Constraint Satisfaction Problems

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Exercise Sheet 9

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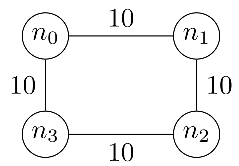
Exercise 9.1 (3+3 points)

We consider the following routing problem in communication networks: We assume that a communication network is given, represented by an undirected, edge-weighted graph $G = (N, E, c)$ with $c(e) > 0$ for each $e \in E$. The value $c(e)$ represents the *capacity* (*maximal bandwidth*) of edge e. A communication request, then, is a triple $r = (x, y, \beta)$ representing that bandwidth β is requested between nodes x and y.

Assume now that a set of requests $\{r_1, \dots, r_m\}$ is given. The routing problem is the problem to assign to each request a route (simple path) between its endpoints such that at no edge the capacity of the edge is exceeded (by the sum of bandwidths of those requests that are routed over the edge). In a solution to this problem requests cannot be split on different routes, but it is possible to reject requests. The objective is to find a solution that maximizes the used bandwidth.

Consider the following instance of the routing problem with requests $r_1 = (n_0, n_3, 8)$, $r_2 = (n_1, n_2, 9)$, $r_3 = (n_0, n_2, 5)$ and $r_4 = (n_0, n_2, 2)$ on the following communication network:



(a) Represent the example as instance of a constraint optimization problem.

(b) Solve the instance by depth-first branch-and-bound with first-choice evaluation function.

Exercise 9.2 (2+2 points)

Consider the class of CSP instances CSP($\{T_D\}$), where

$$T_D := \{(0,0,1), (0,1,0), (1,0,0)\}.$$

The 1-in-3 SAT problem is to determine, given a propositional logic formula in CNF

$$\varphi = \bigwedge_{i=1}^{n} (l_{i1} \lor l_{i2} \lor l_{i3}),$$

whether there exists an assignment to variables in which in each clause $(l_{i1} \lor l_{i2} \lor l_{i3})$ exactly one literal is true (and thus the other two are false). Note, in propositional logic a literal is a variable or a negation of a variable.

Prove that CSP($\{T_D\}$) corresponds to 1-in-3 SAT. Show

(a) there is a polynomial-time reduction of CSP($\{T_D\}$) to 1-in-3 SAT,

(b) there is a polynomial-time reduction of 1-in-3 SAT to CSP($\{T_D\}$).

Hint: For (a) show that a constraint network with only the relation T_D can be written as a formula φ where only assignments with one true literal per clause are considered. For (b) show that φ can be expressed as a constraint network with only polynomially many (new) variables and the relation T_D.