

# Principles of AI Planning

## 3. Normal forms

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October 23rd, 2015



Similarly to normal forms in propositional logic (DNF, CNF, NNF, ...) we can define **normal forms for effects, operators and planning tasks**.

This is useful because algorithms (and proofs) then only need to deal with effects (resp. operators or tasks) in normal form.

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# Effect normal form

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## Definition (equivalent effects)

Two effects  $e$  and  $e'$  over state variables  $A$  are **equivalent**, written  $e \equiv e'$ , if for all states  $s$  over  $A$ ,  $[e]_s = [e']_s$ .

## Definition (equivalent operators)

Two operators  $o$  and  $o'$  over state variables  $A$  are **equivalent**, written  $o \equiv o'$ , if they are applicable in the same states, and for all states  $s$  where they are applicable,  $app_o(s) = app_{o'}(s)$ .

## Theorem

*Let  $o = \langle \chi, e \rangle$  and  $o' = \langle \chi', e' \rangle$  be operators with  $\chi \equiv \chi'$  and  $e \equiv e'$ . Then  $o \equiv o'$ .*

**Note:** The converse is not true. (Why not?)

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$$e_1 \wedge e_2 \equiv e_2 \wedge e_1 \quad (1)$$

$$(e_1 \wedge e_2) \wedge e_3 \equiv e_1 \wedge (e_2 \wedge e_3) \quad (2)$$

$$\top \wedge e \equiv e \quad (3)$$

$$\chi \triangleright e \equiv \chi' \triangleright e \quad \text{if } \chi \equiv \chi' \quad (4)$$

$$\top \triangleright e \equiv e \quad (5)$$

$$\perp \triangleright e \equiv \top \quad (6)$$

$$\chi_1 \triangleright (\chi_2 \triangleright e) \equiv (\chi_1 \wedge \chi_2) \triangleright e \quad (7)$$

$$\chi \triangleright (e_1 \wedge \dots \wedge e_n) \equiv (\chi \triangleright e_1) \wedge \dots \wedge (\chi \triangleright e_n) \quad (8)$$

$$(\chi_1 \triangleright e) \wedge (\chi_2 \triangleright e) \equiv (\chi_1 \vee \chi_2) \triangleright e \quad (9)$$



We can define a **normal form for effects**:

- Nesting of conditionals, as in  $a \triangleright (b \triangleright c)$ , can be eliminated.
- Effects  $e$  within a conditional effect  $\varphi \triangleright e$  can be restricted to atomic effects ( $a$  or  $\neg a$ ).

Transformation to this effect normal form only gives a small polynomial size increase.

**Compare:** transformation to CNF or DNF may increase formula size exponentially.

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## Definition

An operator  $\langle \chi, e \rangle$  is in **effect normal form (ENF)** if for all occurrences of  $\chi' \triangleright e'$  in  $e$  the effect  $e'$  is either  $a$  or  $\neg a$  for some  $a \in A$ , and there is at most one occurrence of any atomic effect in  $e$ .

## Theorem

*For every operator there is an equivalent one in effect normal form.*

Proof is constructive: we can transform any operator into effect normal form using the equivalence transformations for effects.

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$$\begin{aligned} &(a \triangleright (b \wedge \\ &\quad (c \triangleright (\neg d \wedge e)))) \wedge \\ &(\neg b \triangleright e) \end{aligned}$$

transformed to effect normal form is

$$\begin{aligned} &(a \triangleright b) \wedge \\ &((a \wedge c) \triangleright \neg d) \wedge \\ &((\neg b \vee (a \wedge c)) \triangleright e) \end{aligned}$$

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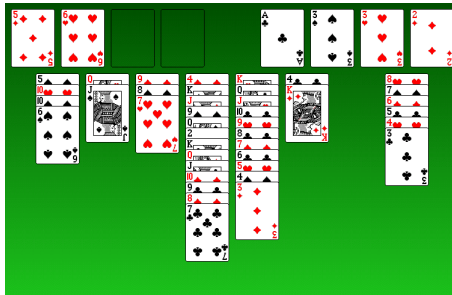
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# Example: Freecell



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## Example (good and bad effects)

If we move a card  $c$  to a free tableau position, the **good effect** is that the card formerly below  $c$  is now available.

The **bad effect** is that we lose one free tableau position.

# What is a good or bad effect?



**Question:** Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- Locking the entrance door is **good** if we want to keep burglars out.
- Locking the entrance door is **bad** if we want to enter.

We will now consider a reformulation of planning tasks that makes the distinction between good and bad effects obvious.

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## Definition (operators in positive normal form)

An operator  $o = \langle \chi, e \rangle$  is in **positive normal form** if it is in effect normal form, no negation symbols appear in  $\chi$ , and no negation symbols appear in any effect condition in  $e$ .

## Definition (planning tasks in positive normal form)

A planning task  $\langle A, I, O, \gamma \rangle$  is in **positive normal form** if all operators in  $O$  are in positive normal form and no negation symbols occur in the goal  $\gamma$ .

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## Theorem (positive normal form)

*Every planning task  $\Pi$  has an equivalent planning task  $\Pi'$  in positive normal form.*

*Moreover,  $\Pi'$  can be computed from  $\Pi$  in polynomial time.*

**Note:** Equivalence here means that the represented transition systems of  $\Pi$  and  $\Pi'$ , limited to the states that can be reached from the initial state, are isomorphic.

We prove the theorem by describing a suitable algorithm.  
(However, we do not prove its correctness or complexity.)

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## Transformation of $\langle A, I, O, \gamma \rangle$ to positive normal form

Convert all operators  $o \in O$  to effect normal form.

Convert all conditions to negation normal form (NNF).

**while** any condition contains a negative literal  $\neg a$ :

Let  $a$  be a variable which occurs negatively in a condition.

$A := A \cup \{\hat{a}\}$  for some new state variable  $\hat{a}$

$I(\hat{a}) := 1 - I(a)$

Replace the effect  $a$  by  $(a \wedge \neg \hat{a})$  in all operators  $o \in O$ .

Replace the effect  $\neg a$  by  $(\neg a \wedge \hat{a})$  in all operators  $o \in O$ .

Replace  $\neg a$  by  $\hat{a}$  in all conditions.

Convert all operators  $o \in O$  to effect normal form (again).

Here, *all conditions* refers to all operator preconditions, operator effect conditions and the goal.

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# Positive normal form: example



## Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike-locked\}$$

$$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0\}$$

$$O = \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\ \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$$

$$\gamma = lecture \wedge bike$$

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# Positive normal form: example



## Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike-locked\}$$

$$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0\}$$

$$O = \{ \langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\ \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle \}$$

$$\gamma = lecture \wedge bike$$

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Identify state variable *a* occurring negatively in conditions.



# Positive normal form: example



## Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$$

$$I = \{home \mapsto 1, bike \mapsto 1, bike\text{-}locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0, bike\text{-}unlocked \mapsto 0\}$$

$$O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \rangle, \\ \langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \triangleright \neg bike) \rangle\}$$

$$\gamma = lecture \wedge bike$$

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Introduce new variable  $\hat{a}$  with complementary initial value.

# Positive normal form: example



## Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$$
$$I = \{home \mapsto 1, bike \mapsto 1, bike\text{-}locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0, bike\text{-}unlocked \mapsto 0\}$$
$$O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \rangle, \\ \langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \triangleright \neg bike) \rangle\}$$
$$\gamma = lecture \wedge bike$$

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Identify effects on variable *a*.

# Positive normal form: example



## Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1,$   
 $uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\}$

$O = \{ \langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle,$   
 $\langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle,$   
 $\langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle,$   
 $\langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle \}$

$\gamma = lecture \wedge bike$

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Introduce complementary effects for  $\hat{a}$ .

# Positive normal form: example



## Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike\text{-}locked \mapsto 1,$   
 $uni \mapsto 0, lecture \mapsto 0, bike\text{-}unlocked \mapsto 0\}$

$O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle,$   
 $\langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \wedge bike\text{-}unlocked \rangle,$   
 $\langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \wedge \neg bike\text{-}unlocked \rangle,$   
 $\langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \triangleright \neg bike) \rangle\}$

$\gamma = lecture \wedge bike$

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Identify negative conditions for  $a$ .

# Positive normal form: example



## Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1,$   
 $uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\}$

$O = \{ \langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle,$   
 $\langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle,$   
 $\langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle,$   
 $\langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle \}$

$\gamma = lecture \wedge bike$

Replace by positive condition  $\hat{a}$ .

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# Positive normal form: example



## Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$$
$$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0, bike-unlocked \mapsto 0\}$$
$$O = \{ \langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\ \langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle \}$$
$$\gamma = lecture \wedge bike$$

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# Why positive normal form is interesting



In positive normal form, good and bad effects are easy to distinguish:

- Effects that make state variables true are good (**add effects**).
- Effects that make state variables false are bad (**delete effects**).

This is of high relevance for some planning techniques that we will see later in this course.

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# STRIPS operators

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## Definition

An operator  $\langle \chi, e \rangle$  is a **STRIPS operator** if

- $\chi$  is a conjunction of atoms, and
- $e$  is a conjunction of atomic effects.

Hence every STRIPS operator is of the form

$$\langle a_1 \wedge \dots \wedge a_n, l_1 \wedge \dots \wedge l_m \rangle$$

where  $a_i$  are atoms and  $l_j$  are atomic effects.

**Note:** Sometimes we allow conjunctions of **literals** as preconditions. We denote this as **STRIPS with negative preconditions**.

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- STRIPS operators are **particularly simple**, yet expressive enough to capture general planning problems.
- In particular, STRIPS planning is **no easier** than general planning problems.
- Most algorithms in the planning literature are **only presented for STRIPS operators** (generalization is often, but not always, obvious).

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## STRIPS

Stanford Research Institute Planning System  
(Fikes & Nilsson, 1971)



- Not every operator is equivalent to a STRIPS operator.
- However, each operator can be transformed into a **set** of STRIPS operators whose “combination” is equivalent to the original operator. (How?)
- However, this transformation may exponentially increase the number of required operators. There are planning tasks for which such a blow-up is unavoidable.
- There are polynomial transformations of planning tasks to STRIPS, but these do not preserve the structure of the transition system (e. g., length of shortest plans may change).

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- **Effect normal form** simplifies structure of operator effects: conditional effects contain only atomic effects; there is at most one occurrence of any atomic effect.
- **Positive normal form** allows to distinguish good and bad effects.
- The form of **STRIPS operators** is even more restrictive than effect normal form, forbidding complex preconditions and conditional effects.
- All three forms are expressive enough to capture general planning problems.
- Transformation to effect normal form and positive normal form possible with polynomial size increase.
- Structure preserving transformations of planning tasks to STRIPS can increase the number of operators exponentially.

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