Constraint Satisfaction Problems

B. Nebel, C. Becker-Asano, S. Wölfl
Wintersemester 2014/15

University of Freiburg
Department of Computer Science

Exercise Sheet 4

Due: 19.11.2014

Exercise 4.1 (Zebra puzzle; 3+1+2+2 points)

The Zebra puzzle is a famous constraint satisfaction problem. Here it is quoted from Wikipedia:

1. *There are five houses.*
2. *The Englishman lives in the red house.*
3. *The Spaniard owns the dog.*
4. *Coffee is drunk in the green house.*
5. *The Ukrainian drinks tea.*
6. *The green house is immediately to the right of the ivory house.*
7. *The Old Gold smoker owns snails.*
8. *Kools are smoked in the yellow house.*
9. *Milk is drunk in the middle house.*
10. *The Norwegian lives in the first house.*
11. *The man who smokes Chesterfields lives in the house next to the man with the fox.*
12. *Kools are smoked in the house next to the house where the horse is kept.*
13. *The Lucky Strike smoker drinks orange juice.*
14. *The Japanese smokes Parliaments.*
15. *The Norwegian lives next to the blue house.*

Now, who drinks water? Who owns the zebra?

In the interest of clarity, it must be added that each of the five houses is painted a different color, and their inhabitants are of different national extractions, own different pets, drink different beverages and smoke different brands of American cigarets [...]

(a) Formulate the zebra problem as a binary constraint network $\langle V, D, C\rangle$. Provide the variables V, domains D, and constraints C.

(b) Draw its primal constraint graph.

(c) Is your formalization arc-consistent? If not, provide an equivalent arc-consistent constraint network.

(d) Is your formalization path-consistent? If not, provide an equivalent path-consistent network.

Exercise 4.2 (2+2 points)

Consider the network N with three variables v_1, v_2, v_3, v_4, each with domain $D = \{0, \ldots, 5\}$, and the following constraints:

$$
\begin{aligned}
&v_1 + v_2 + v_3 \geq 10 \\
&v_1 + v_4 \leq 4 \\
&v_1 \leq v_2 \\
&v_1 < v_3 \\
&v_2 \leq v_3 \\
&v_4 < v_2
\end{aligned}
$$

(a) Apply the PC2 algorithm to N. Provide the state of the queue and the result of Revise3 once for every iteration of the while loop. (We assume that the queue is initially in lexicographic order; and that the queue is processed *first in, first out*).

(b) Is the resulting network 3-consistent?