Principles of AI Planning

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Winter Semester 2014/2015

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Exercise Sheet 5

Due: Friday, November 28th, 2014

Exercise 5.1 (Domination lemma, 3 points)
Let $s, s' : A \to \{0, 1\}$ be valuations for a set A of state variables and let χ be a negation-free formula over A. Show by structural induction on χ: If $s \models \chi$ and s' dominates s, then $s' \models \chi$.

Exercise 5.2 (Delete relaxation, 1+2 points)
Consider the planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form with

$$\begin{aligned}
A &= \{\mathit{haveCake}, \mathit{eatenCake}, \mathit{haveNoCake}\},\\
I &= \{\mathit{have\text{-}cake} \mapsto 0, \mathit{eatenCake} \mapsto 0, \mathit{haveNoCake} \mapsto 1\}\\
O &= \{\mathit{eatCake}, \mathit{bakeCake}\},\\
\mathit{eatCake} &= \langle \mathit{haveCake}, \neg \mathit{haveCake} \wedge \mathit{haveNoCake} \wedge \mathit{eatenCake} \rangle,\\
\mathit{bakeCake} &= \langle \mathit{haveNoCake}, \mathit{haveCake} \wedge \neg \mathit{haveNoCake} \rangle \text{ und}\\
\gamma &= \mathit{haveCake} \wedge \mathit{eatenCake}.
\end{aligned}$$

(a) Give the relaxation Π^+ of Π.
(b) Give a sequence π of operators (as short as possible) from O such that π is *not* a plan of Π, but π^+ is a plan of Π^+.

Exercise 5.3 (h^+ heuristic, 2+2 points)
A 15-puzzle planning task $\Pi = \langle A, I, O, \gamma \rangle$ is given as

$$\begin{aligned}
A &= \{\mathit{empty}(p_{i,j}) \mid 0 \le i, j \le 3\} \cup \{\mathit{at}(t_k, p_{i,j}) \mid 0 \le i, j \le 3, 0 \le k \le 14\},\\
O &= \{\mathit{move}(t_m, p_{i,j}, p_{k,l}) \mid 0 \le i, j, k, l \le 3, 0 \le m \le 14,\\
&\quad (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1)\},\\
\gamma &= \bigwedge_{0 \le m \le 14} \mathit{at}(t_m, p_{\lfloor m/4 \rfloor, m\%4})
\end{aligned}$$

Action $\mathit{move}(t_m, p_{i,j}, p_{k,l})$ moves tile t_m from position $p_{i,j}$ to position $p_{k,l}$:

$$\begin{aligned}
\mathit{move}(t_m, p_{i,j}, p_{k,l}) = \langle &\mathit{at}(t_m, p_{i,j}) \wedge \mathit{empty}(p_{k,l}),\\
&\mathit{at}(t_m, p_{k,l}) \wedge \mathit{empty}(p_{i,j}) \wedge \neg \mathit{at}(t_m, p_{i,j}) \wedge \neg \mathit{empty}(p_{k,l}) \rangle
\end{aligned}$$

A syntactically possible state is *legal* if each tile t_m is at some position p_{ij}, if no two tiles are at the same position and if the remaining position is the only one that is *empty*. The initial state is an arbitrary state that is legal.

One possible heuristic for the 15-puzzle is the Manhattan-distance heuristic $h^{Manhattan}$: It sums the Manhattan distances of all tiles from their current positions to their target positions, where the Manhattan distance between position $p_{i,j}$ and $p_{k,l}$ is given as $|i - k| + |j - l|$.
The h^+ heuristic estimates the distance of state s to the closest goal state as the length of the optimal plan in the relaxed planning task (with initial state s).

(a) Show that $h^+(s) \ge h^{Manhattan}(s)$ for each legal state s of a 15-puzzle planning task.
(b) Show that $h^+(s) > h^{Manhattan}(s)$ for at least one state s of a 15-puzzle planning task.

You can and should solve the exercise sheets in groups of two. Please state both names on your solution.