

Principles of AI Planning

2. Transition systems and planning tasks

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1 Transition systems



- Definition
- Blocks world

Transition systems

Definition

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Summary

Definition (transition system)

A **transition system** is a 5-tuple $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ where

- S is a finite set of **states**,
- L is a finite set of (transition) **labels**,
- $T \subseteq S \times L \times S$ is the **transition relation**,
- $s_0 \in S$ is the **initial state**, and
- $S_\star \subseteq S$ is the set of **goal states**.

We say that \mathcal{T} **has the transition** $\langle s, l, s' \rangle$ if $\langle s, l, s' \rangle \in T$.

We also write this $s \xrightarrow{l} s'$, or $s \rightarrow s'$ when not interested in l .

Note: Transition systems are also called **state spaces**.

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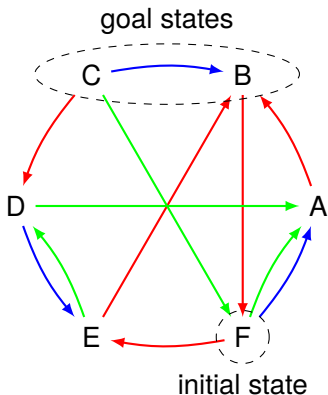
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Transition systems: example



Transition systems are often depicted as **directed arc-labeled graphs** with marks to indicate the initial state and goal states.



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We use common graph theory terms for transition systems:

- s' **successor** of s if $s \rightarrow s'$
- s **predecessor** of s' if $s \rightarrow s'$
- s' **reachable** from s if there exists a sequence of transitions

$$s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n \text{ s.t. } s^0 = s \text{ and } s^n = s'$$

- **Note:** $n = 0$ possible; then $s = s'$
- $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called **path** from s to s'
- s^0, \dots, s^n is also called **path** from s to s'
- **length** of that path is n
- additional terms: **strongly connected**, **weakly connected**, **strong/weak connected components**, ...

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Some additional terminology:

- **s' reachable** (without reference state) means reachable from initial state s_0
- **solution** or **goal path** from s : path from s to some $s' \in S_*$
 - if s is omitted, $s = s_0$ is implied
- transition system **solvable** if a goal path from s_0 exists

Definition (deterministic transition system)

A transition system with transitions T is called **deterministic** if for all states s and labels ℓ , there is **at most one** state s' with $s \xrightarrow{\ell} s'$.

Example: previously shown transition system

- Throughout the course, we will often use the **blocks world** domain as an example.
- In the blocks world, a number of differently coloured blocks are arranged on our table.
- Our job is to rearrange them according to a given goal.

Location on the table does not matter.

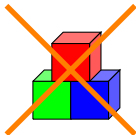


Location on a block does not matter.

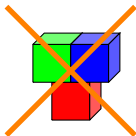


Blocks world rules (ctd.)

At most one block may be below a block.



At most one block may be on top of a block.



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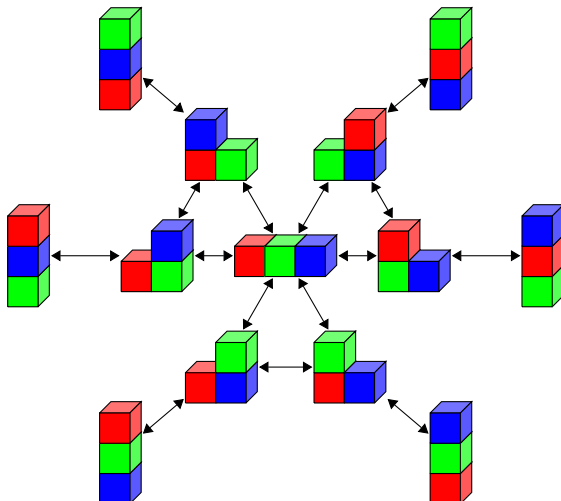
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Blocks world transition system for three blocks



(Transition labels omitted for clarity.)



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blocks	states	blocks	states
1	1	10	58941091
2	3	11	824073141
3	13	12	12470162233
4	73	13	202976401213
5	501	14	3535017524403
6	4051	15	65573803186921
7	37633	16	1290434218669921
8	394353	17	26846616451246353
9	4596553	18	588633468315403843

- Finding a solution is **polynomial time** in the number of blocks (move everything onto the table and then construct the goal configuration).
- Finding a **shortest solution** is **NP-complete** (for a compact description of the problem).

2 Planning tasks

- State variables
- Propositional logic
- Operators
- Deterministic planning tasks

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Summary

- Classical (i. e., deterministic) planning is in essence the problem of finding solutions in **huge** transition systems.
- The transition systems we are usually interested in are too large to explicitly enumerate all states or transitions.
- Hence, the input to a planning algorithm must be given in a more **concise** form.
- In the rest of chapter, we discuss how to represent planning tasks in a suitable way.

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How to represent huge state sets without enumerating them?

- represent different aspects of the world in terms of different **state variables**

↪ a state is a **valuation of state variables**

- n state variables with m possible values each induce m^n different states

↪ **exponentially more compact** than “flat” representations

- **Example:** n variables suffice for blocks world with n blocks

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Blocks world with finite-domain state variables

Describe blocks world state with three state variables:

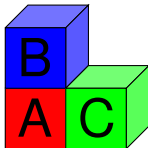
- *location-of-A*: {B, C, table}
- *location-of-B*: {A, C, table}
- *location-of-C*: {A, B, table}

Example

$s(\textit{location-of-A}) = \textit{table}$

$s(\textit{location-of-B}) = \textit{A}$

$s(\textit{location-of-C}) = \textit{table}$



Not all valuations correspond to intended blocks world states.

Example: s with $s(\textit{location-of-A}) = \textit{B}$, $s(\textit{location-of-B}) = \textit{A}$.

Problem:

- How to **succinctly** represent **transitions** and **goal states**?

Idea: Use **propositional logic**

- **state variables**: propositional variables (0 or 1)
- **goal states**: defined by a propositional formula
- **transitions**: defined by **actions** given by
 - **precondition**: when is the action applicable?
 - **effect**: how does it change the valuation?

Note: general finite-domain state variables can be compactly encoded as Boolean variables

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Example

$$s(A\text{-on-}B) = 0$$

$$s(A\text{-on-}C) = 0$$

$$s(A\text{-on-table}) = 1$$

$$s(B\text{-on-}A) = 1$$

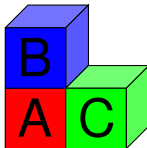
$$s(B\text{-on-}C) = 0$$

$$s(B\text{-on-table}) = 0$$

$$s(C\text{-on-}A) = 0$$

$$s(C\text{-on-}B) = 0$$

$$s(C\text{-on-table}) = 1$$



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Definition (propositional formula)

Let A be a set of **atomic propositions** (here: state variables).

The **propositional formulae** over A are constructed by finite application of the following rules:

- \top and \perp are propositional formulae (**truth** and **falsity**).
- For all $a \in A$, a is a propositional formula (**atom**).
- If φ is a propositional formula, then so is $\neg\varphi$ (**negation**).
- If φ and ψ are propositional formulas, then so are $(\varphi \vee \psi)$ (**disjunction**) and $(\varphi \wedge \psi)$ (**conjunction**).

Note: We often omit the word “propositional”.

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Abbreviations:

- $(\varphi \rightarrow \psi)$ is short for $(\neg\varphi \vee \psi)$ (**implication**)
- $(\varphi \leftrightarrow \psi)$ is short for $((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi))$ (**equivalence**)
- parentheses omitted when not necessary
- (\neg) binds more tightly than binary connectives
- (\wedge) binds more tightly than (\vee) than (\rightarrow) than (\leftrightarrow)

Definition (propositional valuation)

A **valuation** of propositions A is a function $v : A \rightarrow \{0, 1\}$.

Define the notation $v \models \varphi$ (v **satisfies** φ ; v is a **model** of φ ; φ is **true** under v) for valuations v and formulae φ by

- $v \models \top$
- $v \not\models \perp$
- $v \models a$ iff $v(a) = 1$, for $a \in A$.
- $v \models \neg\varphi$ iff $v \not\models \varphi$
- $v \models \varphi \vee \psi$ iff $v \models \varphi$ or $v \models \psi$
- $v \models \varphi \wedge \psi$ iff $v \models \varphi$ and $v \models \psi$

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- A propositional formula φ is **satisfiable** if there is at least one valuation v so that $v \models \varphi$.
- Otherwise it is **unsatisfiable**.
- A propositional formula φ is **valid** or a **tautology** if $v \models \varphi$ for all valuations v .
- A propositional formula ψ is a **logical consequence** of a propositional formula φ , written $\varphi \models \psi$, if $v \models \psi$ for all valuations v with $v \models \varphi$.
- Two propositional formulae φ and ψ are **logically equivalent**, written $\varphi \equiv \psi$, if $\varphi \models \psi$ and $\psi \models \varphi$.

Question: How to phrase these in terms of **models**?

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- A propositional formula that is a proposition a or a negated proposition $\neg a$ for some $a \in A$ is a **literal**.
- A formula that is a disjunction of literals is a **clause**. This includes **unit clauses** / consisting of a single literal, and the **empty clause** \perp consisting of zero literals.

Normal forms: NNF, CNF, DNF

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Transitions for state sets described by propositions A can be concisely represented as **operators** or **actions** $\langle \chi, e \rangle$ where

- the **precondition** χ is a propositional formula over A describing the set of states in which the transition can be taken (states in which a transition starts), and
- the **effect** e describes how the resulting successor states are obtained from the state where the transitions is taken (where the transition goes).

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Blocks world operators

To model blocks world operators conveniently, we use auxiliary state variables *A-clear*, *B-clear*, and *C-clear* to denote that there is nothing on top of a given block.

Then blocks world operators can be modeled as:

- $\langle A\text{-clear} \wedge A\text{-on-}T \wedge B\text{-clear}, A\text{-on-}B \wedge \neg A\text{-on-}T \wedge \neg B\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}T \wedge C\text{-clear}, A\text{-on-}C \wedge \neg A\text{-on-}T \wedge \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}B, A\text{-on-}T \wedge \neg A\text{-on-}B \wedge B\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}C, A\text{-on-}T \wedge \neg A\text{-on-}C \wedge C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}B \wedge C\text{-clear}, A\text{-on-}C \wedge \neg A\text{-on-}B \wedge B\text{-clear} \wedge \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}C \wedge B\text{-clear}, A\text{-on-}B \wedge \neg A\text{-on-}C \wedge C\text{-clear} \wedge \neg B\text{-clear} \rangle$
- ...

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Definition (effects)

(Deterministic) **effects** are recursively defined as follows:

- If $a \in A$ is a state variable, then a and $\neg a$ are effects (**atomic effect**).
- If e_1, \dots, e_n are effects, then $e_1 \wedge \dots \wedge e_n$ is an effect (**conjunctive effect**).
The special case with $n = 0$ is the empty effect \top .
- If χ is a propositional formula and e is an effect, then $\chi \triangleright e$ is an effect (**conditional effect**).

Atomic effects a and $\neg a$ are best understood as assignments $a := 1$ and $a := 0$, respectively.

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$\chi \triangleright e$ means that change e takes place if χ is true in the current state.

Example

Increment 4-bit number $b_3b_2b_1b_0$ represented as four state variables b_0, \dots, b_3 :

$$\begin{aligned} &(\neg b_0 \triangleright b_0) \wedge \\ &((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ &((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ &((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \triangleright (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

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Definition (changes caused by an operator)

For each effect e and state s , we define the **change set** of e in s , written $[e]_s$, as the following set of literals:

- $[a]_s = \{a\}$ and $[\neg a]_s = \{\neg a\}$ for atomic effects a , $\neg a$
- $[e_1 \wedge \dots \wedge e_n]_s = [e_1]_s \cup \dots \cup [e_n]_s$
- $[\chi \triangleright e]_s = [e]_s$ if $s \models \chi$ and $[\chi \triangleright e]_s = \emptyset$ otherwise

Definition (applicable operators)

Operator $\langle \chi, e \rangle$ is **applicable in a state s** iff $s \models \chi$ and $[e]_s$ is consistent (i. e., does not contain two complementary literals).

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Definition (successor state)

The **successor state** $app_o(s)$ of s with respect to operator $o = \langle \chi, e \rangle$ is the state s' with $s' \models [e]_s$ and $s'(v) = s(v)$ for all state variables v not mentioned in $[e]_s$.

This is defined only if o is applicable in s .

Example

Consider the operator $\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle$ and the state $s = \{a \mapsto 1, b \mapsto 1, c \mapsto 1, d \mapsto 1\}$.

The operator is applicable because $s \models a$ and $[\neg a \wedge (\neg c \triangleright \neg b)]_s = \{\neg a\}$ is consistent.

Applying the operator results in the successor state $app_{\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle}(s) = \{a \mapsto 0, b \mapsto 1, c \mapsto 1, d \mapsto 1\}$.

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Definition (deterministic planning task)

A **deterministic planning task** is a 4-tuple $\Pi = \langle A, I, O, \gamma \rangle$ where

- A is a finite set of **state variables** (propositions),
- I is a valuation over A called the **initial state**,
- O is a finite set of **operators** over A , and
- γ is a formula over A called the **goal**.

Note:

- When we talk about deterministic planning tasks, we usually omit the word “deterministic”.
- When we will talk about nondeterministic planning tasks later, we will explicitly qualify them as “nondeterministic”.

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Definition (induced transition system of a planning task)

Every planning task $\Pi = \langle A, I, O, \gamma \rangle$ induces a corresponding deterministic transition system $\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle$:

- S is the set of all valuations of A ,
- L is the set of operators O ,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' = \text{app}_o(s) \}$,
- $s_0 = I$, and
- $S_\star = \{ s \in S \mid s \models \gamma \}$

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- Terminology for transitions systems is also applied to the planning tasks that induce them.
- For example, when we speak of the **states of Π** , we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a goal path of $\mathcal{T}(\Pi)$ is called a **plan** of Π .

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Summary

By **planning**, we mean the following two algorithmic problems:

Definition (satisficing planning)

Given: a planning task Π

Output: a plan for Π , or **unsolvable** if no plan for Π exists

Definition (optimal planning)

Given: a planning task Π

Output: a plan for Π with minimal length among all plans for Π , or **unsolvable** if no plan for Π exists

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- **Transition systems** are (typically huge) directed graphs that encode how the state of the world can change.
- **Planning tasks** are compact representations for transition systems, suitable as input for planning algorithms.
- Planning tasks are based on concepts from **propositional logic**, enhanced to model state change.
- **States** of planning tasks are propositional valuations.
- **Operators** of planning tasks describe **when** (precondition) and **how** (effect) to change the current state of the world.
- In **satisficing planning**, we must find a solution to planning tasks (or show that no solution exists).
- In **optimal planning**, we additionally guarantee that generated solutions are of the shortest possible length.

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