Principles of AI Planning

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Exercise Sheet 7

Due: Friday, December 12th, 2014

Exercise 7.1 (Finite-domain representation, 1+1+3 points)
Consider the propositional Blocksworld planning task $\Pi = \langle A, I, O, \gamma \rangle$, with

- the set of variables

$$A = \{A\text{-}clear, B\text{-}clear, C\text{-}clear, A\text{-}on\text{-}B, A\text{-}on\text{-}C, A\text{-}on\text{-}T, \\ B\text{-}on\text{-}A, B\text{-}on\text{-}C, B\text{-}on\text{-}T, C\text{-}on\text{-}A, C\text{-}on\text{-}B, C\text{-}on\text{-}T\}$$

- $I(a) = 1$ for $a \in \{B\text{-}on\text{-}T, A\text{-}on\text{-}B, A\text{-}clear, C\text{-}on\text{-}T, C\text{-}clear\}$,
  $I(a) = 0$, else.

- O contains the actions

$$\begin{aligned}
\text{move-}X\text{-}Y\text{-}Z &= \langle X\text{-}on\text{-}Y \wedge X\text{-}clear \wedge Z\text{-}clear, \\
&\qquad \neg X\text{-}on\text{-}Y \wedge Y\text{-}clear \wedge X\text{-}on\text{-}Z \wedge \neg Z\text{-}clear \rangle \\
\text{move-}X\text{-Table-}Z &= \langle X\text{-}on\text{-}T \wedge X\text{-}clear \wedge Z\text{-}clear, \\
&\qquad \neg X\text{-}on\text{-}T \wedge X\text{-}on\text{-}Z \wedge \neg Z\text{-}clear \rangle \\
\text{move-}X\text{-}Y\text{-Table} &= \langle X\text{-}on\text{-}Y \wedge X\text{-}clear, \\
&\qquad \neg X\text{-}on\text{-}Y \wedge Y\text{-}clear \wedge X\text{-}on\text{-}T \rangle
\end{aligned}$$

  for pair-wise distinct $X, Y, Z \in \{A, B, C\}$

- $\gamma = B\text{-}on\text{-}C \wedge C\text{-}on\text{-}A$.

(a) The following mutex groups can be found for Π:

$$\begin{aligned}
L_1 &= \{B\text{-}on\text{-}A, C\text{-}on\text{-}A, A\text{-}clear\} \\
L_2 &= \{A\text{-}on\text{-}B, C\text{-}on\text{-}B, B\text{-}clear\} \\
L_3 &= \{A\text{-}on\text{-}C, B\text{-}on\text{-}C, C\text{-}clear\} \\
L_4 &= \{A\text{-}on\text{-}B, A\text{-}on\text{-}C, A\text{-}on\text{-}T\} \\
L_5 &= \{B\text{-}on\text{-}A, B\text{-}on\text{-}C, B\text{-}on\text{-}T\} \\
L_6 &= \{C\text{-}on\text{-}A, C\text{-}on\text{-}B, C\text{-}on\text{-}T\}
\end{aligned}$$

Specify a planning task Π' that is equivalent to Π and in finite-domain representation by using these mutex groups. Please name the variables in a reasonable way (e.g., analogously to the examples given in the lecture).

(b) Specify the propositional planning task Π'' that is induced by Π'.

(c) A planning task $\Pi' = \langle V, I', O', \gamma' \rangle$ in finite-domain representation is equivalent to a propositional planning task Π if there is an isomorphism between $\Pi'' = \langle A'', I'', O'', \gamma'' \rangle$ and Π, where Π'' is the propositional planning task induced by Π.

There is an isomorphism between Π'' and Π if there are injective mappings $f : S \mapsto S''$ and $g : O \mapsto O''$ (where S is the set of reachable states in Π and S'' the set of states in Π'') with:

- $I'' = f(I)$
- For reachable states s_1 and s_2, if $s_2 = app_o(s_1)$ then $f(s_2) = app_{g(o)}(f(s_1))$.
- For all reachable states $s \in S$ it is true that $s \models \gamma$ iff $f(s) \models \gamma''$.

Show that the planning task Π' from exercise (a) is equivalent to Π by specifiying $f : S \mapsto S''$ and $g : O \mapsto O''$ and showing that they have the required properties.

Exercise 7.2 (Abstraction heuristics, 2+3 points)
A state of a 15-puzzle planning task is given as a permutation $\langle b, t_1, \ldots, t_{15}\rangle$ of $\{1, \ldots, 16\}$, where b denotes the empty tile (blank) and all other components denote the positions of the tiles.
Let $T^1 = \{t_1^1, \ldots, t_n^1\}, T^2 = \{t_1^2, \ldots, t_m^2\}$ with $1 \leq n, m \leq 14$ be a partitioning of $\{t_1, \ldots, t_{15}\}$ (i.e., $T^1 \cup T^2 = \{t_1, \ldots, t_{15}\}$ and $T^1 \cap T^2 = \emptyset$). Consider the following abstractions:

- $\alpha_1(\langle b, t_1, \ldots, t_{15}\rangle) = \langle b, t_1^1, \ldots, t_m^1\rangle$
- $\alpha_2(\langle b, t_1, \ldots, t_{15}\rangle) = \langle b, t_1^2, \ldots, t_n^2\rangle$
- $\alpha_3(\langle b, t_1, \ldots, t_{15}\rangle) = \langle t_1^1, \ldots, t_m^1\rangle$
- $\alpha_4(\langle b, t_1, \ldots, t_{15}\rangle) = \langle t_1^2, \ldots, t_n^2\rangle$

For $1 \leq i \leq 4$, the heuristic estimates of h_i are equal to lengths of optimal plans in the respective abstractions (e.g., $h_i(s) = h^*(\alpha_i(s))$. Show that:

(a) $h_1 + h_2$ is not admissible.

(b) $h_3 + h_4$ is admissible.
Hint: A heuristic is admissible if it is goal-aware and consistent.

You can and should solve the exercise sheets in groups of two. Please state both names on your solution.