Principles of AI Planning

Prof. Dr. B. Nebel, Dr. R. Mattmüller
D. Speck
Winter Semester 2015/2016

University of Freiburg
Department of Computer Science

Exercise Sheet 5

Due: Friday, November 27th, 2015

Exercise 5.1 (Delete relaxation, 2+2 points)
Consider the planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form with

$$\begin{aligned}
A &= \{\mathit{haveCake}, \mathit{eatenCake}, \mathit{haveNoCake}\}, \\
I &= \{\mathit{have\text{-}cake} \mapsto 0, \mathit{eatenCake} \mapsto 0, \mathit{haveNoCake} \mapsto 1\} \\
O &= \{\mathit{eatCake}, \mathit{bakeCake}\}, \\
\mathit{eatCake} &= \langle \mathit{haveCake}, \neg \mathit{haveCake} \wedge \mathit{haveNoCake} \wedge \mathit{eatenCake} \rangle, \\
\mathit{bakeCake} &= \langle \mathit{haveNoCake}, \mathit{haveCake} \wedge \neg \mathit{haveNoCake} \rangle \text{ und} \\
\gamma &= \mathit{haveCake} \wedge \mathit{eatenCake}.
\end{aligned}$$

(a) Give the relaxation Π^+ of Π.
(b) Give a sequence π of operators (as short as possible) from O such that π is *not* a plan of Π, but π^+ is a plan of Π^+.

Exercise 5.2 (h^+ heuristic, 2+2 points)
A 15-puzzle planning task $\Pi = \langle A, I, O, \gamma \rangle$ is given as

$$\begin{aligned}
A &= \{\mathit{empty}(p_{i,j}) \mid 0 \leq i, j \leq 3\} \cup \{\mathit{at}(t_k, p_{i,j}) \mid 0 \leq i, j \leq 3, 0 \leq k \leq 14\}, \\
O &= \{\mathit{move}(t_m, p_{i,j}, p_{k,l}) \mid 0 \leq i, j, k, l \leq 3, 0 \leq m \leq 14, \\
&\quad (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1)\}, \\
\gamma &= \bigwedge_{0 \leq m \leq 14} \mathit{at}(t_m, p_{\lfloor m/4 \rfloor, m\%4})
\end{aligned}$$

Action $\mathit{move}(t_m, p_{i,j}, p_{k,l})$ moves tile t_m from position $p_{i,j}$ to position $p_{k,l}$:

$$\begin{aligned}
\mathit{move}(t_m, p_{i,j}, p_{k,l}) = \langle &\mathit{at}(t_m, p_{i,j}) \wedge \mathit{empty}(p_{k,l}), \\
&\mathit{at}(t_m, p_{k,l}) \wedge \mathit{empty}(p_{i,j}) \wedge \neg \mathit{at}(t_m, p_{i,j}) \wedge \neg \mathit{empty}(p_{k,l}) \rangle
\end{aligned}$$

A syntactically possible state is *legal* if each tile t_m is at some position p_{ij}, if no two tiles are at the same position and if the remaining position is the only one that is *empty*. The initial state is an arbitrary state that is legal.
One possible heuristic for the 15-puzzle is the Manhattan-distance heuristic $h^{\mathit{Manhattan}}$: It sums the Manhattan distances of all tiles from their current positions to their target positions, where the Manhattan distance between position $p_{i,j}$ and $p_{k,l}$ is given as $|i - k| + |j - l|$.
The h^+ heuristic estimates the distance of state s to the closest goal state as the length of the optimal plan in the relaxed planning task (with initial state s).

(a) Show that $h^+(s) \geq h^{\mathit{Manhattan}}(s)$ for each legal state s of a 15-puzzle planning task.
(b) Show that $h^+(s) > h^{\mathit{Manhattan}}(s)$ for at least one state s of a 15-puzzle planning task.

Exercise 5.3 (Inaccuracy of $h_{\max}$, 2 points)
Prove that the heuristic $h_{\max}$ is arbitrarily inaccurate, i.e., for all $c \in \mathbb{R}^+$ there exists a relaxed planning task $\Pi = \langle A, I, O^+, \gamma \rangle$ such that $c \cdot h_{\max}(I) \leq h^+(I)$.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.