Principles of AI Planning

15. Strong nondeterministic planning

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Strong planning

In this chapter, we will consider the simplest case of nondeterministic planning by restricting attention to strong plans.

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1 Concepts

- Strong plans
- Images
- Weak preimages
- Strong preimages

Strong plans

Recall the definition of strong plans:

Definition (strong plan)

Let S be the set of states of a planning task Π. Then a strong plan for Π is a function $\pi : S_\pi \to O$ for some subset $S_\pi \subseteq S$ such that

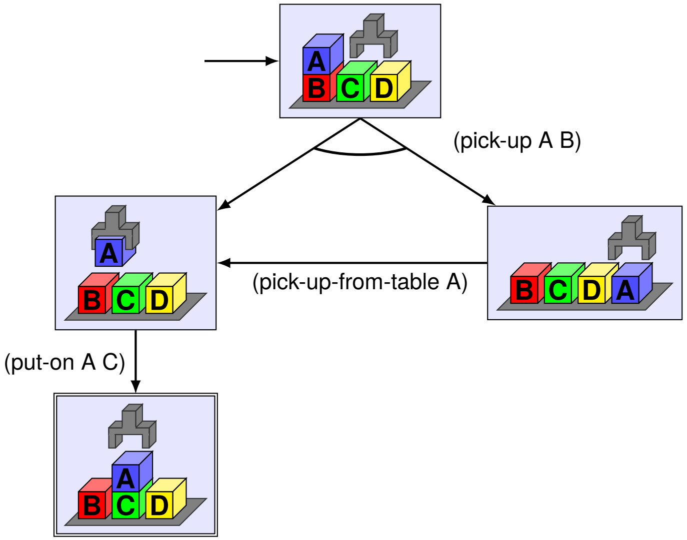
- $\pi(s)$ is applicable in s for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi \cup S_\star$ (π is closed),
- $S_\pi(s') \cap S_\star \neq \emptyset$ for all $s' \in S_\pi(s_0)$ (π is proper), and
- there is no state $s' \in S_\pi(s_0)$ such that s' is reachable from s' following π in a strictly positive number of steps (π is acyclic).

Strong plans

Execution of a strong plan

1. Determine the current state s.
2. If s is a goal state then terminate.
3. Execute action $\pi(s)$.
4. Repeat from first step.

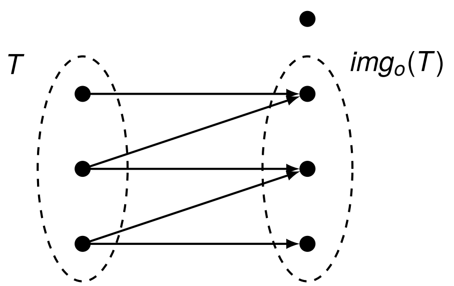
Strong plans



Images

Image

The image of a set T of states with respect to an operator o is the set of those states that can be reached by executing o in a state in T.



Images

Definition (image of a state)

$img_o(s) = \{s' \in S | s \xrightarrow{o} s'\} = app_o(s)$

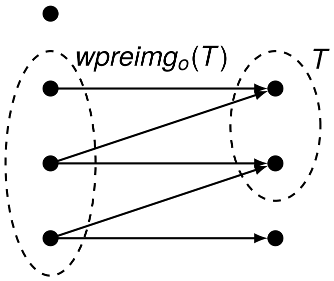
Definition (image of a set of states)

$img_o(T) = \bigcup_{s \in T} img_o(s)$

Weak preimages

Weak preimage

The weak preimage of a set T of states with respect to an operator o is the set of those states from which a state in T can be reached by executing o.



Weak preimages

Definition (weak preimage of a state)

$wpreimg_o(s') = \{s \in S | s \xrightarrow{o} s'\}$

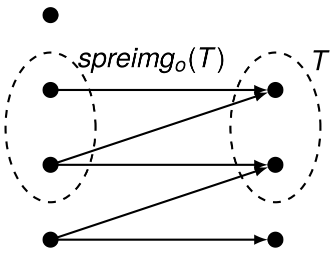
Definition (weak preimage of a set of states)

$wpreimg_o(T) = \bigcup_{s \in T} wpreimg_o(s)$.

Strong preimages

Strong preimage

The strong preimage of a set T of states with respect to an operator o is the set of those states from which a state in T is always reached when executing o.



Strong preimages

Definition (strong preimage of a set of states)

$spreimg_o(T) = \{s \in S \mid \exists s' \in T : s \xrightarrow{o} s' \wedge img_o(s) \subseteq T\}$

2 Algorithms

- Regression
- Efficient implementation of regression
- Progression

Algorithms for strong planning

1. Dynamic programming (backward)
   Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.
   1. Zero actions needed for goal states.
   2. If states with i actions to goals are known, states with $\leq i+1$ actions to goals can be easily identified.

   Automatic reuse of plan suffixes already found.
2. Heuristic search (forward)
   Strong planning can be viewed as AND/OR graph search.

   OR nodes: Choice between operators
   AND nodes: Choice between effects

   Heuristic AND/OR search algorithms:
   AO*, Proof Number Search, ...

Dynamic programming

Planning by dynamic programming

If for all successors of state s with respect to operator o a plan exists, assign operator o to s.

- Base case $i = 0$: In goal states there is nothing to do.
- Inductive case $i \geq 1$: If $\pi(s)$ is still undefined and there is $o \in O$ such that for all $s' \in img_o(s)$, the state s' is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

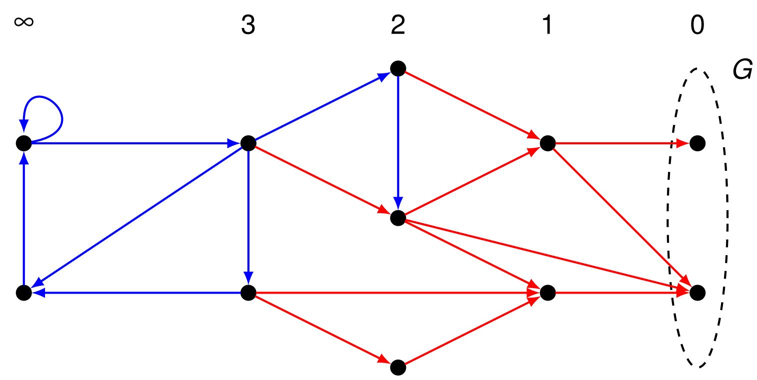
Backward distances

If s is assigned a value on iteration $i \geq 1$, then the backward distance of s is i. The dynamic programming algorithm essentially computes the backward distances of states.

Backward distances

Example

distance to G



Backward distances

Definition (backward distance sets)

Let G be a set of states and O a set of operators.
The backward distance sets D_i^{bwd} for G and O consist of those states for which there is a guarantee of reaching a state in G with at most i operator applications using operators in O:

$$D_0^{bwd} := G$$

$$D_i^{bwd} := D_{i-1}^{bwd} \cup \bigcup_{o \in O} spreimg_o(D_{i-1}^{bwd}) \text{ for all } i \geq 1$$

Backward distances

Definition (backward distance)

Let G be a set of states and O a set of operators, and let $D_0^{bwd}, D_1^{bwd}, \ldots$ be the backward distance sets for G and O.
Then the backward distance of a state s for G and O is

$$\delta_G^{bwd}(s) = \min\{i \in \mathbb{N} \mid s \in D_i^{bwd}\}$$

(where $\min \emptyset = \infty$).

Strong plans based on distances

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states $S_\star$.

Extraction of a strong plan from distance sets

1. Let $S' \subseteq S$ be those states having a finite backward distance for $G = S_\star$ and O.
2. Let $s \in S'$ be a state with distance $i = \delta_G^{bwd}(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $img_o(s) \subseteq D_{i-1}^{bwd}$. Hence o decreases the backward distance by at least one.

Then π is a strong plan for $\mathcal{T}$ iff $I \in S'$.

Question: What is the worst-case runtime of the algorithm?
Question: What is the best-case runtime of the algorithm if most states have a finite backward distance?

Making the algorithm a logic-based algorithm

- An algorithm that represents the states explicitly stops being feasible at about 10^8 or 10^9 states.
- For planning with bigger transition systems structural properties of the transition system have to be taken advantage of.
- As before, representing state sets as propositional formulae (or BDDs) often allows taking advantage of the structural properties: a formula (or BDD) that represents a set of states or a transition relation that has certain regularities may be very small in comparison to the set or relation.
- In the following, we will present an algorithm using a boolean-formula representation (without going into the details of how to implement it using BDDs).

Making the algorithm a logic-based algorithm

Remark: The following algorithm assumes a propositional representation of the state space as opposed to a finite-domain representation. We have already seen how to translate an FDR encoding into a propositional encoding in Chapter 9 (cf. definition of the "induced propositional planning task").

Therefore, for the rest of the present section, we will assume without loss of generality that all $v \in V$ are propositional variables with domain $\mathscr{D}_v = \{0,1\}$.

Breadth-first search with progression and state sets (deterministic case)

Progression breadth-first search

def bfs-progression(V, I, O, γ):
  goal := *formula-to-set*(γ)
  reached := $\{I\}$
  loop:
    if *reached* $\cap$ *goal* $\neq \emptyset$:
      return solution found
    new-reached := *reached* $\cup \bigcup_{o \in O} img_o(reached)$
    if *new-reached* = *reached*:
      return no solution exists
    reached := *new-reached*

$\rightsquigarrow$ This can easily be transformed into a regression algorithm.

Breadth-first search with regression and state sets (deterministic case)

Regression breadth-first search

def bfs-regression(V, I, O, γ):
  init := I
  reached := *formula-to-set*(γ)
  loop:
    if *init* $\in$ *reached*:
      return solution found
    new-reached := *reached* $\cup \bigcup_{o \in O} wpreimg_o(reached)$
    if *new-reached* = *reached*:
      return no solution exists
    reached := *new-reached*

- This algorithm is very similar to the dynamic programming algorithm for the nondeterministic case!

Breadth-first search with regression and state sets (strong nondeterministic case)

Regression breadth-first search

def bfs-regression(V, I, O, γ):
  init := I
  reached := *formula-to-set*(γ)
  loop:
    if *init* $\in$ *reached*:
      return solution found
    new-reached := *reached* $\cup \bigcup_{o \in O} spreimg_o(reached)$
    if *new-reached* = *reached*:
      return no solution exists
    reached := *new-reached*

- How do we define *spreimg* with logic (or BDD) operations?

Transition formula for nondeterministic operators

Let V be the set of state variables and $V' := \{v' \mid v \in V\}$ a set of primed copies of the variables in V. Intuition:

- Variables in V describe the current state s.
- Variables in V' describe the next state s'.

We would like to define a formula $\tau_V(o)$ that describes the transitions labeled with o between states s (over V) and s' (over V') in terms of V and V'.

Transition formula for nondeterministic operators

The formula $\tau_V(o)$ must express

- the conditions for applicability of o,
- how o changes state variables, and
- which state variables o does not change.

A significant difficulty lies in the third requirement because different variables may be affected depending on nondeterministic choices.

Transition formula for nondeterministic operators

$\tau_V(o)$ for deterministic operators $o = \langle \chi, e \rangle$

$$\tau_V(o) = \chi \wedge \bigwedge_{v \in V} ((EPC_v(e) \vee (v \wedge \neg EPC_{\neg v}(e))) \leftrightarrow v')$$
$$\wedge \bigwedge_{v \in V} \neg(EPC_v(e) \wedge EPC_{\neg v}(e))$$

Assume that $e = \bigwedge_{a \in A} a \wedge \bigwedge_{d \in D} \neg d$ for $A = \{a_1, \dots, a_k\}$ and $D = \{d_1, \dots, d_l\}$ with $A \cap D = \emptyset$. Then this becomes simpler.

$\tau_V(o)$ for STRIPS operators $o = \langle \chi, \bigwedge_{a \in A} a \wedge \bigwedge_{d \in D} \neg d \rangle$

$$\tau_V(o) = \chi \wedge \bigwedge_{a \in A} a' \wedge \bigwedge_{d \in D} \neg d' \wedge \bigwedge_{v \in V \setminus (A \cup D)} (v \leftrightarrow v')$$

Transition formula for nondeterministic operators

For nondeterministic operators $o = \langle \chi, \{e_1, \dots, e_n\} \rangle$ with corresponding add and delete lists A_i and D_i of e_i such that $A_i \cap D_i = \emptyset$, $i = 1, \dots, n$, we get:

$\tau_V(o)$ for nondeterministic operators $o = \langle \chi, \{e_1, \dots, e_n\} \rangle$

$$\tau_V(o) = \chi \wedge \bigvee_{i=1}^{n} \left(\bigwedge_{a \in A_i} a' \wedge \bigwedge_{d \in D_i} \neg d' \wedge \bigwedge_{v \in V \setminus (A_i \cup D_i)} (v \leftrightarrow v') \right)$$

Example

Let $V = \{a, b\}$, $V' = \{a', b'\}$, and $o = \langle \neg a, \{a, a \wedge \neg b\} \rangle$. Then

$$\tau_V(o) = \neg a \wedge \Big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \Big).$$

Computing strong preimages

Definition (substitution)

Let $\varphi, t_1, \dots, t_n$ be propositional formulas and $v_1, \dots, v_n$ atomic propositions.

We denote the formula obtained from φ by simultaneous replacement of all variables v_i by the corresponding formulas t_i, $i = 1, \dots, n$, by $\varphi[t_1, \dots, t_n / v_1, \dots, v_n]$.

Computing strong preimages

Definition (existential abstraction)

Let φ be a propositional formula and $v_1, \dots, v_n$ be atomic propositions. Then the existential abstraction of φ wrt. $v_1, \dots, v_n$ is recursively defined as follows:

$$\exists v.\varphi := \varphi[\top/v] \vee \varphi[\bot/v]$$
$$\exists v_1 \dots \exists v_n.\varphi := \exists v_1 \dots \exists v_{n-1}.(\varphi[\top/v_n] \vee \varphi[\bot/v_n])$$

For a set of variables $V = \{v_1, \dots, v_n\}$ we use the abbreviation

$$\exists V.\varphi := \exists v_1 \dots \exists v_n.\varphi.$$

Note: Even with intermediate formula simplifications this can lead to an exponential blowup. BDDs can be useful here.

Computing strong preimages

Strong preimages

$$\begin{aligned}
spreimg_o(T) &= \{s \in S \mid \exists s' \in T : s \xrightarrow{o} s' \wedge img_o(s) \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad \{s' \in S \mid s \xrightarrow{o} s'\} \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad (\forall s' \in S : s \xrightarrow{o} s' \Rightarrow s' \in T)\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad (\neg \exists s' \in S : s \xrightarrow{o} s' \wedge \neg(s' \in T))\}
\end{aligned}$$

Computing strong preimages with boolean function operations

$$\begin{aligned}
spreimg_o(T) = \{s \in S \mid (&\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&(\neg \exists s' \in S : s \xrightarrow{o} s' \wedge \neg(s' \in T))\}
\end{aligned}$$

Strong preimages with boolean functions

For formula φ characterizing set T of strongly backward-reached states:

$$\begin{aligned}
spreimg_o(\varphi) = &\left(\exists V'.(\tau_V(o) \wedge \varphi[v'_1, \dots, v'_n / v_1, \dots, v_n])\right) \wedge \\
&\left(\neg \exists V'.(\tau_V(o) \wedge \neg\varphi[v'_1, \dots, v'_n / v_1, \dots, v_n])\right)
\end{aligned}$$

We can use this regression formula for efficient symbolic regression search. BDDs support all necessary operations (atomic propositions, $\neg$, $\wedge$, $\vee$, substitution, $\exists$, ...).

Computing strong preimages with boolean function operations

Example

Let $V = \{a, b\}$, $V' = \{a', b'\}$, and

$$o = \langle \neg a, \{a, a \wedge \neg b\} \rangle, \quad \text{i.e.,}$$

$$\tau_V(o) = \neg a \wedge \Big(\big(a' \wedge (b \leftrightarrow b')\big) \vee (a' \wedge \neg b') \Big).$$

Moreover, let $\varphi = a$. Then

$$\begin{aligned} spreimg_o(\varphi) = {} & \exists a' \exists b'. \Big(\neg a \wedge \big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \big) \wedge a' \Big) \wedge \\ & \neg \exists a' \exists b'. \Big(\neg a \wedge \big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \big) \wedge \neg a' \Big) \\ \equiv {} & \neg a \end{aligned}$$

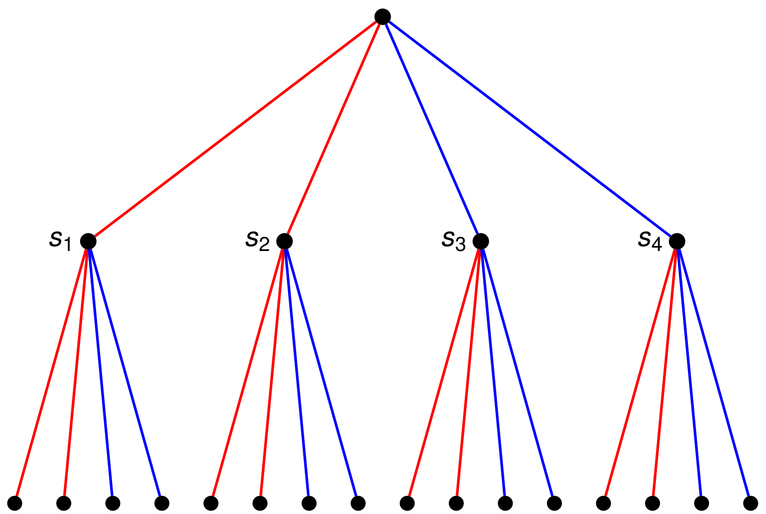
Progression Search

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

- We saw a generalization of regression search to strong planning.
- However, this search is uninformed (breadth-first search).
- Is there an analogue to A* search for strong planning?
- Yes: AO* search
  - Progression search (like A*)
  - Guided by a heuristic (like A*)
  - Guaranteed optimality (under certain conditions, like A*)

AND/OR search

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary



AND/OR search

Concepts
Algorithms
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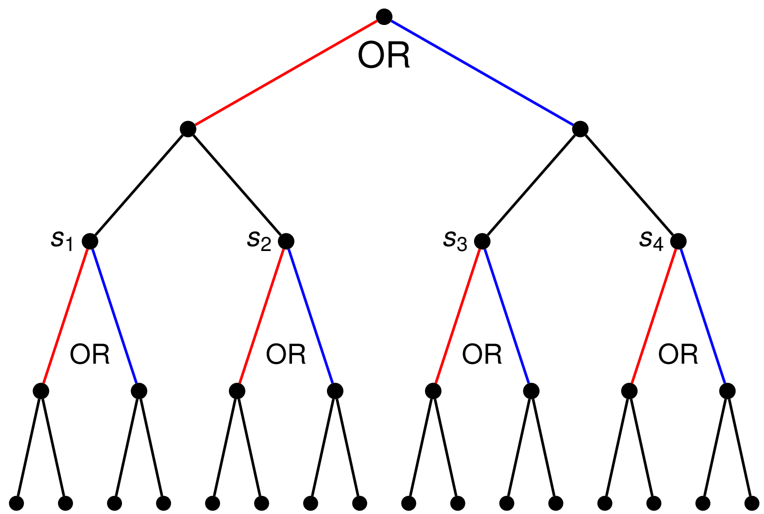
Summary



Progression Search

- We describe AO* on a graph representation without intermediate nodes, i.e., as in the first figure.
- There are different variants of AO*, depending on whether the graph that is being searched is an AND/OR tree, an AND/OR DAG, or a general, possibly cyclic, AND/OR graph.
- The graphs we want to search, $\mathcal{T}(\Pi)$, are in general cyclic.
- However, AO* becomes a bit more involved when dealing with cycles, so we only discuss AO* under the assumption of acyclicity and leave the generalization to cyclic state spaces as an exercise.

AO* Search

- The search is over $\mathcal{T}(\Pi)$.
- For ease of presentation, we do not distinguish between states of $\mathcal{T}(\Pi)$ and search nodes.
- Also, for ease of presentation, we do not handle the case that no strong plan exists.

AO* Search

Definition (solution graph)

A solution graph for a nondeterministic transition system $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ is an acyclic subgraph of $\mathcal{T}$ (viewed as a graph), $\mathcal{T}' = \langle S', L, T' \rangle$, such that

- $s_0 \in S'$,
- for each $s' \in S' \setminus S_\star$, there is exactly one label $l \in L$ s.t.
  - T' contains at least one outgoing transition from s' labeled with l,
  - T' contains all outgoing transitions from s' labeled with l (and S' contains the states reached via such transitions),
  - T' contains no outgoing transitions from s' labeled with any $\tilde{l} \neq l$, and
- every directed path in $\mathcal{T}'$ terminates at a goal state.

AO* Search

Conceptually, there are three graphs/transition systems:

- The induced transitions system $\mathcal{T} = \mathcal{T}(\Pi)$, which only exists as a mathematical object, but is in general not made explicit completely during AO* search,
- The current portion of $\mathcal{T}$ explicitly represented by the search algorithm, $\mathcal{T}_e$, and
- The current portion of $\mathcal{T}_e$ considered by the algorithm as the cheapest/best current partial solution graph, $\mathcal{T}_p$.

AO* Search

Definition (partial solution graph)

A partial solution graph for a nondeterministic transition system $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ is an acyclic subgraph of $\mathcal{T}$ (viewed as a graph), $\mathcal{T}_p = \langle S_p, L, T_p \rangle$, s.t.

- $s_0 \in S_p$,
- for each $s' \in S_p$ that is not an unexpanded leaf node in $\mathcal{T}_p$ there is exactly one label $l \in L$ such that
  - T_p contains at least one outgoing transition from s' labeled with l,
  - T_p contains all outgoing transitions from s' labeled with l (and S_p contains the states reached via such transitions),
  - T_p contains no outgoing transitions from s' labeled with any $\tilde{l} \neq l$, and
- every directed path in $\mathcal{T}_p$ terminates at a goal state or an unexpanded leaf node in $\mathcal{T}_p$.

AO* Search

Definition (cost of a partial solution graph)

Let $h : S \to \mathbb{N} \cup \{\infty\}$ be a heuristic function for the state space S of $\mathcal{T}$, and let $\mathcal{T}_p = \langle S_p, L, T_p \rangle$ be a partial solution graph. The cost labeling of $\mathcal{T}_p$ is the solution to the following system of equations over the states S_p of $\mathcal{T}_p$:

$$f(s) = \begin{cases} 0 & \text{if } s \text{ is a goal state} \\ h(s) & \text{if } s \text{ is an unexpanded non-goal} \\ 1 + \max_{s \xrightarrow{o} s'} f(s') & \text{for the unique outgoing action} \\ & o \text{ of } s \text{ in } \mathcal{T}_p\text{, otherwise.} \end{cases}$$

The cost of $\mathcal{T}_p$ is the cost labeling of its root.

AO* search keeps track of a cheapest partial solution graph by marking for each expanded state s an outgoing action o minimizing $1 + \max_{s \xrightarrow{o} s'} f(s')$.

AO* Search

Procedure ao-star

def ao-star($\mathcal{T}$):
  let $\mathcal{T}_e$ initially consist of the initial state s_0.
  while $\mathcal{T}_p$ has unexpanded non-goal node:
    expand unexpanded non-goal node s of $\mathcal{T}_p$
    add new successor states to $\mathcal{T}_e$
    for all new states s' added to $\mathcal{T}_e$:
      $f(s') \leftarrow h(s')$
    $Z \leftarrow s$ and its ancestors in $\mathcal{T}_e$ along marked actions.
    while Z is not empty:
      remove from Z a state s w/o descendant in Z.
      $f(s) \leftarrow \min_{o \text{ applicable in } s}(1 + \max_{s \xrightarrow{o} s'} f(s'))$.
      mark the best outgoing action for s
        (this may implicitly change $\mathcal{T}_p$).
  return an optimal solution graph.

AO* Search

Correctness (proof sketch)

- Solution graphs directly correspond to strong plans.
- Algorithm eventually terminates (finite number of possible node expansions).
- Acyclicity guarantees that extraction of $\mathcal{T}_p$ and dynamic programming back-propagation of f values always terminates.
- Marking makes sure that existing solutions are eventually marked.

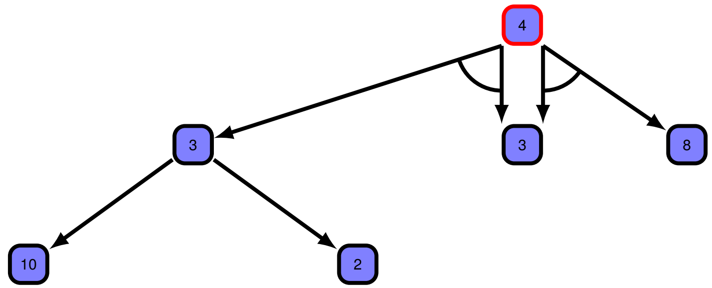
AO* Search

Details

- Pseudocode omits bookkeeping of solved states (can improve performance).
- Choice of unexpanded non-goal node of best partial solution graph is unspecified.
  - Correctness/optimality not affected.
  - One possibility: choose node with lowest cost estimate.
  - Alternative: expand several nodes simultaneously.
- Algorithm can be extended to deal with cycles in the AND/OR graph.

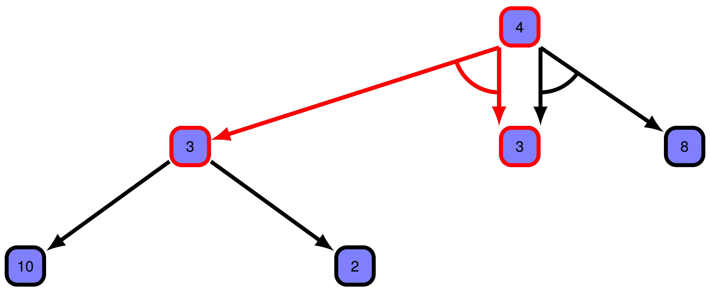
AO* Search

Example



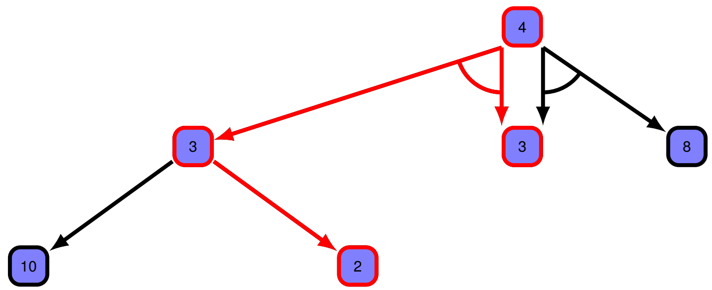
AO* Search

Example



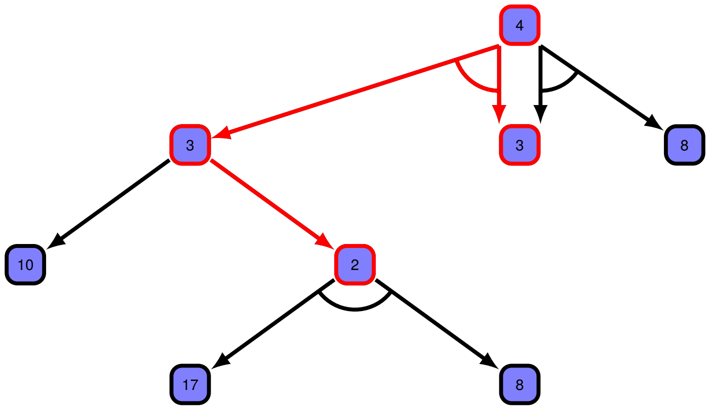
AO* Search

Example



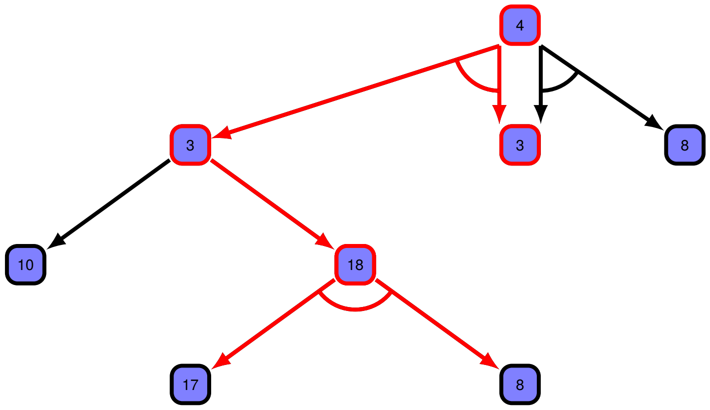
AO* Search

Example



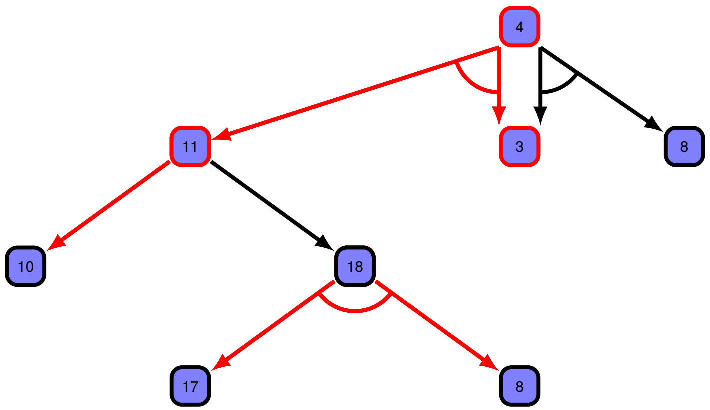
AO* Search

Example



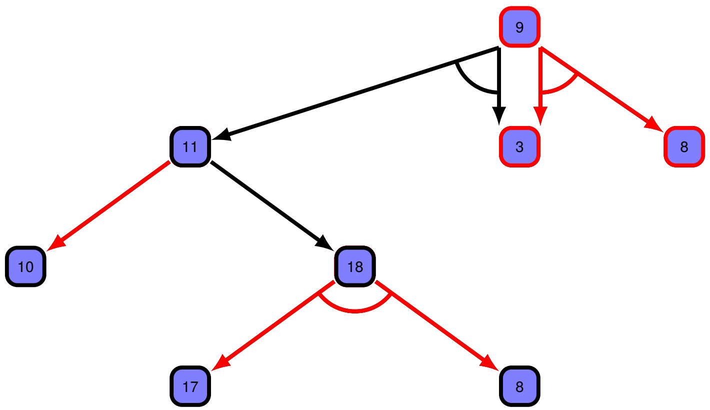
AO* Search

Example



AO* Search

Example



AO* Search

Heuristic Evaluation Function

- Desireable: informative, domain-independent heuristic to initialize cost estimates.
- Heuristic should estimate (strong) goal distances.
- Heuristic does not necessarily have to be admissible (unless we seek optimal solutions).
- We can adapt many heurstics we already know from classical planning (details omitted).

3 Summary

Summary

- We have considered the special case of nondeterministic planning where
  - planning tasks are fully observable and
  - we are interested in strong plans.
- We have introduced important concepts also relevant to other variants of nondeterministic planning such as
  - images and
  - weak and strong preimages.
- We have discussed some basic classes of algorithms:
  - backward induction by dynamic programming, and
  - forward search in AND/OR graphs.