Constraint Satisfaction Problems

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Wintersemester 2014/15 — Department of Computer Science

Exercise Sheet 10

Due: 28.1.2015

Exercise 10.1 (1+1+3 points)

Provide the relations expressed by the following gadgets:

(a) Domain $D = \{0, 1, 2, 3, 4\}$ with the usual ordering of the natural numbers $<$, variables $\{u, v, w, z\}$, constraints $((u, v), <)$, $((v, w), <)$ and $((w, z), <)$, and construction site (z, u).

(b) Domain $D = \{0, 1, 2, 3\}$, variables $\{a, b, c, d\}$, constraints $((a, b), \neq)$, $((a, c), \neq)$, $((a, d), \neq)$, $((b, c), \neq)$, $((b, d), \neq)$, and construction site (c, d).

(c) Domain $D = \{0, 1, 2\}$, variables $\{x, y, z, v, w\}$, constraints $x + 2y + 3z \equiv 1 \pmod 3$, $2x + z + 3w \equiv 1 \pmod 3$, $z \equiv 1 \pmod 3$, $v + 4y \equiv 2 \pmod 3$ and $x + y + z + v + w \equiv 0 \pmod 3$, and construction site $\{w, z, y\}$.

Exercise 10.2 (5 points)

Utilize Shaefer's dichotomy theorem to classify each of the following Boolean constraint languages as polynomial or NP-complete. If you classify the language as polynomial, provide which Schaefer class it falls into and an argument for this. For NP-complete cases, you do not have to provide a proof, nor an argument why it does not fall into any of the Schaefer classes.

(a) *Colorability* (with two colors): only the disequality relation

(b) *Parity*: only k-ary constraints ($k \in \mathbb{N}_1$) that express an even number of variables are assigned 1; i.e. the language has only k-ary relations R such that a tuple $t \in \{0, 1\}^k$ is in R if and only if t has an even number of 1's.

(c) *Balance:* only k-ary constraints (for even $k \in \mathbb{N}_1$) that express that exactly half of the variables are assigned 1, i.e., the language has only k-ary relations R such that a tuple $t \in \{0, 1\}^k$ is in R if and only if t has the same number of 1's and 0's.

(d) *Majority*: consider k-ary constraints (for uneven $k \in \mathbb{N}_1$) that express that the majority of variables are assigned 1.

(e) *One-third*: consider k-ary constraints (for $k \in \mathbb{N}_1$ a multiples of 3) that express that exactly one-third of the variables are assigned 1.