Constraint Satisfaction Problems

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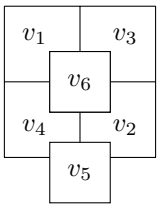
Exercise Sheet 5

Due: 26.11.2014

Exercise 5.1 (2+2+2 points)

Consider the following coloring problem, where every area is either red, green, or blue and every two areas that share a border must not have the same color.



In the following we assume the usual representation of this problem using the binary inequality relation.

(a) How many states are in the *unordered search space*? How many goal states (i.e., solutions) are there?

(b) Draw the *ordered search space* for $\sigma_1 = v_1, v_2, v_3, v_4, v_5, v_6$. Because of the size (and symmetry) of the tree, you may skip drawing the subtrees starting in $v_1 = \mathbf{b}$ and $v_1 = \mathbf{g}$. Mark the dead ends in your drawing. How many states are in the complete (i.e., including the skipped subtrees) ordered search space? How many are dead ends?

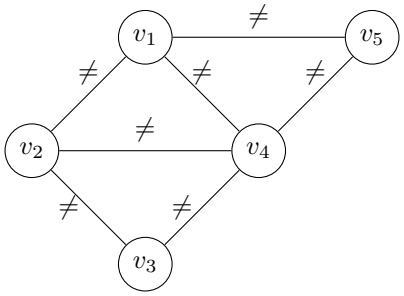
(c) Draw the *ordered search space* for $\sigma_2 = v_6, v_4, v_5, v_2, v_3, v_1$. Because of the size (and symmetry) of the tree, you may skip drawing the subtrees starting in $v_6 = \mathbf{b}$ and $v_6 = \mathbf{g}$. How many states are in the complete ordered search space? How many are dead ends?

Exercise 5.2 (2+2 points)

Consider the constraint network N with variables $v_1, \ldots, v_5$ and domains:

- $D_1 = D_4 = \{1, 2, 3\}$,
- $D_2 = \{2, 3\}$,
- $D_3 = \{0, 2, 3\}$,
- $D_5 = \{1, 3\}$.

The constraints C are given by the following constraint graph:



In the following, use the variable ordering $\sigma = v_1, v_2, v_3, v_4, v_5$ for choosing the next unassigned variable. For value selection use the ordering $0, \ldots, 3$.
Briefly provide each backtracking step and the reduced domains after constraint propagation at each backtracking step (e.g., specify the sets D_i^j for the state of domain D_i at recursion depth j).

(a) Apply the look-ahead algorithm with forward checking to N.

(b) Apply the look-ahead algorithm with real full look-ahead to N.