Constraint Satisfaction Problems

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Exercise Sheet 3

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Exercise 3.1 (2+2 points)

Consider the constraint networkzwerk $\langle V = \{x_1, x_2, x_3\}, D, C = \{R_{12}, R_{23}, R_{31}\}\rangle$:

$$
\begin{aligned}
D_1 &= \{0, \ldots, 3\} \\
D_2 &= \{1, \ldots, 4\} \\
D_3 &= \{2, 3, 4\} \\
R_{12} &= \{(x, y) \in \mathbb{Z}^2 \colon x \geq y\} \\
R_{23} &= \{(x, y) \in \mathbb{Z}^2 \colon 2x \leq y\} \\
R_{31} &= \{(x, y) \in \mathbb{Z}^2 \colon x + y = 4\}
\end{aligned}
$$

Apply the algorithms AC-3 and AC-4 to transform the network into an arc-consistent one. For each of the algorithms detail at least 4 intermediate steps.

Exercise 3.2 (2+1+1+2 points)

Let $\mathcal{C} = \langle V, D, C\rangle$ be a binary constraint network over a finite domain.

(a) Prove that each arc-consistent and normalized binary constraint network is 2-consistent (that is, each locally consistent one-variable assignment $\{v_i\} \to D_i$ can be extended to a locally consistent assignment for variables v_i and v_j).

Is the condition that the network is normalized necessary (i.e., is this claim even true, if we allow networks to be not normalized)?

(b) Prove that for an input constraint network $\mathcal{C}$ the algorithm AC-3 returns an *equivalent* network $\mathcal{C}'$ (i.e., $\mathcal{C}$ and $\mathcal{C}'$ have the same set of solutions).

(c) Prove that the algorithm AC-3 (on normalized, binary constraint networks) achieves *arc-consistent* networks.

(d) Prove that the algorithm AC-4 (applied to normalized, binary constraint networks) runs in time $\mathcal{O}(e \cdot k^2)$ and in space $\mathcal{O}(e \cdot k^2)$.