Principles of AI Planning

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Exercise Sheet 8

Due: Friday, December 18th, 2015

Exercise 8.1 (Abstraction heuristics, 2+2 points)

A state of a 15-puzzle planning task is given as a permutation $\langle b, t_1, \ldots, t_{15} \rangle$ of $\{1, \ldots, 16\}$, where b denotes the empty tile (blank) and all other components denote the positions of the tiles.
Let $T^1 = \{t_1^1, \ldots, t_n^1\}, T^2 = \{t_1^2, \ldots, t_m^2\}$ with $1 \leq n, m \leq 14$ be a partitioning of $\{t_1, \ldots, t_{15}\}$ (i.e., $T^1 \cup T^2 = \{t_1, \ldots, t_{15}\}$ and $T^1 \cap T^2 = \emptyset$). Consider the following abstractions:

- $\alpha_1(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t_1^1, \ldots, t_m^1 \rangle$
- $\alpha_2(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t_1^2, \ldots, t_n^2 \rangle$
- $\alpha_3(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t_1^1, \ldots, t_m^1 \rangle$
- $\alpha_4(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t_1^2, \ldots, t_n^2 \rangle$

For $1 \leq i \leq 4$, the heuristic estimates of h_i are equal to lengths of optimal plans in the respective abstractions (e.g., $h_i(s) = h^*(\alpha_i(s))$. Show that:

(a) $h_1 + h_2$ is not admissible.

(b) $h_3 + h_4$ is admissible.

Hint: A heuristic is admissible if it is goal-aware and consistent.

Exercise 8.2 (Accuracy of pattern database heuristics, 3 points)

Show that PDB heuristics can become arbitrarily inaccurate even if almost all variables are represented in the pattern.

More formally: Show that there exists a family of FDR planning tasks $(\Pi_n)_{n \in \mathbb{N}}$ (that are not trivially unsolvable and that contain no trivially inapplicable operators) with $\Pi_n = (V_n, I_n, O_n, \gamma_n)$ where $|V_n| = \Theta(n)$, $h^*(I_n) = \Omega(n)$, and such that for all patterns $P_n \subsetneq V_n$ with $|P_n| = |V_n| - 1$, we have $h^{P_n}(I_n) = O(1)$.

Hint: Generalize the logistics example from slides 8-10 in `aip12.pdf`.

Exercise 8.3 (Syntactic projections vs. projections, 3 points)

Let Π be a planning task in finite-domain representation and let P be a pattern for Π. Show that: If Π is a SAS$^+$ planning task that is not trivially unsolvable and that has no trivially inapplicable operators, then $\mathcal{T}(\Pi|_P) \overset{\mathrm{G}}{\sim} \mathcal{T}(\Pi)^{\pi_P}$.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.