Constraint Satisfaction Problems

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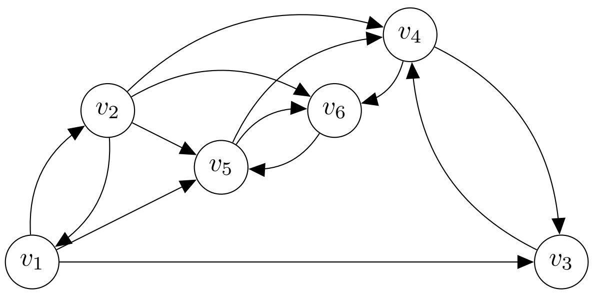
Exercise Sheet 2

Due: 05.11.2014

Exercise 2.1 (2+2 points)

Let $G = \langle V, A \rangle$ be the directed graph given by the following figure.

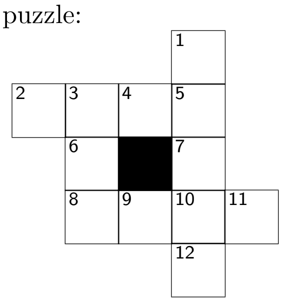


(a) A **strongly connected component** of a directed graph is a maximal strongly connected subgraph. List the strongly connected components of G.

(b) Let $G' = \langle V, E \rangle$ be the undirected simple graph obtained from G by setting $\{v, v'\} \in E$ if and only if $(v, v') \in A \lor (v', v) \in A$. List all cliques of G' of size ≥ 2.

Exercise 2.2 (2+2 Punkte)

Consider the following crossword puzzle:



with fill-in words: ALB, BIER, BAER, BRAEU (usually, all words in the given list need to be filled; for the exercise, it is only required that all words filled-in are from the given list).

(a) Formalize the puzzle as a *constraint network*.

(b) Provide its *primal* and *dual constraint graphs*.

Exercise 2.3 (2+2 points)

A set of n children C has to be split up into k different groups. However, some of the children like each other, others do not.

> **Decision problem**: Given a finite set $C = \{c_1, \ldots, c_n\}$ of children, a binary relation L on C such that $(c_i, c_j) \in L$ if and only if c_i and c_j $(i \neq j)$ like each other. Is it possible to distribute the children into k groups such that no group contains a pair of children that do not like each other.

(a) How difficult is the problem in the case $k = 2$?

(b) How difficult is the problem in the case $k = 3$?

If the problem can be solved in polynomial time, describe a polynomial-time algorithm for the decision problem. If it is NP-complete, provide an appropriate proof.