond to states
   - when the same state is generated along different paths, it is not considered again (duplicate detection)
   - pro: save time to consider same state again
   - con: memory intensive (must maintain closed list)
2. search nodes correspond to operator sequences
   - different operator sequences may lead to identical states (transpositions); search does not notice this
   - pro: can be very memory-efficient
   - con: much wasted work (often exponentially slower)

$\leadsto$ first alternative usually preferable in planning (unlike many classical search benchmarks like 15-puzzle)

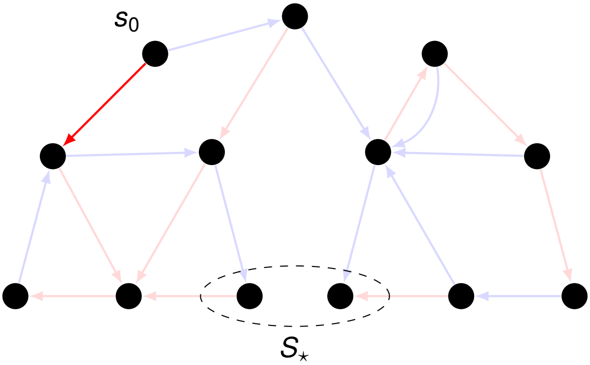
Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



Progression planning example (depth-first search)

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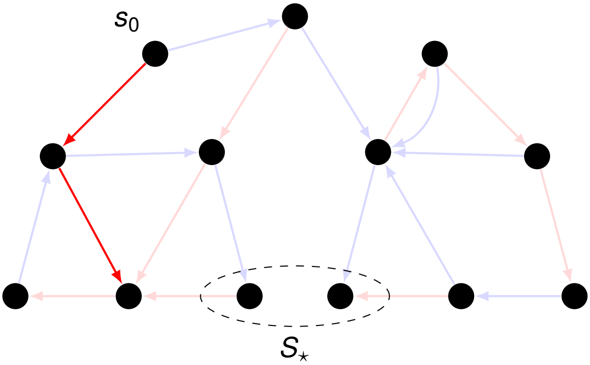

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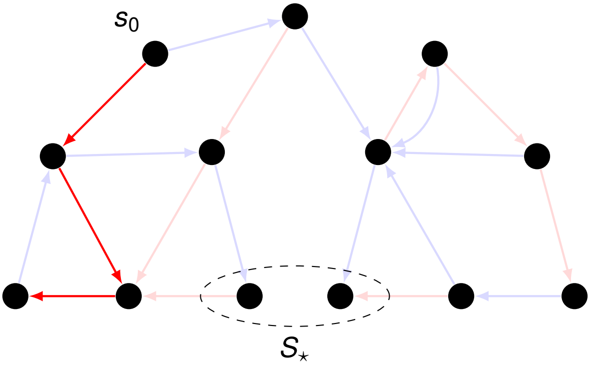

Progression planning example (depth-first search)

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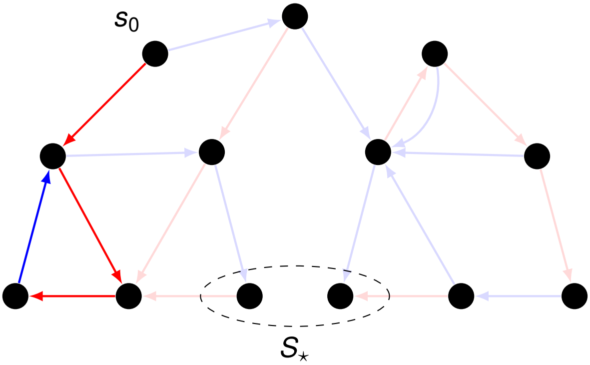
(no duplicate detection)



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Example where search nodes correspond to operator sequences
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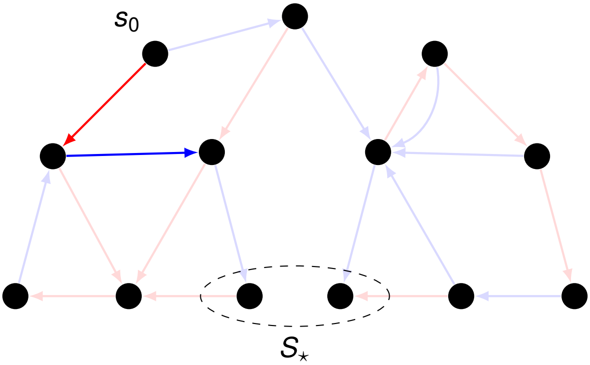

Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
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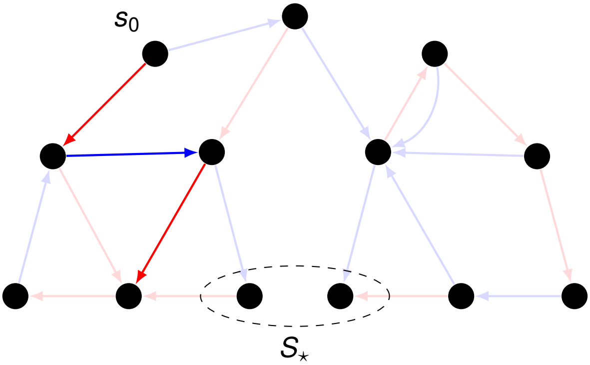

Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
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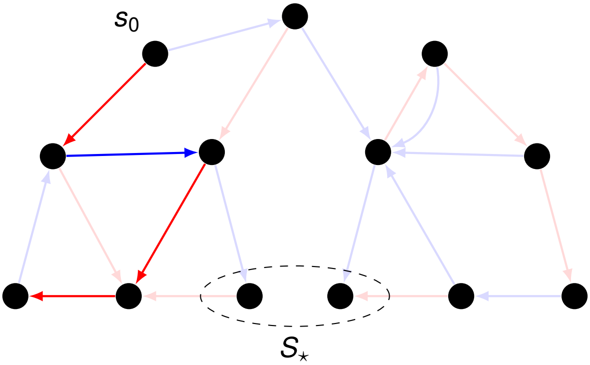

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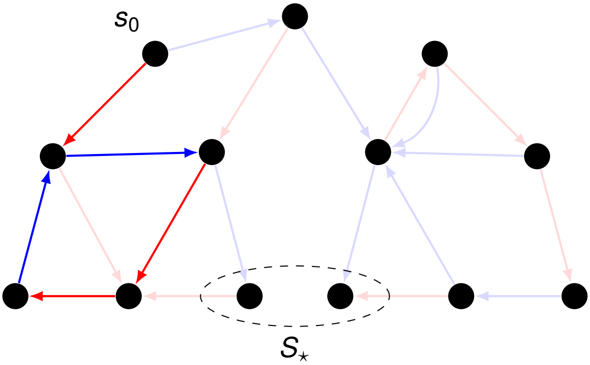

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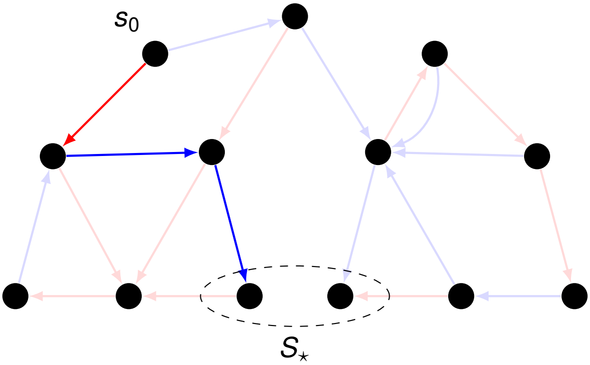


Regression

Forward search vs. backward search

Going through a transition graph in forward and backward directions is not symmetric:

- forward search starts from a single initial state; backward search starts from a set of goal states
- when applying an operator o in a state s in forward direction, there is a unique successor state s'; if we applied operator o to end up in state s', there can be several possible predecessor states s

$\rightsquigarrow$ most natural representation for backward search in planning associates sets of states with search nodes

Planning by backward search: regression

Regression: Computing the possible predecessor states $regr_o(G)$ of a set of states G with respect to the last operator o that was applied.

Regression planners find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated state set and regress it through an operator, generating a new state set
- solution found when a generated state set includes the initial state

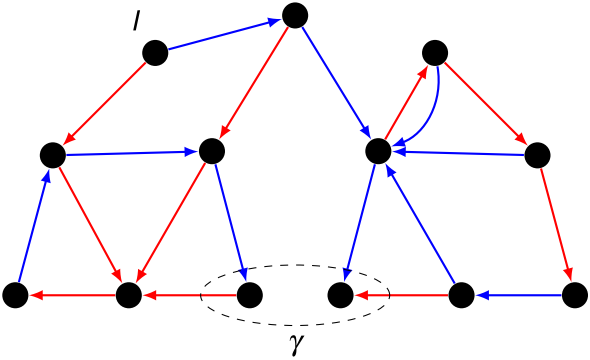
Pro: can handle many states simultaneously
Con: basic operations complicated and expensive

Search space representation in regression planners

identify state sets with logical formulae (again):

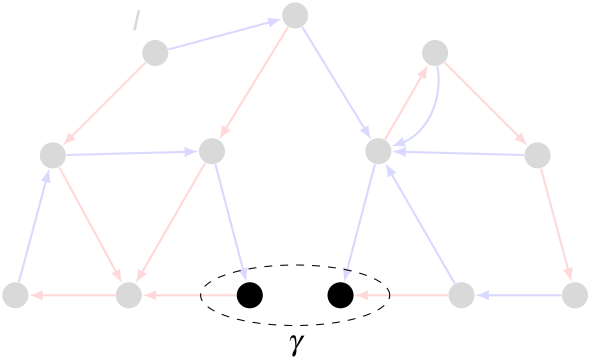
- search nodes correspond to state sets
- each state set is represented by a logical formula: φ represents $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-hard or coNP-hard

Regression planning example (depth-first search)



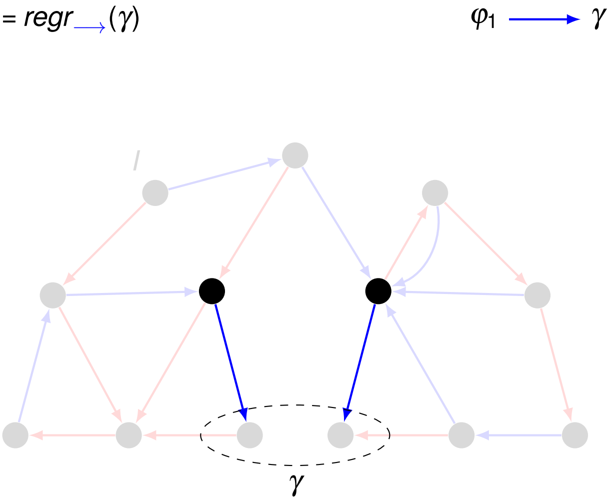
Regression planning example (depth-first search)

γ



Regression planning example (depth-first search)

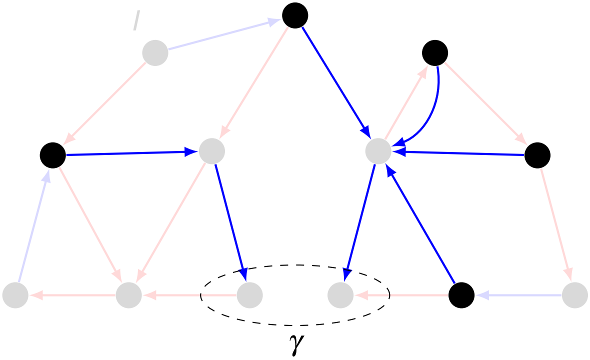
$\varphi_1 = regr_{\longrightarrow}(\gamma)$



Regression planning example (depth-first search)

$$\varphi_1 = regr_{\longrightarrow}(\gamma)$$
$$\varphi_2 = regr_{\longrightarrow}(\varphi_1)$$

$$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$$



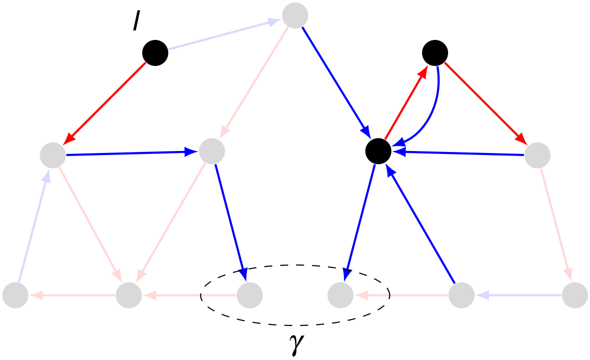
Regression planning example (depth-first search)

$\varphi_1 = regr_{\longrightarrow}(\gamma)$

$\varphi_2 = regr_{\longrightarrow}(\varphi_1)$

$\varphi_3 = regr_{\longrightarrow}(\varphi_2), I \models \varphi_3$

$\varphi_3 \longrightarrow \varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



Regression for STRIPS planning tasks

Definition (STRIPS planning task)

A planning task is a STRIPS planning task if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression for STRIPS planning tasks is very simple:

- Goals are conjunctions of atoms $a_1 \wedge \cdots \wedge a_n$.
- First step: Choose an operator that makes none of $a_1, \ldots, a_n$ false.
- Second step: Remove goal atoms achieved by the operator (if any) and add its preconditions.

$\rightsquigarrow$ Outcome of regression is again conjunction of atoms.

Optimization: only consider operators making some a_i true

STRIPS regression

Definition (STRIPS regression)

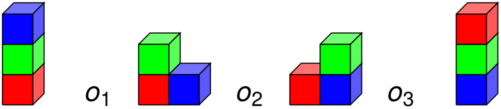
Let $\varphi = \varphi_1 \wedge \cdots \wedge \varphi_n$ be a conjunction of atoms, and let $o = \langle \chi, e \rangle$ be a STRIPS operator which adds the atoms $a_1, \ldots, a_k$ and deletes the atoms $d_1, \ldots, d_l$.

The STRIPS regression of φ with respect to o is

$$sregr_o(\varphi) := \begin{cases} \bot & \text{if } a_i = d_j \text{ for some } i, j \\ \bot & \text{if } \varphi_i = d_j \text{ for some } i, j \\ \chi \wedge \bigwedge(\{\varphi_1, \ldots, \varphi_n\} \setminus \{a_1, \ldots, a_k\}) & \text{otherwise} \end{cases}$$

Note: $sregr_o(\varphi)$ is again a conjunction of atoms, or $\bot$.

STRIPS regression example



Note: Predecessor states are in general not unique.
This picture is just for illustration purposes.

$$o_1 = \langle {\color{blue}\blacksquare} on {\color{green}\blacksquare} \land {\color{blue}\blacksquare} clr, \quad \neg {\color{blue}\blacksquare} on {\color{green}\blacksquare} \land {\color{blue}\blacksquare} onT \land {\color{green}\blacksquare} clr \rangle$$

$$o_2 = \langle {\color{green}\blacksquare} on {\color{red}\blacksquare} \land {\color{green}\blacksquare} clr \land {\color{blue}\blacksquare} clr, \quad \neg {\color{blue}\blacksquare} clr \land \neg {\color{green}\blacksquare} on {\color{red}\blacksquare} \land {\color{green}\blacksquare} on {\color{blue}\blacksquare} \land {\color{red}\blacksquare} clr \rangle$$

$$o_3 = \langle {\color{red}\blacksquare} onT \land {\color{red}\blacksquare} clr \land {\color{green}\blacksquare} clr, \quad \neg {\color{green}\blacksquare} clr \land \neg {\color{red}\blacksquare} onT \land {\color{red}\blacksquare} on {\color{green}\blacksquare} \rangle$$

$$\gamma = {\color{red}\blacksquare} on {\color{green}\blacksquare} \land {\color{green}\blacksquare} on {\color{blue}\blacksquare}$$

$$\varphi_1 = sregr_{o_3}(\gamma) = {\color{red}\blacksquare} onT \land {\color{red}\blacksquare} clr \land {\color{green}\blacksquare} clr \land {\color{green}\blacksquare} on {\color{blue}\blacksquare}$$

$$\varphi_2 = sregr_{o_2}(\varphi_1) = {\color{green}\blacksquare} on {\color{red}\blacksquare} \land {\color{green}\blacksquare} clr \land {\color{blue}\blacksquare} clr \land {\color{red}\blacksquare} onT$$

$$\varphi_3 = sregr_{o_1}(\varphi_2) = {\color{blue}\blacksquare} on {\color{green}\blacksquare} \land {\color{blue}\blacksquare} clr \land {\color{green}\blacksquare} on {\color{red}\blacksquare} \land {\color{red}\blacksquare} onT$$

Regression for general planning tasks

- With disjunctions and conditional effects, things become more tricky. How to regress $a \vee (b \wedge c)$ with respect to $\langle q, d \rhd b \rangle$?
- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.
- We present a general method for doing regression for any formula and any operator.
- Now we extensively use the idea of representing sets of states as formulae.

Effect preconditions

Definition (effect precondition)

The effect precondition $EPC_l(e)$ for literal l and effect e is defined as follows:

$$
\begin{aligned}
EPC_l(l) &= \top \\
EPC_l(l') &= \bot \text{ if } l \neq l' \quad \text{(for literals } l'\text{)} \\
EPC_l(e_1 \wedge \cdots \wedge e_n) &= EPC_l(e_1) \vee \cdots \vee EPC_l(e_n) \\
EPC_l(\chi \rhd e) &= EPC_l(e) \wedge \chi
\end{aligned}
$$

Intuition: $EPC_l(e)$ describes the situations in which effect e causes literal l to become true.

Effect precondition examples

Example

$$
\begin{aligned}
EPC_a(b \land c) &= \bot \lor \bot \equiv \bot \\
EPC_a(a \land (b \rhd a)) &= \top \lor (\top \land b) \equiv \top \\
EPC_a((c \rhd a) \land (b \rhd a)) &= (\top \land c) \lor (\top \land b) \equiv c \lor b
\end{aligned}
$$

Effect preconditions: connection to change sets

Lemma (A)

Let s be a state, l a literal and e an effect.
Then $l \in [e]_s$ if and only if $s \models EPC_l(e)$.

Proof.

Induction on the structure of the effect e.

Base case 1, $e = l$: $l \in [l]_s = \{l\}$ by definition, and $s \models EPC_l(l) = \top$ by definition. Both sides of the equivalence are true.

Base case 2, $e = l'$ for some literal $l' \neq l$: $l \notin [l']_s = \{l'\}$ by definition, and $s \not\models EPC_l(l') = \bot$ by definition. Both sides are false.

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Effect preconditions: connection to change sets

Proof (ctd.)

Inductive case 1, $e = e_1 \wedge \cdots \wedge e_n$:

$$\begin{aligned} I \in [e]_s &\text{ iff } I \in [e_1]_s \cup \cdots \cup [e_n]_s && (\text{Def } [e_1 \wedge \cdots \wedge e_n]_s) \\ &\text{ iff } I \in [e']_s \text{ for some } e' \in \{e_1, \ldots, e_n\} \\ &\text{ iff } s \models EPC_l(e') \text{ for some } e' \in \{e_1, \ldots, e_n\} && (\text{IH}) \\ &\text{ iff } s \models EPC_l(e_1) \vee \cdots \vee EPC_l(e_n) \\ &\text{ iff } s \models EPC_l(e_1 \wedge \cdots \wedge e_n). && (\text{Def } EPC) \end{aligned}$$

Inductive case 2, $e = \chi \rhd e'$:

$$\begin{aligned} I \in [\chi \rhd e']_s &\text{ iff } I \in [e']_s \text{ and } s \models \chi && (\text{Def } [\chi \rhd e']_s) \\ &\text{ iff } s \models EPC_l(e') \text{ and } s \models \chi && (\text{IH}) \\ &\text{ iff } s \models EPC_l(e') \wedge \chi \\ &\text{ iff } s \models EPC_l(\chi \rhd e'). && (\text{Def } EPC) \end{aligned}$$

□

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Effect preconditions: connection to change sets

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&\text{ iff } s \models EPC_l(e_1 \wedge \cdots \wedge e_n). && (\text{Def } EPC)
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&\text{ iff } s \models EPC_l(e') \text{ and } s \models \chi && (\text{IH})\\
&\text{ iff } s \models EPC_l(e') \wedge \chi\\
&\text{ iff } s \models EPC_l(\chi \rhd e'). && (\text{Def } EPC)
\end{aligned}
$$

□

Effect preconditions: connection to normal form

Remark: *EPC* vs. effect normal form

Notice that in terms of $EPC_a(e)$, any operator $\langle \chi, e \rangle$ can be expressed in effect normal form as

$$\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \rhd a) \wedge (EPC_{\neg a}(e) \rhd \neg a)) \right\rangle,$$

where A is the set of all state variables.

Regressing state variables

The formula $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ expresses the value of state variable $a \in A$ after applying o in terms of values of state variables before applying o.

Either:

- a became true, or
- a was true before and it did not become false.

Regressing state variables: examples

Example

Let $e = (b \rhd a) \wedge (c \rhd \neg a) \wedge b \wedge \neg d$.

variable x	$EPC_x(e) \vee (x \wedge \neg EPC_{\neg x}(e))$
a	$b \vee (a \wedge \neg c)$
b	$\top \vee (b \wedge \neg\bot) \equiv \top$
c	$\bot \vee (c \wedge \neg\bot) \equiv c$
d	$\bot \vee (d \wedge \neg\top) \equiv \bot$

Regressing state variables: correctness

Lemma (B)

Let a be a state variable, $o = \langle \chi, e \rangle$ an operator, s a state, and $s' = app_o(s)$. Then $s \models EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ if and only if $s' \models a$.

Proof.

($\Rightarrow$): Assume $s \models EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.
Do a case analysis on the two disjuncts.

1. Assume that $s \models EPC_a(e)$. By Lemma A, we have $a \in [e]_s$ and hence $s' \models a$.
2. Assume that $s \models a \wedge \neg EPC_{\neg a}(e)$. By Lemma A, we have $\neg a \notin [e]_s$. Hence a remains true in s'.

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2. Assume that $s \models a \wedge \neg EPC_{\neg a}(e)$. By Lemma A, we have $\neg a \notin [e]_s$. Hence a remains true in s'.

Regressing state variables: correctness

Proof (ctd.)

(⇐): We showed that if the formula is true in s, then a is true in s'. For the second part, we show that if the formula is false in s, then a is false in s'.

- So assume $s \not\models EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.
- Then $s \models \neg EPC_a(e) \wedge (\neg a \vee EPC_{\neg a}(e))$ (de Morgan).
- Case distinction: a is true or a is false in s.
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Regression: general definition

We base the definition of regression on formulae $EPC_l(e)$.

Definition (general regression)

Let φ be a propositional formula and $o = \langle \chi, e \rangle$ an operator. The regression of φ with respect to o is

$$regr_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$$

where

1. φ_r is obtained from φ by replacing each $a \in A$ by $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$, and
2. $\kappa = \bigwedge_{a \in A} \neg(EPC_a(e) \wedge EPC_{\neg a}(e))$.

The formula κ expresses that operators are only applicable in states where their change sets are consistent.

Regression examples

- $regr_{\langle a,b\rangle}(b) \equiv a \wedge (\top \vee (b \wedge \neg\bot)) \wedge \top \equiv a$
- $regr_{\langle a,b\rangle}(b \wedge c \wedge d)$
  $\equiv a \wedge (\top \vee (b \wedge \neg\bot)) \wedge (\bot \vee (c \wedge \neg\bot)) \wedge (\bot \vee (d \wedge \neg\bot)) \wedge \top$
  $\equiv a \wedge c \wedge d$
- $regr_{\langle a,c\rhd b\rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg\bot)) \wedge \top \equiv a \wedge (c \vee b)$
- $regr_{\langle a,(c\rhd b)\wedge(b\rhd\neg b)\rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg b)) \wedge \neg(c \wedge b)$
  $\equiv a \wedge c \wedge \neg b$
- $regr_{\langle a,(c\rhd b)\wedge(d\rhd\neg b)\rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg d)) \wedge \neg(c \wedge d)$
  $\equiv a \wedge (c \vee b) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d)$
  $\equiv a \wedge (c \vee b) \wedge \neg d$

Regression example: binary counter

$$(\neg b_0 \rhd b_0) \wedge$$
$$((\neg b_1 \wedge b_0) \rhd (b_1 \wedge \neg b_0)) \wedge$$
$$((\neg b_2 \wedge b_1 \wedge b_0) \rhd (b_2 \wedge \neg b_1 \wedge \neg b_0))$$

$$EPC_{b_2}(e) = \neg b_2 \wedge b_1 \wedge b_0$$
$$EPC_{b_1}(e) = \neg b_1 \wedge b_0$$
$$EPC_{b_0}(e) = \neg b_0$$
$$EPC_{\neg b_2}(e) = \bot$$
$$EPC_{\neg b_1}(e) = \neg b_2 \wedge b_1 \wedge b_0$$
$$EPC_{\neg b_0}(e) = (\neg b_1 \wedge b_0) \vee (\neg b_2 \wedge b_1 \wedge b_0) \equiv (\neg b_1 \vee \neg b_2) \wedge b_0$$

Regression replaces state variables as follows:

b_2 by $(\neg b_2 \wedge b_1 \wedge b_0) \vee (b_2 \wedge \neg \bot) \equiv (b_1 \wedge b_0) \vee b_2$

b_1 by $(\neg b_1 \wedge b_0) \vee (b_1 \wedge \neg(\neg b_2 \wedge b_1 \wedge b_0))$
$\equiv (\neg b_1 \wedge b_0) \vee (b_1 \wedge (b_2 \vee \neg b_0))$

b_0 by $\neg b_0 \vee (b_0 \wedge \neg((\neg b_1 \vee \neg b_2) \wedge b_0)) \equiv \neg b_0 \vee (b_1 \wedge b_2)$

General regression: correctness

Theorem (correctness of $regr_o(\varphi)$)

Let φ be a formula, o an operator and s a state. Then $s \models regr_o(\varphi)$ iff o is applicable in s and $app_o(s) \models \varphi$.

Proof.

Let $o = \langle \chi, e \rangle$. Recall that $regr_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$, where φ_r and κ are as defined previously.

If o is inapplicable in s, then $s \not\models \chi \wedge \kappa$, both sides of the "iff" condition are false, and we are done. Hence, we only further consider states s where o is applicable. Let $s' := app_o(s)$.

We know that $s \models \chi \wedge \kappa$ (because o is applicable), so the "iff" condition we need to prove simplifies to:

$$s \models \varphi_r \text{ iff } s' \models \varphi.$$

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Proof (ctd.)

To show: $s \models \varphi_r$ iff $s' \models \varphi$.

We show that for all formulae ψ, $s \models \psi_r$ iff $s' \models \psi$, where ψ_r is ψ with every $a \in A$ replaced by $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.

The proof is by structural induction on ψ.

Induction hypothesis $s \models \psi_r$ if and only if $s' \models \psi$.

Base cases 1 & 2 $\psi = \top$ or $\psi = \bot$: trivial, as $\psi_r = \psi$.

Base case 3 $\psi = a$ for some $a \in A$:
Then $\psi_r = EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.
By Lemma B, $s \models \psi_r$ iff $s' \models \psi$.

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Inductive case 1 $\psi = \neg\psi'$:

$$s \models \psi_r \text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r$$
$$\text{iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi$$

Inductive case 2 $\psi = \psi' \vee \psi''$:

$$s \models \psi_r \text{ iff } s \models (\psi' \vee \psi'')_r \text{ iff } s \models \psi'_r \vee \psi''_r$$
$$\text{iff } s \models \psi'_r \text{ or } s \models \psi''_r$$
$$\text{iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi''$$
$$\text{iff } s' \models \psi' \vee \psi'' \text{ iff } s' \models \psi$$

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Emptiness and subsumption testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $regr_o(\varphi)$ does not represent the empty set (which would mean that search is in a dead end). For example, $regr_{\langle a, \neg p \rangle}(p) \equiv a \wedge \bot \equiv \bot$.
- Test that $regr_o(\varphi)$ does not represent a subset of φ (which would make the problem harder than before). For example, $regr_{\langle b, c \rangle}(a) \equiv a \wedge b$.

Both of these problems are NP-hard.

Formula growth

The formula $regr_{o_1}(regr_{o_2}(\dots regr_{o_{n-1}}(regr_{o_n}(\varphi))))$ may have size $O(|\varphi||o_1||o_2|\dots|o_{n-1}||o_n|)$, i. e., the product of the sizes of φ and the operators.

$\rightsquigarrow$ worst-case exponential size $O(m^n)$

Logical simplifications

- $\bot \wedge \varphi \equiv \bot$, $\top \wedge \varphi \equiv \varphi$, $\bot \vee \varphi \equiv \varphi$, $\top \vee \varphi \equiv \top$
- $a \vee \varphi \equiv a \vee \varphi[\bot/a]$, $\neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a]$, $a \wedge \varphi \equiv a \wedge \varphi[\top/a]$, $\neg a \wedge \varphi \equiv \neg a \wedge \varphi[\bot/a]$
- idempotency, absorption, commutativity, associativity, ...

Restricting formula growth in search trees

Problem very big formulae obtained by regression

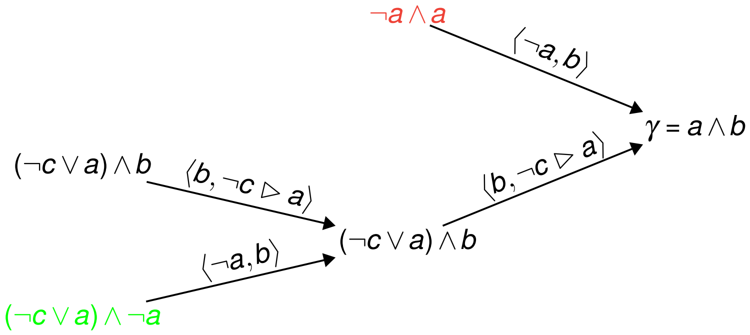
Cause disjunctivity in the (NNF) formulae (formulae without disjunctions easily convertible to small formulae $l_1 \wedge \cdots \wedge l_n$ where l_i are literals and n is at most the number of state variables.)

Idea handle disjunctivity when generating search trees

Unrestricted regression: search tree example

Unrestricted regression: do not treat disjunctions specially

Goal $\gamma = a \land b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.



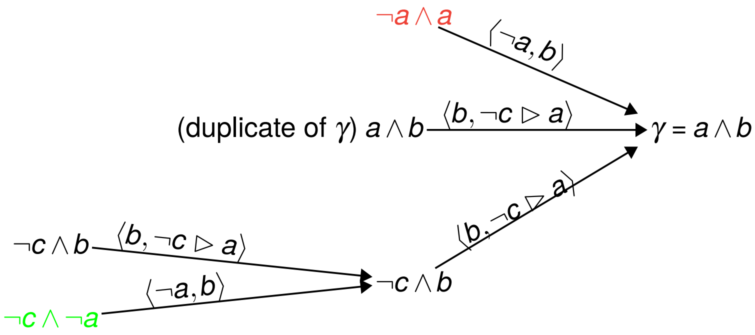
Full splitting: search tree example

Full splitting: always remove all disjunctivity

Goal $\gamma = a \wedge b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.

$(\neg c \vee a) \wedge b$ in DNF: $(\neg c \wedge b) \vee (a \wedge b)$

$\rightsquigarrow$ split into $\neg c \wedge b$ and $a \wedge b$



General splitting strategies

Alternatives:

1. Do nothing (unrestricted regression).
2. Always eliminate all disjunctivity (full splitting).
3. Reduce disjunctivity if formula becomes too big.

Discussion:

- With unrestricted regression the formulae may have size that is exponential in the number of state variables.
- With full splitting search tree can be exponentially bigger than without splitting.
- The third option lies between these two extremes.

Summary

- (Classical) search is a very important planning approach.
- Search-based planning algorithms differ along many dimensions, including
  - search direction (forward, backward)
  - what each search node represents
    (a state, a set of states, an operator sequence)
- Progression search proceeds forwards from the initial state.
  - If we use duplicate detection, each search node corresponds to a unique state.
  - If we do not use duplicate detection, each search node corresponds to a unique operator sequence.

Summary (ctd.)

- Regression search proceeds backwards from the goal.
  - Each search node corresponds to a set of states represented by a formula.
  - Regression is simple for STRIPS operators.
  - The theory for general regression is more complex.
  - When applying regression in practice, additional considerations such as when and how to perform splitting come into play.