Principles of Knowledge Representation and Reasoning

Belief Revision

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January 27, 2016

1 Introduction

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- Revision vs. update

Introductory example

Gärdenfors - 1988

Oscar used to believe that he had given Victoria a gold ring at their wedding. He had bought their two rings at a jewellery in Casablanca. He thought it was a bargain. The merchant had claimed that the rings were made of 24 carat gold. They certainly looked like gold, but to be on the safe side Oscar had taken the rings to the jeweller next door who has testified to their gold content. However, some time after the wedding, Oscar was repairing his boat and he noticed that the sulphuric acid he was using stained his ring. He remembered from his school chemistry that the only acid that affected gold was aqua regia. Somewhat surprised, he verified that the ring was also stained by the acid.

Belief change

- A dual approach to nonmonotonic reasoning is belief change.
- We start with some belief state K. When new information arrives, we change the belief state in order to accommodate the new information.
- In the general case, the changed belief state may not be a superset of the original belief state.
- Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

Two scenarios

- We have a theory about the world, and the new information is meant to correct our theory ...

⇝ Belief revision: change your belief state minimally in order to accommodate the new information

- We have a correct theory about the current state of the world, and the new information is meant to record a change in the world ...

⇝ Belief update: incorporate the change by assuming that the world has changed minimally

Update and revision are different

Assume the new information is consistent with our old beliefs.

- In case of belief revision, we would like to add the new information monotonically to our old beliefs.
- For belief update this is not necessarily the case.
  - Assume we know that the **door is open or the window is open**.
  - Assume we learn that the world has changed and the **door is now closed**.
  - In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that **the window is open**.

2 Belief Revision

- Change Operators
- AGM Postulates
- Base Revision
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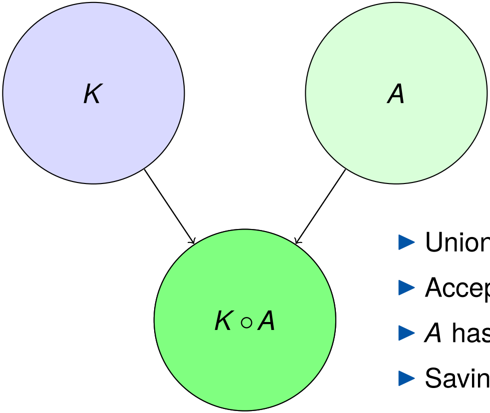
- Conclusions

Belief revision

- How to react to new information? K is the knowledge base, A some new information



- Union $\rightarrow$ inconsistency
- Accept loss of beliefs
- A has priority over K
- Saving the most from K

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Belief change operations

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General assumption:

- A belief state is modeled by a deductively closed theory, i.e., $K = \mathrm{Cn}(K)$ with Cn the consequence operator
- $\mathcal{L}$: logical language (propositional logic)
- $\mathrm{Th}_{\mathcal{L}}$: the set of all deductively closed theories (called belief sets) over $\mathcal{L}$

Belief change operations

Most belief change operations have the form:

$$op : \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \mathrm{Th}_{\mathcal{L}}$$

- Expansion: $K + \psi := \mathrm{Cn}(K \cup \{\psi\})$
- Revision: $K \dotplus \varphi$
- Contraction: $K \dot{-} \varphi$ (removal of some belief)

Revision vs Contraction

How are revision and contraction related to each other?

Given a contraction operator, one can define a revision operator:

Levi identity

$$K \dotplus \varphi \equiv (K \dot{-} \neg\varphi) + \varphi$$

Given a revision operator, one can define a contraction operator:

Harper identity

$$K \dot{-} \varphi \equiv K \cap (K \dotplus \neg\varphi)$$

What is a good revision operator?

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Rationale of revision operator:

- Consistency: a revision has to produce a consistent set of beliefs
- Minimality of change: a revision has to change as few beliefs as possible
- Priority to the new information: the 'new' information is considered more important than the 'old' one

To characterize rational revision operators, Alchourron, Gärdenfors, and Makinson identified conditions that should be satisfied by such an operator.

AGM Postulates:
Constraining the space of revision operations

AGM postulates:

($\dot{+}$1) $K \dot{+} \varphi \in \mathrm{Th}_{\mathcal{L}}$;

($\dot{+}$2) $\varphi \in K \dot{+} \varphi$;

($\dot{+}$3) $K \dot{+} \varphi \subseteq K + \varphi$;

($\dot{+}$4) If $\neg\varphi \notin K$, then $K + \varphi \subseteq K \dot{+} \varphi$;

($\dot{+}$5) $K \dot{+} \varphi = \mathrm{Cn}(\bot)$ only if $\vdash \neg\varphi$;

($\dot{+}$6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;

Supplementary postulates:

($\dot{+}$7) $K \dot{+} (\varphi \wedge \psi) \subseteq (K \dot{+} \varphi) + \psi$;

($\dot{+}$8) If $\neg\psi \notin K \dot{+} \varphi$, then $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$.

Canonical revision operations?

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- AGM postulates do not constrain the operation with respect to varying belief sets!
- The postulates constrain the space to fully rational revision operations, but do not pick a single one.
- Revision operations are closed under intersection, so should we choose the minimum?

Remainder set

Given a belief set K and some new information φ, we are specifically interested in the maximal subtheories consistent with φ:

Definition

Let $A \cup \{\varphi\}$ be a set of formulae. The φ-remainder set of A, denoted by $A \perp \varphi$, is the set of all (inclusion-) maximal subsets B of A that do not entail φ, i.e.:

1. $\varphi \notin Cn(B)$
2. There is no set B' such that $B \subsetneq B' \subseteq A$ with $\varphi \notin Cn(B')$

Canonical revision operations: Full-meet revision

Full-meet contraction/revision

Full-meet contraction: $K \dot{-} \varphi = \bigcap(K \perp \varphi)$ (if $K \perp \varphi \neq \emptyset$; $= K$, else)
Full-meet revision: $K \dot{+} \varphi = (K \dot{-} \neg\varphi) + \varphi$.

- Is full-meet contraction reasonable?
- Easy to show: all AGM postulates are satisfied.
- But: it is far too cautious.
  Given φ is inconsistent with K, we get: $K \dot{+} \varphi = \mathrm{Cn}(\varphi)$
- More reasonable: define contraction by only considering some of the remainders: $\leadsto$ partial meet contraction
- Are there other revision schemes?

Belief revision schemes

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- Preference information (what to keep and what to give up)
- ...may be different for different K's, but independent from the new information φ

$\leadsto$ compose revision operation pointwise for each K

- In general, a belief revision scheme (BRS) is a "recipe" for deriving a revision operation – restricted to a particular set K – from
  - the belief set and
  - preference information over this belief set

Examples

Partial meet revision (AGM): Preference information is given by a selection function γ over the set of maximal subtheories consistent with the new information:

$$K \dotplus \varphi \stackrel{\text{def}}{=} \left(\bigcap \gamma(K \perp \neg\varphi)\right) + \varphi.$$

Cut revision (GM): Preference information is given by a complete preorder $\preceq$ over all $\psi \in K$:

$$K \dotplus \varphi \stackrel{\text{def}}{=} \{\psi \in K \mid \neg\varphi \prec \psi\} + \varphi.$$

Provided $\preceq$ satisfies a number of axioms (epistemic entrenchment), cut revisions correspond to fully rational revision operations.

Revision – Viewed computationally

- We don't want to deal with deductively closed theories . . .
- Consider belief bases (finite sets of propositions) to represent belief sets.
- We don't want to specify an arbitrary amount of preference information . . .
- A theory K over the propositional logic $\mathcal{L}$ with n propositional atoms can have as much as
  - 2^{2^n} different propositions,
  - 2^n different models.
- Consider ways of specifying preference information in a concise way, i.e., polynomial in the size of the belief base.

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Base revision schemes

- Start with a finite belief base A and preference information over the elements of A ...
- We want to generate a revision operation (restricted to $\mathrm{Cn}(A)$)
- Assume a partitioning of A into n priority classes $A_1, \ldots, A_n$ such that the elements of A_i are more important or relevant than those of A_j for $j < i$
- Equivalently, consider a complete preorder $\trianglelefteq$ over A comparing priorities (epistemic relevance)
- Define a (base) revision scheme that keeps as many of the more relevant propositions as possible

$\Rightarrow$ Base revision schemes generate revision operations in the same way as ordinary schemes do.

Example: Prioritized Meet-Base Revision

Let $(A \Downarrow \varphi)$ be the maximal subsets of A that are consistent with $\neg\varphi$ and maximize relevant propositions.

Definition

Let $A \cup \{\varphi\}$ be a set of formulae. The prioritized base-removal $A \Downarrow \varphi$ is the set of all subsets B of A such that:

1. $\varphi \notin Cn(B)$
2. For each $C \subseteq A$ and $1 \leq j \leq n$, if $B \cap \bigcup_{i \geq j} A_i \subsetneq C \cap \bigcup_{i \geq j} A_i$, then $\varphi \in Cn(C \cap \bigcup_{i \geq j} A_i)$.

Note that the 2nd condition is equivalent to:
For each $1 \leq j \leq n$ and each $C \subseteq \bigcup_{i \geq j} A_i$, if $B \cap \bigcup_{i \geq j} A_i \subsetneq C$, then $\varphi \in Cn(C)$.

Example: Prioritized Meet-Base Revision

Prioritized Meet-Base Revision (PMBR):

$$A \oplus \varphi \stackrel{\text{def}}{=} \left(\bigcap_{B \in (A \Downarrow \neg\varphi)} \mathrm{Cn}(B) \right) + \varphi.$$

Define a revision operation $\dot{+}$ on $\mathrm{Cn}(A)$ (that depends on A and the priority information) by

$$\mathrm{Cn}(A) \dot{+} \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

Properties of PMBRs

- Generates partial meet revision, but does not satisfy ($\dotplus$8) in general.
- Deciding whether $A \oplus \varphi \vdash \psi$ is Π_2^p-complete, even for one priority class.
- A revised base can be represented by

$$A \oplus \varphi = \mathrm{Cn}\Big(\big(\bigvee (A \Downarrow \neg\varphi)\big) \wedge \varphi\Big).$$

- A revised base can become exponentially large:

$$A = \{p_1, \dots, p_m, q_1, \dots, q_m\}, \quad \varphi = \bigwedge_{i=1}^{m} (p_i \leftrightarrow \neg q_i)$$

  $(A \Downarrow \varphi)$ has size exponential in $|A|$.
- Worse, in some cases there exists no concise representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).

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Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

- Given K and a revision operation $\dot{+}$, a nonmonotonic consequence relation can be defined as follows: $\varphi \mid\!\sim \psi$ iff $\psi \in K \dot{+} \varphi$.

In this case,

- the rationality postulates correspond to principles of NMR (such as cautious monotonicity, etc.);
- in the case of prerequisite-free, normal defaults D, the cautions conclusions from (W, D) are simply $D \oplus W$ with one priority level;
- a similar relationship holds between Brewka's level default theories and PMBRs.

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NMR Principles and Rationality Postulates

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($\dot{+}2$) $\varphi \in K \dot{+} \varphi$;

- Reflexivity

($\dot{+}3$) $K \dot{+} \varphi \subseteq K + \varphi$;

- Supraclassicality

($\dot{+}6$) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;

- Left Logical Equivalence

($\dot{+}8$) If $\neg\psi \notin K \dot{+} \varphi$,
then $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$;

- Rational Monotonicity

Conclusions from the Correspondence

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- NMR can be thought of as the other side of the same coin.
- NMR (at least for default logic) is as hard as belief revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in polynomial space, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a denser way than classical logic, and with that come higher computational costs.

Outlook & Summary

- While NMR and Belief Revision seem to be the two sides of the same coin, there are notable pragmatic differences:
  - Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use dense representations.
  - A similar argument could be made for the computational complexity.
- NMR and Belief Revision can be thought of as qualitative ways of dealing with uncertainty in a purely logical setting.
- There exists a strong correspondence between NMR and Belief Revision.
- Both are computationally expensive and representational problematic.
- There are cases, though, that are tractable and practical.

3 Literature

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