Principles of Knowledge Representation and Reasoning

Qualitative Representation and Reasoning II: Allen's Interval Calculus

Bernhard Nebel, Stefan Wölfl, and Felix Lindner

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1 Allen's Interval Calculus

- Motivation
- Intervals and Relations Between Them
- Composing Interval Relations

Qualitative Temporal Representation and Reasoning

Often we do not want to talk about precise times:

- NLP – we do not have precise time points
- Planning – we do not want to commit to time points too early
- Scenario descriptions – we do not have the exact times or do not want to state them

What are the primitives in our representation system?

- Time points: actions and events are instantaneous, or we consider their beginning and ending
- Time intervals: actions and events have duration
- Reducibility? Expressiveness? Computational costs for reasoning?

Motivation: Example

Consider a planning scenario for multimedia generation:

P1: Display Picture1
P2: Say "Put the plug in."
P3: Say "The device should be shut off."
P4: Point to Plug-in-Picture1.

Temporal relations between events:

P2	should happen during	P1
P3	should happen during	P1
P2	should happen before or directly precede	P3
P4	should happen during or end together with	P2

- ⇝ P4 happens before or directly precedes P3
- ⇝ We could add the statement "P4 does not overlap with P3" without creating an inconsistency.

Allen's Interval Calculus

- Allen's interval calculus: time intervals and binary relations over them
- Time intervals: $X = (X^-, X^+)$, where X^- and X^+ are interpreted over the reals and $X^- < X^+$ ($\rightsquigarrow$ naïve approach)
- Relations between concrete intervals, e. g.:

  (1.0,2.0) strictly before (3.0,5.5)
  (1.0,3.0) meets (3.0,5.5)
  (1.0,4.0) overlaps (3.0,5.5)
  ...

$\rightsquigarrow$ Which relations are conceivable?

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The base relations

How many ways are there to order the four points of two intervals?

Relation	Symbol	Name
$\{(X,Y) : X^- < X^+ < Y^- < Y^+\}$	$\prec$	before
$\{(X,Y) : X^- < X^+ = Y^- < Y^+\}$	m	meets
$\{(X,Y) : X^- < Y^- < X^+ < Y^+\}$	o	overlaps
$\{(X,Y) : X^- = Y^- < X^+ < Y^+\}$	s	starts
$\{(X,Y) : Y^- < X^- < X^+ = Y^+\}$	f	finishes
$\{(X,Y) : Y^- < X^- < X^+ < Y^+\}$	d	during
$\{(X,Y) : Y^- = X^- < X^+ = Y^+\}$	$\equiv$	equal

and the **converse** relations (obtained by exchanging X and Y)

$\rightsquigarrow$ These relations are JEPD.

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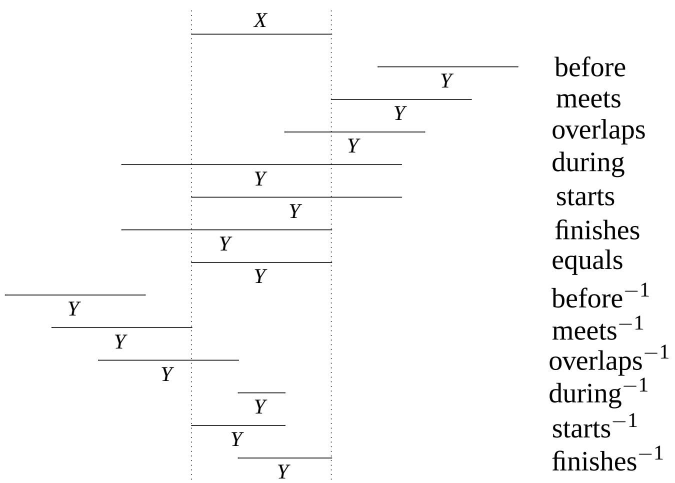
The 13 base relations graphically



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Disjunctive descriptions

- Assumption: We don't have precise information about the relation between X and Y, e. g.:

$$X \text{ o } Y \text{ or } X \text{ m } Y$$

- ...modelled by sets of base relations (meaning the union of the relations):

$$X \{\mathtt{o},\mathtt{m}\} Y$$

$\rightsquigarrow$ 2^{13} imprecise relations (incl. $\emptyset$ and **B**)

Example of an indefinite qualitative description:

$$\left\{ X \{\mathtt{o},\mathtt{m}\} Y, Y \{\mathtt{m}\} Z, X \{\mathtt{o},\mathtt{m}\} Z \right\}$$

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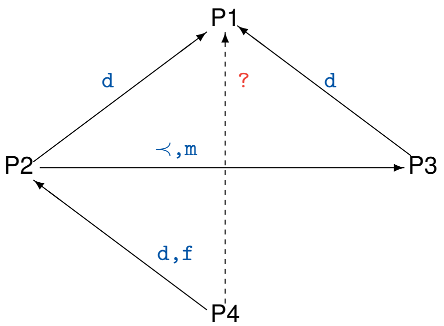
Our example…formally

P1: Display Picture1
P2: Say “Put the plug in.”
P3: Say “The device should be shut off.”
P4: Point to Plug-in-Picture1.



Compose the constraints: $P4\,\{\mathtt{d},\mathtt{f}\}\,P2$ and $P2\,\{\mathtt{d}\}\,P1$
$\rightsquigarrow$ $P4\,\{\mathtt{d}\}\,P1$.

Composition of base relations

	$\prec$	$\succ$	d	d^{-1}	o	o^{-1}	m	m^{-1}	s	s^{-1}	f	f^{-1}
$\prec$	$\prec$	$\mathbf{B}$	$\prec\ o\ m\ d\ s$	$\prec$	$\prec$	$\prec\ o\ m\ d\ s$	$\prec$	$\prec\ o\ m\ d\ s$	$\prec$	$\prec$	$\prec\ o\ m\ d\ s$	$\prec$
$\succ$	$\mathbf{B}$	$\succ$	$\succ\ o^{-1}\ m^{-1}\ d\ f$	$\succ$	$\succ\ o^{-1}\ m^{-1}\ d\ f$	$\succ$	$\succ\ o^{-1}\ m^{-1}\ d\ f$	$\succ$	$\succ\ o^{-1}\ m^{-1}\ d\ f$	$\succ$	$\succ$	$\succ$
d	$\prec$	$\succ$	d	$\mathbf{B}$	$\prec\ o\ m\ d\ s$	$\succ\ o^{-1}\ m^{-1}\ d\ f$	$\prec$	$\succ$	d	$\succ\ o^{-1}\ m^{-1}\ d\ f$	d	$\prec\ o\ m\ d\ s$
d^{-1}	$\prec\ o\ m\ d^{-1}\ f^{-1}$	$\succ\ o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$\mathbf{B}-\ \prec\succ\ m\ m^{-1}$	d^{-1}	$o\ d^{-1}\ f^{-1}$	$o^{-1}\ d^{-1}\ s^{-1}$	$o\ d^{-1}\ f^{-1}$	$o^{-1}\ d^{-1}\ s^{-1}$	$o\ d^{-1}\ f^{-1}$	d^{-1}	$o^{-1}\ d^{-1}\ s^{-1}$	d^{-1}
o	$\prec$	$\succ\ o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$o\ d\ s$	$\prec\ o\ m\ d^{-1}\ f^{-1}$	$\prec\ o\ m$	$\mathbf{B}-\ \prec\succ\ m\ m^{-1}$	$\prec$	$o^{-1}\ d^{-1}\ s^{-1}$	o	$d^{-1}\ f^{-1}\ o$	$d\ s\ o$	$\prec\ o\ m$
o^{-1}	$\prec\ o\ m, d^{-1}\ f^{-1}$	$\succ$	$o^{-1}\ d\ f$	$\succ, o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$\mathbf{B}-\ \prec\succ\ m\ m^{-1}$	$\succ\ o^{-1}\ m^{-1}$	$o\ d^{-1}\ f^{-1}$	$\succ$	$o^{-1}\ d\ f$	$o^{-1}\ \succ\ m^{-1}$	o^{-1}	$o^{-1}\ d^{-1}\ s^{-1}$
m	$\prec$	$\succ\ o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$o\ d\ s$	$\prec$	$\prec$	$o\ d\ s$	$\prec$	$f\ f^{-1}\ \equiv$	m	m	$d\ s\ o$	$\prec$
m^{-1}	$\prec\ o\ m\ d^{-1}\ f^{-1}$	$\succ$	$o^{-1}\ d\ f$	$\succ$	$o^{-1}\ d\ f$	$\succ$	$s\ s^{-1}\ \equiv$	$\succ$	$d\ f\ o^{-1}$	$\succ$	m^{-1}	m^{-1}
s	$\prec$	$\succ$	d	$\prec\ o\ m\ d^{-1}\ f^{-1}$	$\prec\ o\ m$	$o^{-1}\ d\ f$	$\prec$	m^{-1}	s	$s\ s^{-1}\ \equiv$	d	$\prec\ m\ o$
s^{-1}	$\prec\ o\ m\ d^{-1}\ f^{-1}$	$\succ$	$o^{-1}\ .d\ f$	d^{-1}	$o\ d^{-1}\ f^{-1}$	o^{-1}	$o\ d^{-1}\ f^{-1}$	m^{-1}	$s\ s^{-1}\ \equiv$	s^{-1}	o^{-1}	d^{-1}
f	$\prec$	$\succ$	d	$\succ\ o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$o\ d\ s$	$\succ\ o^{-1}\ m^{-1}$	m	$\succ$	d	$\succ\ o^{-1}\ m^{-1}$	f	$f\ f^{-1}\ \equiv$
f^{-1}	$\prec$	$\succ\ o^{-1}\ m^{-1}\ d^{-1}\ s^{-1}$	$o\ d\ s$	d^{-1}	o	$o^{-1}\ d^{-1}\ s^{-1}$	m	$s^{-1}\ o^{-1}\ d^{-1}$	o	d^{-1}	$f\ f^{-1}\ \equiv$	f^{-1}

Outlook

- Using the composition table and the rules about operations on relations, we can deduce new relations between time intervals.
- What would be a systematic approach?
- How costly is that?
- Is that complete?
- If not, could it be complete on a subset of the relation system?

2 Reasoning in Allen's Interval Calculus

- Enforcing path consistency
- NP-Hardness Example
- The Continuous Endpoint Class
- Completeness for the CEP Class

Constraint propagation: The naive algorithm

Enforcing path consistency using the straight-forward method:
Let *Table*$[i,j]$ be an array of size $n \times n$ (n: number of intervals) in which we record the constraints between the intervals.

EnforcePathConsistency1($\mathcal{C}$)

Input: a (binary) CSP $\mathcal{C} = \langle V, D, C \rangle$
Output: an equivalent, but path consistent CSP $\mathcal{C}'$

repeat
  for each pair (i,j), $1 \leq i,j \leq n$
    for each k with $1 \leq k \leq n$
      $Table[i,j] := Table[i,j] \cap (Table[i,k] \circ Table[k,j])$
until no entry in *Table* is changed

$\leadsto$ needs $O(n^5)$ intersections and compositions.

An $O(n^3)$ algorithm

EnforcePathConsistency2($\mathcal{C}$)

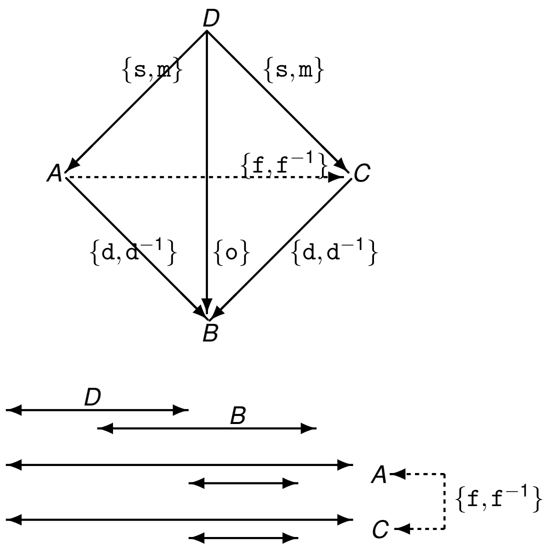
Input: a (binary) CSP $\mathcal{C} = \langle V, D, C \rangle$
Output: an equivalent, but path consistent CSP $\mathcal{C}'$

$Paths(i,j) = \{(i,j,k) : 1 \leq k \leq n\} \cup \{(k,i,j) : 1 \leq k \leq n\}$

$Queue := \bigcup_{i,j} Paths(i,j)$

while $Queue \neq \emptyset$
  select and delete (i,k,j) from *Queue*
  $T := Table[i,j] \cap (Table[i,k] \circ Table[k,j])$
  if $T \neq Table[i,j]$
    $Table[i,j] := T$
    $Table[j,i] := T^{-1}$
    $Queue := Queue \cup Paths(i,j)$

Example for incompleteness



NP-hardness

Theorem (Kautz & Vilain)

CSAT is NP-hard for Allen's interval calculus.

Proof.

Reduction from 3-colorability (original proof using 3Sat).

Let $G = (V,E)$, $V = \{v_1, \ldots, v_n\}$ be an instance of 3-colorability. Then we use the intervals $\{v_1, \ldots, v_n, 1, 2, 3\}$ with the following constraints:

$$\begin{array}{cccl} 1 & \{\mathtt{m}\} & 2 & \\ 2 & \{\mathtt{m}\} & 3 & \\ v_i & \{\mathtt{m}, \equiv, \mathtt{m}^{-1}\} & 2 & \forall v_i \in V \\ v_i & \{\mathtt{m}, \mathtt{m}^{-1}, \prec, \succ\} & v_j & \forall (v_i, v_j) \in E \end{array}$$

This constraint system is satisfiable iff G can be colored with 3 colors. □

Looking for special cases

- Idea: Let us look for polynomial special cases. In particular, let us look for sets of relations (subsets of the entire set of relations) that have an easy CSAT problem.
- Note: Interval formulae $X\ R\ Y$ can be expressed as clauses over atoms of the form $a\ op\ b$, where:
  - a and b are endpoints X^-, X^+, Y^- and Y^+ and
  - $op \in \{<, >, =, \leq, \geq\}$.
- Example: All base relations can be expressed as unit clauses.

Lemma

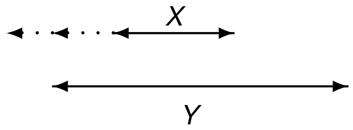
Let $\pi(\Theta)$ be the translation of Θ to clause form. Θ is satisfiable over intervals iff $\pi(\Theta)$ is satisfiable over the rational numbers.

The Continuous Endpoint Class

Continuous Endpoint Class $\mathcal{C}$: This is a subset of $\mathcal{A}$ such that there exists a clause form for each relation containing only unit clauses where $\neg(a = b)$ is forbidden.

Example: All basic relations and $\{\mathtt{d}, \mathtt{o}, \mathtt{s}\}$, because

$$\pi(X\ \{\mathtt{d}, \mathtt{o}, \mathtt{s}\}\ Y) = \{ X^- < X^+, Y^- < Y^+, X^- < Y^+, X^+ > Y^-, X^+ < Y^+ \}$$



Why do we have completeness?

The set $\mathcal{C}$ is closed under intersection, composition, and converse (it is a sub-algebra wrt. these three operations on relations). This can be shown by using a computer program.

Lemma

Each 3-consistent interval CSP over $\mathcal{C}$ is globally consistent.

Theorem (van Beek)

Path consistency solves CMIN($\mathcal{C}$) and decides CSAT($\mathcal{C}$).

(Proof: Follows from the above lemma and the fact that a strongly n-consistent CSP is minimal.)

Corollary

A path consistent interval CSP consisting of base relations only is satisfiable.

Helly's Theorem

Definition

A set $M \subseteq \mathbf{R}^n$ is convex iff for all pairs of points $a, b \in M$, all points on the line connecting a and b belong to M.

Theorem (Helly)

Let F be a finite family of at least $n+1$ convex sets in $\mathbf{R}^n$. If all sub-families of F with $n+1$ sets have a non-empty intersection, then $\bigcap F \neq \emptyset$.

Strong n-consistency (1)

Proof (part 1).

We prove the claim by induction over k with $k \leq n$.

Base case: $k = 1, 2, 3$ $\surd$

Induction assumption: Assume strong $(k-1)$-consistency (and non-emptiness of all relations)

Induction step: From the assumption, it follows that there is an instantiation of $k-1$ variables X_i to pairs (s_i, e_i) satisfying the constraints R_{ij} between the $k-1$ variables.

We have to show that we can extend the instantiation to any kth variable.

Strong n-consistency (2): Instantiating the kth variable

Proof (part 2).

The instantiation of the $k-1$ variables X_i to (s_i, e_i) restricts the instantiation of X_k.

Note: Since $R_{ij} \in \mathcal{C}$ by assumption, these restrictions can be expressed by inequalities of the form:

$$s_i < X_k^+ \wedge e_j \geq X_k^- \wedge \ldots$$

Such inequalities define convex subsets in $\mathbf{R}^2$.

$\rightsquigarrow$ Consider sets of 3 inequalities (= 3 convex sets).

Strong n-consistency (3): Using Helly's Theorem

Proof (part 3).

Case 1: All 3 inequalities mention only X_k^- (or mention only X_k^+). Then it suffices to consider only 2 of these inequalities (the strongest). Because of 3-consistency, there exists at least 1 common point satisfying these 2 inequalities.

Case 2: The inequalities mention X_k^- and X_k^+, but do not contain the inequality $X_k^- < X_k^+$. Then there are at most 2 inequalities with the same variable and we have the same situation as in Case 1.

Case 3: The set contains the inequality $X_k^- < X_k^+$. In this case, only three intervals (incl. X_k) can be involved and by 3-consistency there exists a common point.

$\rightsquigarrow$ With Helly's Theorem, there exists an instantiation consistent with all inequalities.

$\rightsquigarrow$ Strong k-consistency for all $k \leq n$. □

Outlook

- CMIN($\mathcal{C}$) can be computed in $O(n^3)$ time (for n being the number of intervals) using the path consistency algorithm.
- $\mathcal{C}$ is a set of relations occurring “naturally” when observations are uncertain.
- $\mathcal{C}$ contains 83 relations (incl. the impossible and the universal relations).
- Are there larger sets such that path consistency computes minimal CSPs? Probably not.
- Are there larger sets of relations that permit polynomial satisfiability testing? Yes.

3 A Maximal Tractable Sub-Algebra

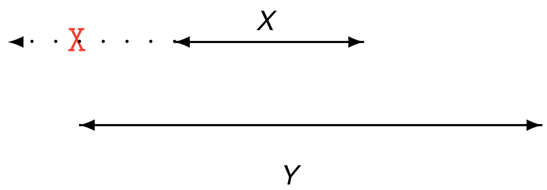
- The Endpoint Subclass
- The ORD-Horn Subclass
- Maximality
- Solving Arbitrary Allen CSPs

The EP-subclass

End-Point Subclass: $\mathcal{P} \subseteq \mathcal{A}$ is the subclass that permits a clause form containing only unit clauses ($a \neq b$ is allowed).

Example: all basic relations and $\{\texttt{d}, \texttt{o}\}$ since

$$\pi(X\{\texttt{d},\texttt{o}\}Y) = \{X^- < X^+, Y^- < Y^+, X^- < Y^+, X^+ > Y^-, X^- \neq Y^-, X^+ < Y^+\}$$



Theorem (Vilain & Kautz 86, Ladkin & Maddux 88)

Enforcing path consistency decides CSAT($\mathcal{P}$).

The ORD-Horn Subclass

ORD-Horn Subclass: $\mathcal{H} \subseteq \mathcal{A}$ is the subclass that permits a clause form containing only Horn clauses where only the following literals are allowed:

$$a \leq b, a = b, a \neq b$$

$\neg a \leq b$ is not allowed!

Example: all $R \in \mathcal{P}$ and $\{\texttt{o}, \texttt{s}, \texttt{f}^{-1}\}$:

$$\pi(X\{\texttt{o},\texttt{s},\texttt{f}^{-1}\}Y) = \{X^- \leq X^+, X^- \neq X^+, Y^- \leq Y^+, Y^- \neq Y^+, X^- \leq Y^-, X^- \leq Y^+, X^- \neq Y^+, Y^- \leq X^+, X^+ \neq Y^-, X^+ \leq Y^+, X^- \neq Y^- \vee X^+ \neq Y^+\}.$$

Partial orders: The *ORD* theory

Let *ORD* be the following theory:

$$\begin{array}{llll}
\forall x,y,z: & x \leq y \wedge y \leq z & \rightarrow x \leq z & \text{(transitivity)} \\
\forall x: & x \leq x & & \text{(reflexivity)} \\
\forall x,y: & x \leq y \wedge y \leq x & \rightarrow x = y & \text{(anti-symmetry)} \\
\forall x,y: & x = y & \rightarrow x \leq y & \text{(weakening of =)} \\
\forall x,y: & x = y & \rightarrow y \leq x & \text{(weakening of =).}
\end{array}$$

- *ORD* describes partially ordered sets, $\leq$ being the ordering relation.
- *ORD* is a Horn theory
- What is missing wrt. dense and linear orders?

Satisfiability over partial orders

Proposition

Let Θ be a CSP over $\mathcal{H}$. Θ is satisfiable over interval interpretations iff $\pi(\Theta) \cup ORD$ is satisfiable over arbitrary interpretations.

Proof.

$\Rightarrow$: Since the reals form a partially ordered set (i. e., satisfy *ORD*), this direction is trivial.
$\Leftarrow$: Each extension of a partial order to a linear order satisfies all formulae of the form $a \leq b$, $a = b$, and $a \neq b$ which have been satisfied over the original partial order. □

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Complexity of CSAT($\mathcal{H}$)

Let $ORD_{\pi(\Theta)}$ be the propositional theory resulting from instantiating all axioms with the endpoints occurring in $\pi(\Theta)$.

Proposition

$ORD \cup \pi(\Theta)$ is satisfiable iff $ORD_{\pi(\Theta)} \cup \pi(\Theta)$ is so.

Proof idea: Herbrand expansion!

Theorem

CSAT($\mathcal{H}$) can be decided in polynomial time.

Proof.

CSAT($\mathcal{H}$) instances can be translated into a propositional Horn theory with blowup $O(n^3)$ according to the previous Prop., and such a theory is decidable in polynomial time. □

$\mathcal{C} \subset \mathcal{P} \subset \mathcal{H}$ with $|\mathcal{C}| = 83$, $|\mathcal{P}| = 188$, $|\mathcal{H}| = 868$

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Path consistency and the OH-class

Lemma

Let Θ be a path-consistent set over $\mathcal{H}$. Then

$$(X\{\}Y) \notin \Theta \text{ iff } \Theta \text{ is satisfiable}$$

Proof idea: One can show that $ORD_{\pi(\Theta)} \cup \pi(\Theta)$ is closed wrt. positive unit resolution. Since this inference rule is refutation complete for Horn theories, the claim follows.

Theorem

Enforcing path consistency decides CSAT($\mathcal{H}$).

- ⇝ Maximality of $\mathcal{H}$?
- ⇝ Do we have to check all $8192 - 868$ extensions?

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Complexity of sub-algebras

Let $\hat{S}$ be the closure of $S \subseteq \mathcal{A}$ under converse, intersection, and composition (i.e., the carrier of the least sub-algebra generated by S).

Theorem

CSAT($\hat{S}$) can be polynomially transformed to CSAT(S).

Proof Idea.

All relations in $\hat{S} - S$ can be modeled by a fixed number of compositions, intersections, and conversions of relations in S, introducing perhaps some fresh variables. □

- ⇝ Polynomiality of S extends to $\hat{S}$.
- ⇝ NP-hardness of $\hat{S}$ is inherited by all generating sets S.
- ⇝ Note: $\mathcal{H} = \hat{\mathcal{H}}$.

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Minimal extensions of the $\mathcal{H}$-subclass

A computer-aided case analysis leads to the following result:

Lemma

There are only two minimal sub-algebras that strictly contain $\mathcal{H}$: $\mathcal{X}_1, \mathcal{X}_2$

$$N_1 = \{\mathtt{d}, \mathtt{d}^{-1}, \mathtt{o}^{-1}, \mathtt{s}^{-1}, \mathtt{f}\} \in \mathcal{X}_1$$
$$N_2 = \{\mathtt{d}^{-1}, \mathtt{o}, \mathtt{o}^{-1}, \mathtt{s}^{-1}, \mathtt{f}^{-1}\} \in \mathcal{X}_2$$

The clause form of these relations contain "proper" disjunctions!

Theorem

CSAT($\mathcal{H} \cup \{N_i\}$) is NP-complete.

Question: Are there other maximal tractable subclasses?

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"Interesting" subclasses

Interesting subclasses of $\mathcal{A}$ should contain all basic relations.

A computer-aided case analysis reveals:
For $S \supseteq \{\{B\} : B \in \mathbf{B}\}$ it holds that

1. $\hat{S} \subseteq \mathcal{H}$, or
2. N_1 or N_2 is in $\hat{S}$.

In case 2, one can show: CSAT(S) is NP-complete.

⇝ $\mathcal{H}$ is the only interesting maximal tractable subclass.

If we include non-interesting subalgebras, there exist exactly 18 tractable classes.

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Relevance?

Theory: ⊕ We now know the boundary between polynomial and NP-hard reasoning problems along the dimension expressiveness.

Practice: ⊖ All known applications either need only $\mathcal{P}$ or they need more than $\mathcal{H}$!

Backtracking methods might profit from the result by reducing the branching factor.

⇝ How difficult is CSAT($\mathcal{A}$) in practice?

⇝ What are the relevant branching factors?

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Solving general Allen CSPs

- Backtracking algorithm using path consistency as a forward-checking method
- Relies on tractable fragments of Allen's calculus: split relations into relations of a tractable fragment, and backtrack over these.
- Refinements and evaluation of different heuristics

⇝ Which tractable fragment should one use?

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Branching factors

- If the labels are split into base relations, then on average a label is split into

  6.5 relations

- If the labels are split into pointizable relations ($\mathcal{P}$), then on average a label is split into

  2.955 relations

- If the labels are split into ORD-Horn relations ($\mathcal{H}$), then on average a label is split into

  2.533 relations

⇝ A difference of 0.422

⇝ This makes a difference for “hard” instances.

Summary

- Allen's interval calculus is often adequate for describing relative orders of events that have duration.
- The satisfiability problem for CSPs using the relations is NP-complete.
- For the continuous endpoint class, minimal CSPs can be computed using the path-consistency method.
- For the larger ORD-Horn class, CSAT is still decided by the path-consistency method.
- Can be used in practice for backtracking algorithms.

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4 Literature

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