

# Principles of AI Planning

## 15. Nondeterministic planning

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# 1 Motivation



Motivation

Transition  
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planning  
tasks

Plans



- The world is not predictable.
- AI robotics:
  - imprecise movement of the robot
  - other robots
  - human beings, animals
  - machines (cars, trains, airplanes, lawn-mowers, ...)
  - natural phenomena (wind, water, snow, temperature, ...)
- Games: other players are outside our control.
  - To win a game (reaching a goal state) with certainty, all possible actions by the other players have to be anticipated (a **winning strategy** of a game).
  - The world is not predictable because it is unknown: we cannot **observe** everything.

Motivation

Transition  
systems and  
planning  
tasks

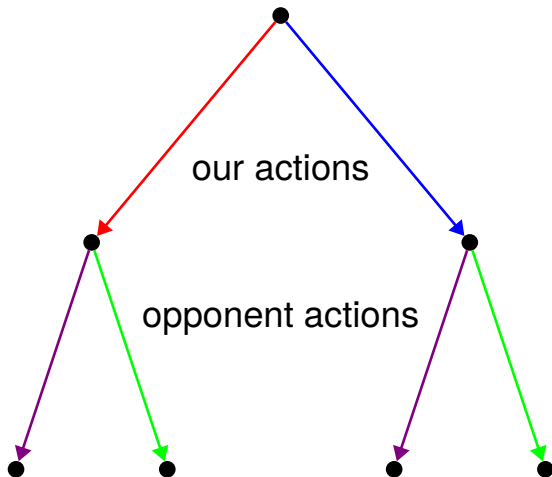
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In this lecture, we will only deal with uncertain operator outcomes, not with partial observability.

Motivation

Transition  
systems and  
planning  
tasks

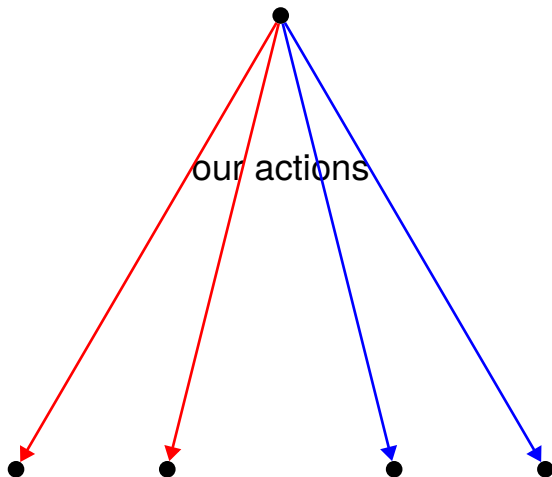
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Motivation

Transition  
systems and  
planning  
tasks

Plans





- In **deterministic planning** we have assumed that the only changes taking place in the world are those caused by us and that we can **exactly predict** the results of our actions.
- **Other agents** and processes, beyond our control, are formalized as **nondeterminism**.
- Implications:
  - 1 The future state of the world cannot be predicted.
  - 2 We cannot reliably plan ahead: no single operator sequence achieves the goals.
  - 3 In some cases it is not possible to achieve the goals with certainty no matter which outcomes the actions have, but only under certain fairness assumptions.

Motivation

Transition  
systems and  
planning  
tasks

Plans

## 2 Transition systems and planning tasks



- Transition systems
- Operators
- Planning tasks

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans

# Transition systems with nondeterminism (cf. Chapter 2)

## Definition (transition system)

A **nondeterministic transition system** is a 5-tuple

$\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$  where

- $S$  is a finite set of **states**,
- $L$  is a finite set of (transition) **labels**,
- $T \subseteq S \times L \times S$  is the **transition relation**,
- $s_0 \in S$  is the **initial state**, and
- $S_\star \subseteq S$  is the set of **goal states**.

**Note:**  $T \subseteq S \times L \times S$  allows **nondeterministic operators** with more than one possible outcome.

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans



## Definition (nondeterministic operator)

Let  $V$  be a set of finite-domain state variables. A nondeterministic operator in unary nondeterminism normal form with conjunctive precondition and unconditional effects, or **nondeterministic operator** for short, is a pair  $o = \langle \chi, E \rangle$ , where

- $\chi$  is a conjunction of atoms over  $V$  (the **precondition**), and
- $E = \{e_1, \dots, e_n\}$  is a finite set of possible **effects** of  $o$ , each  $e_i$  being a conjunction of atomic finite-domain effects over  $V$ .

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans

## Definition (nondeterministic operator application)

Let  $o = \langle \chi, E \rangle$  be a nondeterministic operator and  $s$  a state.

Applicability of  $o$  in  $s$  is defined as in the deterministic case, i.e.,  $o$  is **applicable** in  $s$  iff  $s \models \chi$  and the change set of each effect  $e \in E$  is consistent.

If  $o$  is applicable in  $s$ , then the **application** of  $o$  in  $s$  leads to one of the states in the set  $app_o(s) := \{app_{\langle \chi, e \rangle}(s) \mid e \in E\}$  nondeterministically.

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans

## Example

*put-on-block*( $A, B$ ) =  $\langle \chi, \{e_1, e_2\} \rangle$  where

- $\chi = \{handempty \mapsto false, clear-B \mapsto true, pos-A \mapsto hand\}$ ,
- $e_1 = \{handempty \mapsto true, clear-B \mapsto false, pos-A \mapsto on-B\}$ ,
- $e_2 = \{handempty \mapsto true, pos-A \mapsto table\}$ .

Applied to a state where the agent is holding block  $A$  and block  $B$  is clear, this operator leads to one of two possible successor states. Either  $A$  gets stacked on  $B$  successfully, or  $A$  is dropped to the table.

Motivation

Transition systems and  
planning tasks

Transition systems  
Operators

Planning tasks

Plans

## Definition (nondeterministic planning task)

A (fully observable) **nondeterministic planning task** is a 4-tuple  $\Pi = \langle V, I, O, \gamma \rangle$  where

- $V$  is a finite set of **finite-domain state variables**,
- $I$  is an **initial state** over  $V$ ,
- $O$  is a finite set of **nondeterministic operators** over  $V$ , and
- $\gamma$  is a conjunctions of atoms over  $V$  describing the **goal states**.

**Remark:** In the following, we will always assume that our nondeterministic planning tasks are fully observable.

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans

## Definition (induced transition system)

Every nondeterministic planning task  $\Pi = \langle V, I, O, \gamma \rangle$  induces a corresponding nondeterministic transition system

$$\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle:$$

- $S$  is the set of all states over  $V$ ,
- $L$  is the set of operators  $O$ ,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' \in \text{app}_o(s) \}$ ,
- $s_0 = I$ , and
- $S_\star = \{ s \in S \mid s \models \gamma \}$

Motivation

Transition systems and planning tasks

Transition systems

Operators

Planning tasks

Plans

# 3 Plans



- Motivation
- Definition

Motivation

Transition systems and planning tasks

**Plans**

Motivation

Definition

# What is a plan?



In nondeterministic planning, plans are more complicated objects than in the deterministic case:

The best action to take may **depend on nondeterministic effects** of previous operators.

Nondeterministic plans thus often require **branching**.  
Sometimes, they even require **looping**.

Motivation

Transition systems and planning tasks

Plans

Motivation

Definition

## Example (Branching)

(Part of) a plan for winning the game **Connect Four** can be described as follows:

- Place a tile in the 4th column.
  - If opponent places a tile in the 1st, 4th or 7th column, place a tile in the 4th column.
  - If opponent places a tile in the 2nd or 5th column, place a tile in the 2nd column.
  - If opponent places a tile in the 3rd or 6th column, place a tile in the 6th column.

There is no **non-branching** plan that solves the task  
(= is guaranteed to win the game).

Motivation

Transition  
systems and  
planning  
tasks

Plans

Motivation

Definition



# What is a plan?



## Example (Looping)

A plan for building a card house can be described as follows:

- 1 Build a wall with two cards.  
If the structure falls apart, redo from start.
- 2 Build a second wall with two cards.  
If the structure falls apart, redo from start.
- 3 Build a ceiling on top of the walls with a fifth card.  
If the structure falls apart, redo from start.
- 4 Build a wall on top of the ceiling with two cards.  
If the structure falls apart, redo from start.

There is no **non-looping** plan that solves the task (unless the planning agent is very dextrous).

Motivation

Transition  
systems and  
planning  
tasks

Plans

Motivation  
Definition

# What is a plan?

- Plans should be allowed to **branch**. Otherwise, most interesting nondeterministic planning tasks cannot be solved.
- We may or may not allow plans to **loop**.
  - Non-looping plans are preferable because they **guarantee** that the goal is reached within a bounded number of steps.
  - Where non-looping plans are not possible, looping plans may be adequate because they at least guarantee that the goal will be reached **eventually** unless nature is **unfair**.

We will now introduce the formal concepts necessary to define branching and looping plans.

Motivation

Transition  
systems and  
planning  
tasks

Plans

Motivation  
Definition

## Definition (strategy)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ .

A **strategy** for  $\Pi$  is a function  $\pi : S_\pi \rightarrow O$  for some subset  $S_\pi \subseteq S$  such that for all states  $s \in S_\pi$  the action  $\pi(s)$  is applicable in  $s$ .

The set of states reachable in  $\mathcal{T}(\Pi)$  starting in state  $s$  and following  $\pi$  is denoted by  $S_\pi(s)$ .

Motivation

Transition systems and planning tasks

Plans

Motivation

Definition

## Definition (weak, closed, proper, and acyclic strategies)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ , and let  $\pi$  be a strategy for  $\Pi$ .

Then  $\pi$  is called

- **weak** iff  $S_\pi(s_0) \cap S_* \neq \emptyset$ ,
- **closed** iff  $S_\pi(s_0) \subseteq S_\pi \cup S_*$ ,
- **proper** iff  $S_\pi(s') \cap S_* \neq \emptyset$  for all  $s' \in S_\pi(s_0)$ , and
- **acyclic** iff there is no state  $s' \in S_\pi(s_0)$  such that  $s'$  is reachable from  $s'$  following  $\pi$  in a strictly positive number of steps.

Motivation

Transition systems and planning tasks

Plans

Motivation

Definition



- **Strategies** in nondeterministic planning correspond to **applicable operator sequences** in deterministic planning.
- In deterministic planning, a **plan** is an applicable operator sequence that results in a goal state.
- In nondeterministic planning, we define different notions of “resulting in a goal state”.

Motivation

Transition  
systems and  
planning  
tasks

Plans

Motivation

Definition

## Definition

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ .

- A strategy for  $\Pi$  is called a **weak plan** for  $\Pi$  iff it is weak.
- A strategy for  $\Pi$  is called a **strong cyclic plan** for  $\Pi$  iff it is closed and proper.
- A strong cyclic plan for  $\Pi$  is called a **strong plan** for  $\Pi$  iff it is acyclic.

Motivation

Transition systems and planning tasks

Plans

Motivation

Definition



We extended the deterministic (**classical**) planning formalism:

- **operators** can be nondeterministic

**Remark:** We could also introduce nondeterminism in the initial situation by allowing more than one initial state, but this can be easily compiled into our formalism. (**How?**)

As a consequence, **plans** can contain

- **branches** and
- **loops**.

In the following chapter, we consider the **strong planning** problem and the **strong cyclic planning** problem and discuss some algorithms.

Motivation

Transition systems and planning tasks

Plans

Motivation

Definition