Principles of AI Planning

5. Planning as search: progression and regression

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1 Planning as (classical) search

- Introduction
- Classification of search-based planners

What do we mean by search?

- Search is a very generic term.

⇝ Every algorithm that tries out various alternatives can be said to "search" in some way.

- Here, we mean classical search algorithms.
  - Search nodes are expanded to generate successor nodes.
  - Examples: breadth-first search, A*, hill-climbing, ...
- To be brief, we just say search in the following (not "classical search").

Do you know this stuff already?

- We assume prior knowledge of basic search algorithms:
  - uninformed vs. informed
  - systematic vs. local
- There will be a small refresher in the next chapter.
- Background: Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)

Search in planning

- search: one of the big success stories of AI
- many planning algorithms based on classical AI search (we'll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)

Satisficing or optimal planning?

Must carefully distinguish two different problems:

- satisficing planning: any solution is OK (although shorter solutions typically preferred)
- optimal planning: plans must have shortest possible length

Both are often solved by search, but:

- details are very different
- almost no overlap between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners

Planning by search

How to apply search to planning? $\rightsquigarrow$ many choices to make!

Choice 1: Search direction

- progression: forward from initial state to goal
- regression: backward from goal states to initial state
- bidirectional search

Planning by search

How to apply search to planning? $\rightsquigarrow$ many choices to make!

Choice 2: Search space representation

- search nodes are associated with states ($\rightsquigarrow$ state-space search)
- search nodes are associated with sets of states

Planning by search

How to apply search to planning? ⇝ many choices to make!

Choice 3: Search algorithm

- uninformed search:
  depth-first, breadth-first, iterative depth-first, . . .
- heuristic search (systematic):
  greedy best-first, A^*, Weighted A^*, IDA^*, . . .
- heuristic search (local):
  hill-climbing, simulated annealing, beam search, . . .

Planning by search

Search
Introduction
Classification
Progression
Regression
Summary

How to apply search to planning? ⇝ many choices to make!

Choice 4: Search control

- heuristics for informed search algorithms
- pruning techniques: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, . . .

Search-based satisficing planners

FF (Hoffmann & Nebel, 2001)

- search direction: forward search
- search space representation: single states
- search algorithm: enforced hill-climbing (informed local)
- heuristic: FF heuristic (inadmissible)
- pruning technique: helpful actions (incomplete)

⇝ one of the best satisficing planners

Search-based optimal planners

Fast Downward Stone Soup (Helmert et al., 2011)

- search direction: forward search
- search space representation: single states
- search algorithm: A^* (informed systematic)
- heuristic: multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, . . .)
- pruning technique: none

⇝ one of the best optimal planners

Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   ⇝ this chapter
2. search space representation: states/sets of states
   ⇝ this chapter
3. search algorithm: uninformed/heuristic; systematic/local
   ⇝ next chapter
4. search control: heuristics, pruning techniques
   ⇝ following chapters

2 Progression

- Overview
- Example

Planning by forward search: progression

Progression: Computing the successor state $app_o(s)$ of a state s with respect to an operator o.

Progression planners find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and progress it through an operator, generating a new state
- solution found when a goal state generated

pro: very easy and efficient to implement

Search space representation in progression planners

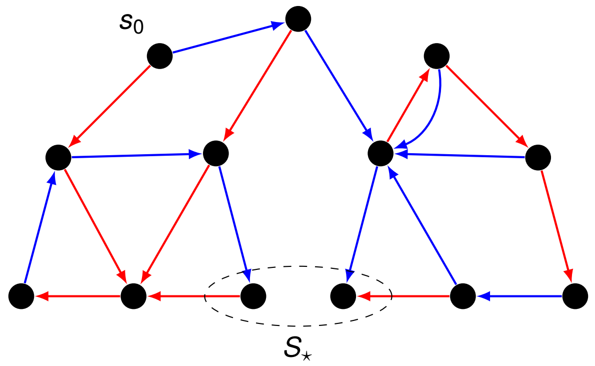
Two alternative search spaces for progression planners:

1. search nodes correspond to states
   - when the same state is generated along different paths, it is not considered again (duplicate detection)
   - pro: save time to consider same state again
   - con: memory intensive (must maintain closed list)
2. search nodes correspond to operator sequences
   - different operator sequences may lead to identical states (transpositions); search does not notice this
   - pro: can be very memory-efficient
   - con: much wasted work (often exponentially slower)

⇝ first alternative usually preferable in planning
(unlike many classical search benchmarks like 15-puzzle)

Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



Progression planning example (depth-first search)

UNI FREIBURG
Search
Progression
Overview
Example
Regression
Summary

Example where search nodes correspond to operator sequences
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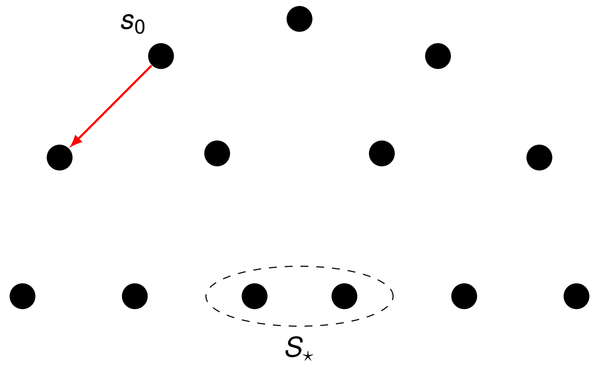

Progression planning example (depth-first search)

UNI FREIBURG
Search
Progression
Overview
Example
Regression
Summary

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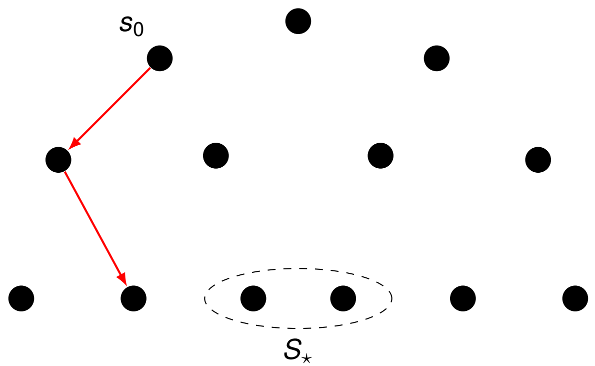

Progression planning example (depth-first search)

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Search
Progression
Overview
Example
Regression
Summary

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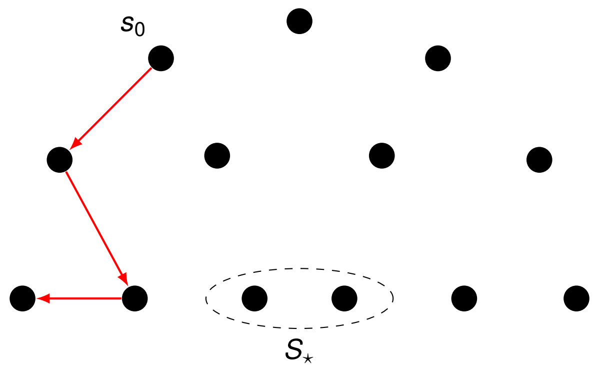
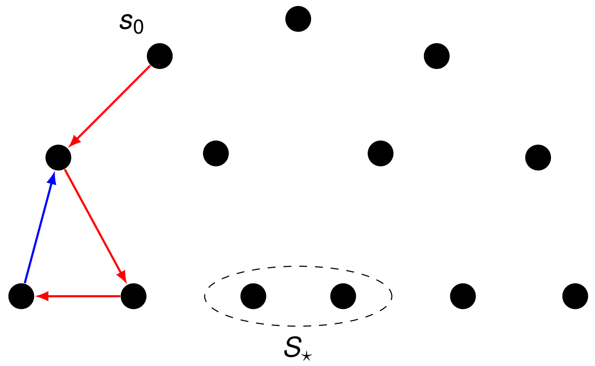


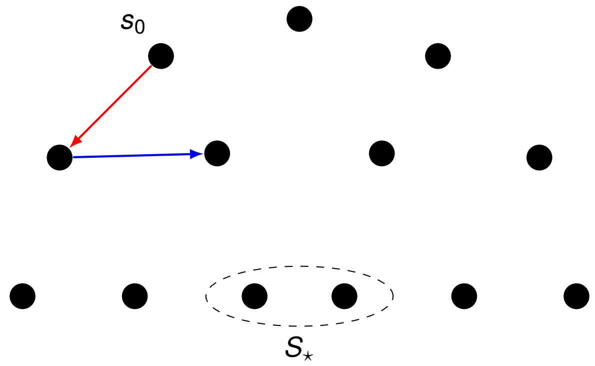
Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



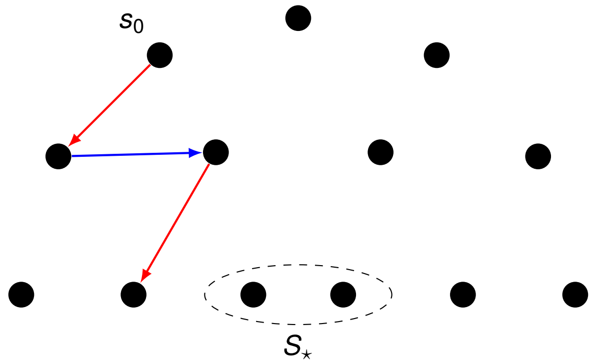
Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



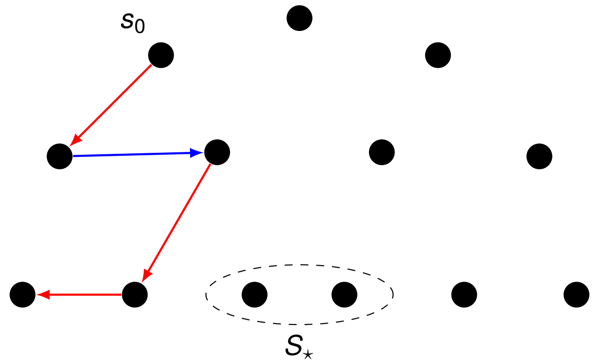
Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences
(no duplicate detection)



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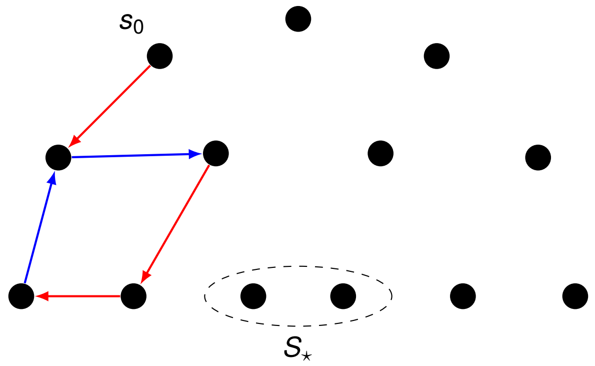

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Example where search nodes correspond to operator sequences
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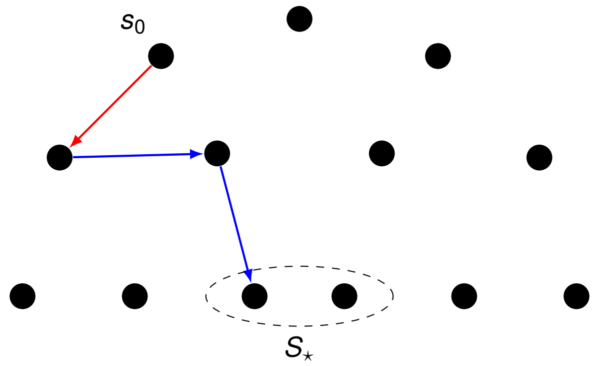


3 Regression

- Overview
- Example
- Regression for STRIPS tasks
- Regression for general planning tasks
- Practical issues

Forward search vs. backward search

Going through a transition graph in forward and backward directions is not symmetric:

- forward search starts from a single initial state; backward search starts from a set of goal states
- when applying an operator o in a state s in forward direction, there is a unique successor state s'; if we applied operator o to end up in state s', there can be several possible predecessor states s

$\rightsquigarrow$ most natural representation for backward search in planning associates sets of states with search nodes

Planning by backward search: regression

Regression: Computing the possible predecessor states $regr_o(G)$ of a set of states G with respect to the last operator o that was applied.

Regression planners find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated state set and regress it through an operator, generating a new state set
- solution found when a generated state set includes the initial state

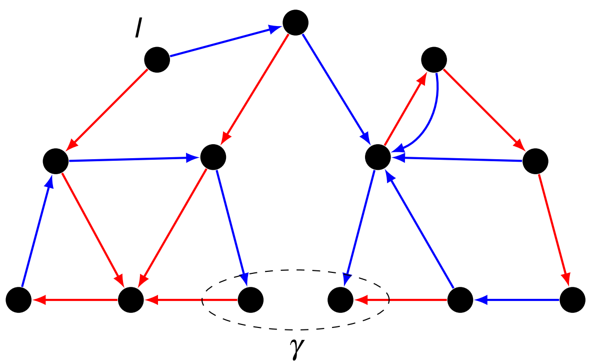
Pro: can handle many states simultaneously
Con: basic operations complicated and expensive

Search space representation in regression planners

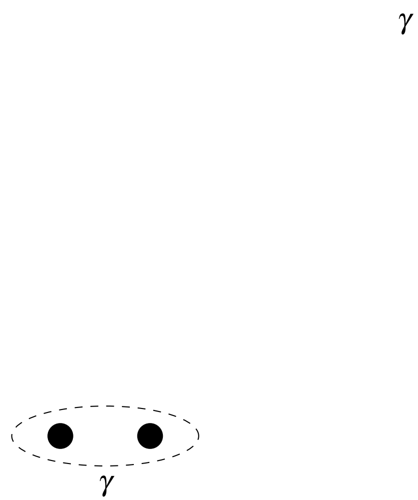
identify state sets with logical formulae (again):

- search nodes correspond to state sets
- each state set is represented by a logical formula: φ represents $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-hard or coNP-hard

Regression planning example (depth-first search)



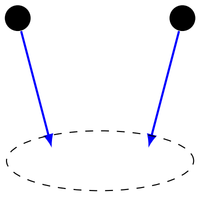
Regression planning example (depth-first search)



Regression planning example (depth-first search)

$\varphi_1 = regr_{\longrightarrow}(\gamma)$

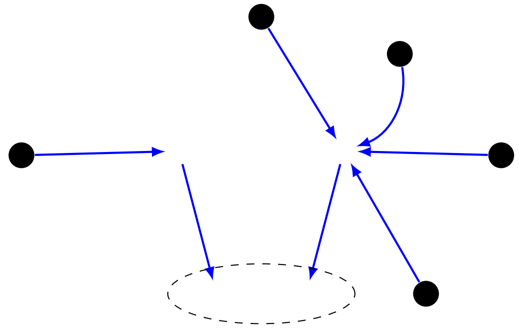
$\varphi_1 \longrightarrow \gamma$



Regression planning example (depth-first search)

$\varphi_1 = regr_{\longrightarrow}(\gamma)$
$\varphi_2 = regr_{\longrightarrow}(\varphi_1)$

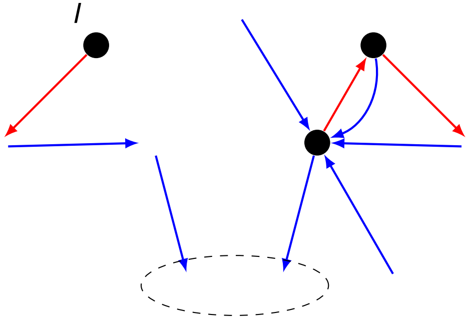
$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



Regression planning example (depth-first search)

$\varphi_1 = regr_{\longrightarrow}(\gamma)$
$\varphi_2 = regr_{\longrightarrow}(\varphi_1)$
$\varphi_3 = regr_{\longrightarrow}(\varphi_2), I \models \varphi_3$

$\varphi_3 \longrightarrow \varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



Regression for STRIPS planning tasks

Definition (STRIPS planning task)

A planning task is a STRIPS planning task if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression for STRIPS planning tasks is very simple:

- Goals are conjunctions of atoms $a_1 \wedge \cdots \wedge a_n$.
- First step: Choose an operator that makes none of $a_1, \ldots, a_n$ false.
- Second step: Remove goal atoms achieved by the operator (if any) and add its preconditions.

⇝ Outcome of regression is again conjunction of atoms.

Optimization: only consider operators making some a_i true

STRIPS regression

Definition (STRIPS regression)

Let $\varphi = \varphi_1 \wedge \cdots \wedge \varphi_n$ be a conjunction of atoms, and let $o = \langle \chi, e \rangle$ be a STRIPS operator which adds the atoms $a_1, \ldots, a_k$ and deletes the atoms $d_1, \ldots, d_l$.

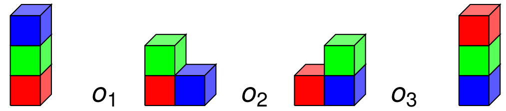
The STRIPS regression of φ with respect to o is

$$sregr_o(\varphi) := \begin{cases} \bot & \text{if } a_i = d_j \text{ for some } i,j \\ \bot & \text{if } \varphi_i = d_j \text{ for some } i,j \\ \chi \wedge \bigwedge(\{\varphi_1, \ldots, \varphi_n\} \setminus \{a_1, \ldots, a_k\}) & \text{otherwise} \end{cases}$$

Note: $sregr_o(\varphi)$ is again a conjunction of atoms, or $\bot$.

STRIPS regression example

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary



Note: Predecessor states are in general not unique.
This picture is just for illustration purposes.

$o_1 = \langle {\color{blue}\blacksquare} on {\color{green}\blacksquare} \wedge {\color{blue}\blacksquare} clr, \quad \neg {\color{blue}\blacksquare} on {\color{green}\blacksquare} \wedge {\color{blue}\blacksquare} onT \wedge {\color{green}\blacksquare} clr \rangle$

$o_2 = \langle {\color{green}\blacksquare} on {\color{red}\blacksquare} \wedge {\color{green}\blacksquare} clr \wedge {\color{blue}\blacksquare} clr, \quad \neg {\color{blue}\blacksquare} clr \wedge \neg {\color{green}\blacksquare} on {\color{red}\blacksquare} \wedge {\color{green}\blacksquare} on {\color{blue}\blacksquare} \wedge {\color{red}\blacksquare} clr \rangle$

$o_3 = \langle {\color{red}\blacksquare} onT \wedge {\color{red}\blacksquare} clr \wedge {\color{green}\blacksquare} clr, \quad \neg {\color{green}\blacksquare} clr \wedge \neg {\color{red}\blacksquare} onT \wedge {\color{red}\blacksquare} on {\color{green}\blacksquare} \rangle$

$\gamma = {\color{red}\blacksquare} on {\color{green}\blacksquare} \wedge {\color{green}\blacksquare} on {\color{blue}\blacksquare}$

$\varphi_1 = sregr_{o_3}(\gamma) = {\color{red}\blacksquare} onT \wedge {\color{red}\blacksquare} clr \wedge {\color{green}\blacksquare} clr \wedge {\color{green}\blacksquare} on {\color{blue}\blacksquare}$

$\varphi_2 = sregr_{o_2}(\varphi_1) = {\color{green}\blacksquare} on {\color{red}\blacksquare} \wedge {\color{green}\blacksquare} clr \wedge {\color{blue}\blacksquare} clr \wedge {\color{red}\blacksquare} onT$

$\varphi_3 = sregr_{o_1}(\varphi_2) = {\color{blue}\blacksquare} on {\color{green}\blacksquare} \wedge {\color{blue}\blacksquare} clr \wedge {\color{green}\blacksquare} on {\color{red}\blacksquare} \wedge {\color{red}\blacksquare} onT$

Regression for general planning tasks

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

- With disjunctions and conditional effects, things become more tricky. How to regress $a \vee (b \wedge c)$ with respect to $\langle q, d \rhd b \rangle$?
- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.
- We present a general method for doing regression for any formula and any operator.
- Now we extensively use the idea of representing sets of states as formulae.

Effect preconditions

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Definition (effect precondition)

The effect precondition $EPC_l(e)$ for literal l and effect e is defined as follows:

$$\begin{aligned} EPC_l(l) &= \top \\ EPC_l(l') &= \bot \text{ if } l \neq l' \quad (\text{for literals } l') \\ EPC_l(e_1 \wedge \cdots \wedge e_n) &= EPC_l(e_1) \vee \cdots \vee EPC_l(e_n) \\ EPC_l(\chi \rhd e) &= EPC_l(e) \wedge \chi \end{aligned}$$

Intuition: $EPC_l(e)$ describes the situations in which effect e causes literal l to become true.

Effect precondition examples

Example

$$
\begin{aligned}
EPC_a(b \wedge c) &= \bot \vee \bot \equiv \bot \\
EPC_a(a \wedge (b \rhd a)) &= \top \vee (\top \wedge b) \equiv \top \\
EPC_a((c \rhd a) \wedge (b \rhd a)) &= (\top \wedge c) \vee (\top \wedge b) \equiv c \vee b
\end{aligned}
$$

Effect preconditions: connection to change sets

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Lemma (A)

Let s be a state, l a literal and e an effect. Then $l \in [e]_s$ if and only if $s \models EPC_l(e)$.

Proof.

Induction on the structure of the effect e.
Base case 1, $e = l$: $l \in [l]_s = \{l\}$ by definition, and $s \models EPC_l(l) = \top$ by definition. Both sides of the equivalence are true.
Base case 2, $e = l'$ for some literal $l' \neq l$: $l \notin [l']_s = \{l'\}$ by definition, and $s \not\models EPC_l(l') = \bot$ by definition. Both sides are false.

Effect preconditions: connection to change sets

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Proof (ctd.)

Inductive case 1, $e = e_1 \wedge \dots \wedge e_n$:

$$
\begin{aligned}
l \in [e]_s &\text{ iff } l \in [e_1]_s \cup \dots \cup [e_n]_s && (\text{Def } [e_1 \wedge \dots \wedge e_n]_s) \\
&\text{ iff } l \in [e']_s \text{ for some } e' \in \{e_1, \dots, e_n\} \\
&\text{ iff } s \models EPC_l(e') \text{ for some } e' \in \{e_1, \dots, e_n\} && (\text{IH}) \\
&\text{ iff } s \models EPC_l(e_1) \vee \dots \vee EPC_l(e_n) \\
&\text{ iff } s \models EPC_l(e_1 \wedge \dots \wedge e_n). && (\text{Def } EPC)
\end{aligned}
$$

Inductive case 2, $e = \chi \rhd e'$:

$$
\begin{aligned}
l \in [\chi \rhd e']_s &\text{ iff } l \in [e']_s \text{ and } s \models \chi && (\text{Def } [\chi \rhd e']_s) \\
&\text{ iff } s \models EPC_l(e') \text{ and } s \models \chi && (\text{IH}) \\
&\text{ iff } s \models EPC_l(e') \wedge \chi \\
&\text{ iff } s \models EPC_l(\chi \rhd e'). && (\text{Def } EPC)
\end{aligned}
$$

□

Effect preconditions: connection to normal form

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Remark: *EPC* vs. effect normal form

Notice that in terms of $EPC_a(e)$, any operator $\langle \chi, e \rangle$ can be expressed in effect normal form as

$$
\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \rhd a) \wedge (EPC_{\neg a}(e) \rhd \neg a)) \right\rangle,
$$

where A is the set of all state variables.

Regressing state variables

The formula $EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$ expresses the value of state variable $a \in A$ after applying o in terms of values of state variables before applying o.

Either:

- a became true, or
- a was true before and it did not become false.

Regressing state variables: examples

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Example

Let $e = (b \rhd a) \land (c \rhd \neg a) \land b \land \neg d$.

variable x	$EPC_x(e) \lor (x \land \neg EPC_{\neg x}(e))$
a	$b \lor (a \land \neg c)$
b	$\top \lor (b \land \neg \bot) \equiv \top$
c	$\bot \lor (c \land \neg \bot) \equiv c$
d	$\bot \lor (d \land \neg \top) \equiv \bot$

Regressing state variables: correctness

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Lemma (B)

Let a be a state variable, $o = \langle \chi, e \rangle$ an operator, s a state, and $s' = app_o(s)$. Then $s \models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$ if and only if $s' \models a$.

Proof.

($\Rightarrow$): Assume $s \models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$. Do a case analysis on the two disjuncts.

1. Assume that $s \models EPC_a(e)$. By Lemma A, we have $a \in [e]_s$ and hence $s' \models a$.
2. Assume that $s \models a \land \neg EPC_{\neg a}(e)$. By Lemma A, we have $\neg a \notin [e]_s$. Hence a remains true in s'.

Regressing state variables: correctness

Search
Progression
Regression
Overview
Example
STRIPS
General case
Practical issues
Summary

Proof (ctd.)

($\Leftarrow$): We showed that if the formula is true in s, then a is true in s'. For the second part, we show that if the formula is false in s, then a is false in s'.

- So assume $s \not\models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$.
- Then $s \models \neg EPC_a(e) \land (\neg a \lor EPC_{\neg a}(e))$ (de Morgan).
- Case distinction: a is true or a is false in s.
  1. Assume that $s \models a$. Now $s \models EPC_{\neg a}(e)$ because $s \models \neg a \lor EPC_{\neg a}(e)$. Hence by Lemma A $\neg a \in [e]_s$ and we get $s' \not\models a$.
  2. Assume that $s \not\models a$. Because $s \models \neg EPC_a(e)$, by Lemma A we get $a \notin [e]_s$ and hence $s' \not\models a$.

  Therefore in both cases $s' \not\models a$.

□

Regression: general definition

We base the definition of regression on formulae $EPC_l(e)$.

Definition (general regression)

Let φ be a propositional formula and $o = \langle \chi, e \rangle$ an operator.
The regression of φ with respect to o is

$$regr_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$$

where

1. φ_r is obtained from φ by replacing each $a \in A$ by $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$, and
2. $\kappa = \bigwedge_{a \in A} \neg(EPC_a(e) \wedge EPC_{\neg a}(e))$.

The formula κ expresses that operators are only applicable in states where their change sets are consistent.

Regression examples

- $regr_{\langle a,b \rangle}(b) \equiv a \wedge (\top \vee (b \wedge \neg\bot)) \wedge \top \equiv a$
- $regr_{\langle a,b \rangle}(b \wedge c \wedge d)$
  $\equiv a \wedge (\top \vee (b \wedge \neg\bot)) \wedge (\bot \vee (c \wedge \neg\bot)) \wedge (\bot \vee (d \wedge \neg\bot)) \wedge \top$
  $\equiv a \wedge c \wedge d$
- $regr_{\langle a, c \rhd b \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg\bot)) \wedge \top \equiv a \wedge (c \vee b)$
- $regr_{\langle a, (c \rhd b) \wedge (b \rhd \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg b)) \wedge \neg(c \wedge b)$
  $\equiv a \wedge c \wedge \neg b$
- $regr_{\langle a, (c \rhd b) \wedge (d \rhd \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg d)) \wedge \neg(c \wedge d)$
  $\equiv a \wedge (c \vee b) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d)$
  $\equiv a \wedge (c \vee b) \wedge \neg d$

Regression example: binary counter

$$(\neg b_0 \rhd b_0) \wedge$$
$$((\neg b_1 \wedge b_0) \rhd (b_1 \wedge \neg b_0)) \wedge$$
$$((\neg b_2 \wedge b_1 \wedge b_0) \rhd (b_2 \wedge \neg b_1 \wedge \neg b_0))$$

$EPC_{b_2}(e) = \neg b_2 \wedge b_1 \wedge b_0$
$EPC_{b_1}(e) = \neg b_1 \wedge b_0$
$EPC_{b_0}(e) = \neg b_0$
$EPC_{\neg b_2}(e) = \bot$
$EPC_{\neg b_1}(e) = \neg b_2 \wedge b_1 \wedge b_0$
$EPC_{\neg b_0}(e) = (\neg b_1 \wedge b_0) \vee (\neg b_2 \wedge b_1 \wedge b_0) \equiv (\neg b_1 \vee \neg b_2) \wedge b_0$

Regression replaces state variables as follows:

b_2 by $(\neg b_2 \wedge b_1 \wedge b_0) \vee (b_2 \wedge \neg\bot) \equiv (b_1 \wedge b_0) \vee b_2$
b_1 by $(\neg b_1 \wedge b_0) \vee (b_1 \wedge \neg(\neg b_2 \wedge b_1 \wedge b_0))$
$\equiv (\neg b_1 \wedge b_0) \vee (b_1 \wedge (b_2 \vee \neg b_0))$
b_0 by $\neg b_0 \vee (b_0 \wedge \neg((\neg b_1 \vee \neg b_2) \wedge b_0)) \equiv \neg b_0 \vee (b_1 \wedge b_2)$

General regression: correctness

Theorem (correctness of $regr_o(\varphi)$)

*Let φ be a formula, o an operator and s a state.
Then $s \models regr_o(\varphi)$ iff o is applicable in s and $app_o(s) \models \varphi$.*

Proof.

Let $o = \langle \chi, e \rangle$. Recall that $regr_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$, where φ_r and κ are as defined previously.

If o is inapplicable in s, then $s \not\models \chi \wedge \kappa$, both sides of the "iff" condition are false, and we are done. Hence, we only further consider states s where o is applicable. Let $s' := app_o(s)$.

We know that $s \models \chi \wedge \kappa$ (because o is applicable), so the "iff" condition we need to prove simplifies to:

$$s \models \varphi_r \text{ iff } s' \models \varphi.$$

General regression: correctness

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Proof (ctd.)

To show: $s \models \varphi_r$ iff $s' \models \varphi$.

We show that for all formulae ψ, $s \models \psi_r$ iff $s' \models \psi$, where ψ_r is ψ with every $a \in A$ replaced by $EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$.

The proof is by structural induction on ψ.

Induction hypothesis $s \models \psi_r$ if and only if $s' \models \psi$.

Base cases 1 & 2 $\psi = \top$ or $\psi = \bot$: trivial, as $\psi_r = \psi$.

Base case 3 $\psi = a$ for some $a \in A$:
Then $\psi_r = EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$.
By Lemma B, $s \models \psi_r$ iff $s' \models \psi$.

General regression: correctness

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Proof (ctd.)

Inductive case 1 $\psi = \neg\psi'$:

$$s \models \psi_r \text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r$$
$$\text{iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi$$

Inductive case 2 $\psi = \psi' \lor \psi''$:

$$s \models \psi_r \text{ iff } s \models (\psi' \lor \psi'')_r \text{ iff } s \models \psi'_r \lor \psi''_r$$
$$\text{iff } s \models \psi'_r \text{ or } s \models \psi''_r$$
$$\text{iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi''$$
$$\text{iff } s' \models \psi' \lor \psi'' \text{ iff } s' \models \psi$$

Inductive case 3 $\psi = \psi' \land \psi''$: Very similar to inductive case 2, just with $\land$ instead of $\lor$ and “and” instead of “or”.

Emptiness and subsumption testing

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The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $regr_o(\varphi)$ does not represent the empty set (which would mean that search is in a dead end). For example, $regr_{\langle a, \neg p \rangle}(p) \equiv a \land \bot \equiv \bot$.
- Test that $regr_o(\varphi)$ does not represent a subset of φ (which would make the problem harder than before). For example, $regr_{\langle b, c \rangle}(a) \equiv a \land b$.

Both of these problems are NP-hard.

Formula growth

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The formula $regr_{o_1}(regr_{o_2}(\ldots regr_{o_{n-1}}(regr_{o_n}(\varphi))))$ may have size $O(|\varphi||o_1||o_2|\ldots|o_{n-1}||o_n|)$, i. e., the product of the sizes of φ and the operators.
$\leadsto$ worst-case exponential size $O(m^n)$

Logical simplifications

- $\bot \land \varphi \equiv \bot$, $\top \land \varphi \equiv \varphi$, $\bot \lor \varphi \equiv \varphi$, $\top \lor \varphi \equiv \top$
- $a \lor \varphi \equiv a \lor \varphi[\bot/a]$, $\neg a \lor \varphi \equiv \neg a \lor \varphi[\top/a]$, $a \land \varphi \equiv a \land \varphi[\top/a]$, $\neg a \land \varphi \equiv \neg a \land \varphi[\bot/a]$
- idempotency, absorption, commutativity, associativity, …

Restricting formula growth in search trees

Problem: very big formulae obtained by regression

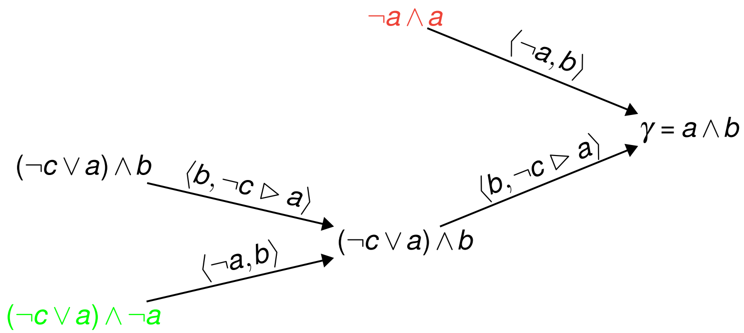
Cause: disjunctivity in the (NNF) formulae
(formulae without disjunctions easily convertible to small formulae $l_1 \wedge \dots \wedge l_n$ where l_i are literals and n is at most the number of state variables.)

Idea: handle disjunctivity when generating search trees

Unrestricted regression: search tree example

Unrestricted regression: do not treat disjunctions specially

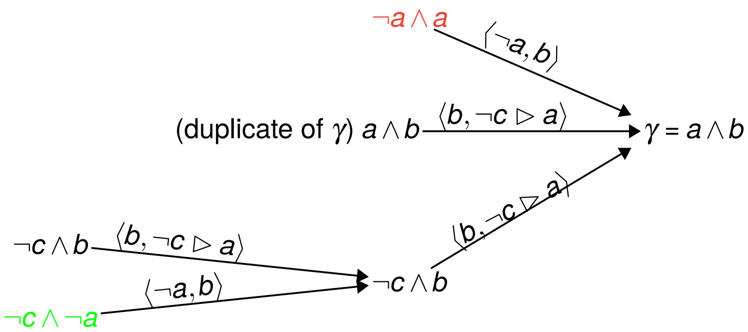
Goal $\gamma = a \wedge b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.



Full splitting: search tree example

Full splitting: always remove all disjunctivity

Goal $\gamma = a \wedge b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.
$(\neg c \vee a) \wedge b$ in DNF: $(\neg c \wedge b) \vee (a \wedge b)$
$\rightsquigarrow$ split into $\neg c \wedge b$ and $a \wedge b$



General splitting strategies

Alternatives:

1. Do nothing (unrestricted regression).
2. Always eliminate all disjunctivity (full splitting).
3. Reduce disjunctivity if formula becomes too big.

Discussion:

- With unrestricted regression the formulae may have size that is exponential in the number of state variables.
- With full splitting search tree can be exponentially bigger than without splitting.
- The third option lies between these two extremes.

Summary

- (Classical) search is a very important planning approach.
- Search-based planning algorithms differ along many dimensions, including
  - search direction (forward, backward)
  - what each search node represents
    (a state, a set of states, an operator sequence)
- Progression search proceeds forwards from the initial state.
  - If we use duplicate detection, each search node corresponds to a unique state.
  - If we do not use duplicate detection, each search node corresponds to a unique operator sequence.

Summary (ctd.)

- Regression search proceeds backwards from the goal.
  - Each search node corresponds to a set of states represented by a formula.
  - Regression is simple for STRIPS operators.
  - The theory for general regression is more complex.
  - When applying regression in practice, additional considerations such as when and how to perform splitting come into play.