

# Principles of AI Planning

## 13. Planning as search: Partial-Order Reduction

Albert-Ludwigs-Universität Freiburg



**UNI  
FREIBURG**

Bernhard Nebel and Robert Mattmüller

January 7th, 2015



# Motivation

Motivation

Preliminaries

Stubborn  
Sets

Conclusion



- **Worst case:** Heuristic search may explore **exponentially** more states than necessary, even if heuristic is **almost perfect**.
- **Example:** A\* search in GRIPPER domain explores all permutations of ball transportations if heuristic is off by a small constant.
- **Idea:** Complement heuristic search with **orthogonal technique** to reduce size of explored state space.
- **Desired properties of this technique:** preservation of **completeness** and, if possible, **optimality**.

Motivation

Preliminaries

Stubborn  
Sets

Conclusion

## Idea:

- Enforce particular ordering among operators.
- Ignore all other orderings.

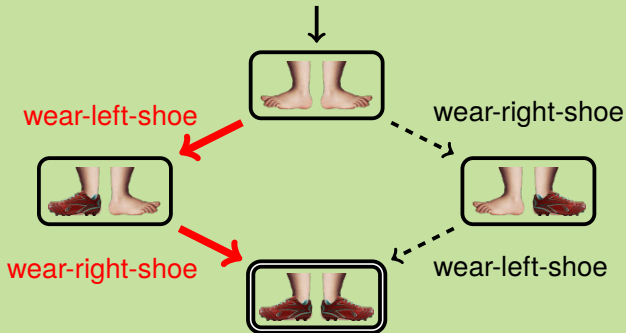
Motivation

Preliminaries

Stubborn  
Sets

Conclusion

## Example





# Preliminaries

Motivation

**Preliminaries**

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion

**Assumption:** For the rest of the chapter, we assume that all planning tasks are SAS<sup>+</sup> planning tasks  $\Pi = (V, I, O, \gamma)$ .

For convenience, we assume that operators have the form  $o = \langle pre(o), eff(o) \rangle$ , where  $pre(o)$  and  $eff(o)$  are both **partial states** over  $V$ , i.e., partial functions mapping variables  $v$  to values in  $\mathcal{D}_v$ . Similarly, we assume that  $\gamma$  is a partial state describing the goal.

## Example

Operator  $o = \langle pre(o), eff(o) \rangle$  with

- $pre(o) = \{v_1 \mapsto d_1, v_5 \mapsto d_5\}$  and
- $eff(o) = \{v_2 \mapsto d_2, v_3 \mapsto d_3\}$

corresponds to  $o = \langle \chi, e \rangle$  with

$\chi = (v_1 = d_1 \wedge v_5 = d_5)$  and  $e = (v_2 := d_2 \wedge v_3 := d_3)$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion



## Definition (Operators)

Let  $\Pi = (V, I, O, \gamma)$  be a SAS<sup>+</sup> planning task and  $o \in O$  an operator. Then

- $prevars(o) := vars(pre(o))$  are the variables that occur in the precondition of  $o$ .
- $effvars(o) := vars(eff(o))$  are the variables that occur in the effect of  $o$ .
- $o$  **reads**  $v \in V$  iff  $v \in prevars(o)$ .
- $o$  **modifies**  $v \in V$  iff  $v \in effvars(o)$ .

Variable  $v \in V$  is **goal-related** iff  $v \in vars(\gamma)$ .

**Assumption:**  $effvars(o) \neq \emptyset$  for all  $o \in O$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion



## Definition (Domain transition graph)

Let  $\Pi = (V, I, O, \gamma)$  be a SAS<sup>+</sup> planning task and  $v \in V$ . The **domain transition graph** for  $v$  is the directed graph

$DTG(v) = \langle \mathcal{D}_v, E \rangle$  where  $(d, d') \in E$  iff there is an operator  $o \in O$  with

- $eff(o)(v) = d'$ , and
- $v \notin prevars(o)$  or  $pre(o)(v) = d$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion



## Example

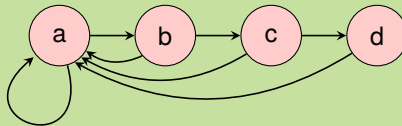
$\text{move-a-b} = \langle \text{pos} = \text{a}, \text{pos} := \text{b} \rangle$

$\text{move-b-c} = \langle \text{pos} = \text{b}, \text{pos} := \text{c} \rangle$

$\text{move-c-d} = \langle \text{pos} = \text{c}, \text{pos} := \text{d} \rangle$

$\text{reset} = \langle \top, \text{pos} := \text{a} \wedge \text{othervar} := \text{othervar} \rangle$

Then  $DTG(\text{pos})$ :



Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion

## Definition (Operator dependencies)

Let  $\Pi = \langle V, O, I, \gamma \rangle$  be a planning task and  $o, o' \in O$ .

- 1  $o$  **disables**  $o'$  iff there exists  $v \in \text{effvars}(o) \cap \text{prevars}(o')$  such that  $\text{eff}(o)(v) \neq \text{pre}(o')(v)$ .
- 2  $o$  **enables**  $o'$  iff there exists  $v \in \text{effvars}(o) \cap \text{prevars}(o')$  such that  $\text{eff}(o)(v) = \text{pre}(o')(v)$ .
- 3  $o$  and  $o'$  **conflict** iff there is  $v \in \text{effvars}(o) \cap \text{effvars}(o')$  such that  $\text{eff}(o)(v) \neq \text{eff}(o')(v)$ .
- 4  $o$  and  $o'$  **interfere** iff  $o$  disables  $o'$ , or  $o'$  disables  $o$ , or  $o$  and  $o'$  conflict.
- 5  $o$  and  $o'$  are **commutative** iff  $o$  and  $o'$  do not interfere, and neither  $o$  enables  $o'$ , nor  $o'$  enables  $o$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator  
Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn

Sets

Conclusion

## Example

wear-left =  $\langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$   
wear-right =  $\langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$   
go-to-uni =  $\langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$   
go-to-gym =  $\langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{gym} \rangle$

Then:

- go-to-uni and go-to-gym disable wear-left and wear-right.
- wear-left and wear-right enable go-to-uni and go-to-gym.
- go-to-uni and go-to-gym conflict.
- wear-left and wear-right are commutative.

Motivation

Preliminaries

Setting

Basic Definitions

Operator  
Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn  
Sets

Conclusion



## Definition (Active operators)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and let  $s$  be a state. The set of **active operators**  $Act(s) \subseteq O$  in  $s$  is defined as the set of operators such that for all  $o \in Act(s)$ :

- For every variable  $v \in prevars(o)$ , there is a path in  $DTG(v)$  from  $s(v)$  to  $pre(o)(v)$ . If  $v$  is goal-related, then there is also a path from  $pre(o)(v)$  to the goal value  $\gamma(v)$ .
- For every goal-related variable  $v \in effvars(o)$ , there is a path in  $DTG(v)$  from  $eff(o)(v)$  to the goal value  $\gamma(v)$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

**Active Operators**

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion



## Proposition

- 1  $Act(s)$  can be identified efficiently for a given state  $s$  by considering paths in the projection of  $\Pi$  onto  $v$ .
- 2 Operators not in  $Act(s)$  can be treated as nonexistent when reasoning about  $s$  because they are not applicable in all states reachable from  $s$ , or they lead to a dead-end from  $s$ .

## Proof

- 1 Homework: Specify efficient algorithm for identification of  $Act(s)$ .
- 2 Obvious. □

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

**Active Operators**

Necessary

Enabling Sets and

Disjunctive Action

Landmarks

Stubborn

Sets

Conclusion



## Definition (Necessary enabling set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task,  $s$  a state, and  $o \in O$  an operator that is not applicable in  $s$ . A set  $N$  of operators is a **necessary enabling set** (NES) for  $o$  in  $s$  if all operator sequences that lead from  $s$  to a goal state and include  $o$  contain an operator in  $N$  before the first occurrence of  $o$ .

**Note:** NESs not uniquely determined for given  $o$  and  $s$ . (E.g., supersets of NESs are still NESs.)

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn

Sets

Conclusion

# Necessary Enabling Sets and Disjunctive Action Landmarks



## Definition (Disjunctive action landmark)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A **disjunctive action landmark** (DAL)  $L$  in  $s$  is a set of operators such that all operator sequences that lead from  $s$  to a goal state contain some operator in  $L$ .

## Observation

For state  $s$  and operator  $o$  that is not applicable in  $s$ , disjunctive action landmarks for task  $\langle V, I, O, pre(o) \rangle$  are necessary enabling sets for  $o$  in  $s$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn

Sets

Conclusion

# Necessary Enabling Sets and Disjunctive Action Landmarks



## Proof

Let  $L$  be such a disjunctive action landmark.

Then each operator sequence that leads from  $s$  to a state satisfying  $pre(o)$  contains some operator in  $L$ .

Thus, each operator sequence that leads from  $s$  to a goal state and includes  $o$  contains an operator in  $L$  before the first occurrence of  $o$ .

Therefore,  $L$  is an NES for  $o$  in  $s$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn  
Sets

Conclusion





## Proof

Let  $L$  be such a disjunctive action landmark.

Then each operator sequence that leads from  $s$  to a state satisfying  $pre(o)$  contains some operator in  $L$ .

Thus, each operator sequence that leads from  $s$  to a goal state and includes  $o$  contains an operator in  $L$  before the first occurrence of  $o$ .

Therefore,  $L$  is an NES for  $o$  in  $s$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn  
Sets

Conclusion



## Proof

Let  $L$  be such a disjunctive action landmark.

Then each operator sequence that leads from  $s$  to a state satisfying  $pre(o)$  contains some operator in  $L$ .

Thus, each operator sequence that leads from  $s$  to a goal state and includes  $o$  contains an operator in  $L$  before the first occurrence of  $o$ .

Therefore,  $L$  is an NES for  $o$  in  $s$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn  
Sets

Conclusion



## Proof

Let  $L$  be such a disjunctive action landmark.

Then each operator sequence that leads from  $s$  to a state satisfying  $pre(o)$  contains some operator in  $L$ .

Thus, each operator sequence that leads from  $s$  to a goal state and includes  $o$  contains an operator in  $L$  before the first occurrence of  $o$ .

Therefore,  $L$  is an NES for  $o$  in  $s$ .

Motivation

Preliminaries

Setting

Basic Definitions

Operator

Dependencies

Active Operators

Necessary  
Enabling Sets and  
Disjunctive Action  
Landmarks

Stubborn  
Sets

Conclusion



# Stubborn Sets

Motivation

Preliminaries

**Stubborn  
Sets**

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Back to the motivation:

If, in state  $s$ , some set of operators can be **applied in any order** and the order does not matter, we want to **commit to one such order** and **ignore all other orders**.

## One idea:

Identify operators that can be “postponed” since they are independent of all operators that are not “postponed”.  
E.g., wear-right-shoe could be postponed, since it is independent of wear-left-shoe (that is not postponed).

## Second idea (roughly):

Identify operators that **have** to be applied and cannot be postponed because they are not independent of other operators also not postponed.

Motivation

Preliminaries

**Stubborn Sets**

Strong Stubborn Sets

Weak Stubborn Sets

Algorithms

Properties of Stubborn Sets

Some Experiments

Conclusion



Following the second idea:

First attempt at a definition:

## Definition (Strong stubborn set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_s \subseteq O$  is a **strong stubborn set in  $s$**  if

- 1  $T_s$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_s$  that are applicable in  $s$ ,  $T_s$  contains all operators that interfere with  $o$ , and
- 3 for all  $o \in T_s$  that are not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $o$  and  $s$ .

Instead of applying all applicable operators in  $s$  only apply those that are applicable and contained in  $T_s$ .

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



Following the second idea:

Improved attempt at a definition:

## Definition (Strong stubborn set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_s \subseteq O$  is a **strong stubborn set in  $s$**  if

- 1  $T_s$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_s$  that are applicable in  $s$ ,  $T_s$  contains all operators that **are active in  $s$  and** interfere with  $o$ , and
- 3 for all  $o \in T_s$  that are not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $o$  and  $s$ .

Instead of applying all applicable operators in  $s$  only apply those that are applicable and contained in  $T_s$ .

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion

**Remark 1:** Even when excluding **inactive** operators, this preserves completeness and even optimality of a search algorithm (see proof below).

**Remark 2:** Excluding **inactive** operators can “cascade” in the sense that additional **active** operators need not be considered.





### Example

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task with the following components:

- $V = \{u_1, u_2, v, w\}$
- $O = \{o_1, o_2, o_3\}$
- $pre(o_1) = \{u_1 \mapsto 0\}$ ,  $eff(o_1) = \{u_1 \mapsto 1, w \mapsto 2\}$
- $pre(o_2) = \{u_2 \mapsto 0\}$ ,  $eff(o_2) = \{u_2 \mapsto 1, w \mapsto 2\}$
- $pre(o_3) = \{u_1 \mapsto 0, u_2 \mapsto 0\}$ ,  $eff(o_3) = \{v \mapsto 1, w \mapsto 1\}$
- $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$
- $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



### Example

- **Case 1** (first attempt at definition where non-active interfering operators are included in  $T_S$ ):

- Include  $o_1$  (or  $o_2$ ) in  $T_S$  as disjunctive action landmark.
- Include  $o_3$  in  $T_S$  since it interferes with  $o_1$  (or  $o_2$ ).
- Include  $o_2$  (or  $o_1$ ) in  $T_S$  since it interferes with  $o_3$ .

⇒ all applicable operators included in  $T_S$ , no pruning.

- **Case 2** (improved attempt without non-active interfering operators):

- $o_3$  is not active in any reachable state.
- $T_S = \{o_1\}$  strong stubborn set in  $I$ .
- Even **active** operator  $o_2$  is not included in  $T_S = \{o_1\}$ .

⇒ nice amount of pruning occurs.

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



With **weak** stubborn sets, some operators that disable an operator in  $T_S$  need not be included in  $T_S$ .

Therefore, weak stubborn sets potentially allow more pruning than strong stubborn sets.

## Definition (Weak stubborn set)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a planning task and  $s$  a state. A set  $T_S \subseteq O$  is a **weak stubborn set in  $s$**  if

- 1  $T_S$  contains a disjunctive action landmark in  $s$ , and
- 2 for all  $o \in T_S$  that are applicable in  $s$ ,  $T_S$  contains the active operators in  $s$  that have conflicting effects with  $o$  or that are disabled by  $o$ , and
- 3 for all  $o \in T_S$  that are not applicable in  $s$ ,  $T_S$  contains a necessary enabling set for  $o$  and  $s$ .

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



For weak stubborn sets, it suffices to include active operators  $o'$  that **are disabled** or **conflict** with applicable operators  $o \in T_S$ . However,  $o'$  **does not need to be included** if  $o'$  **disables** an applicable operator  $o \in T_S$ .

**No computational overhead** of computing weak stubborn sets over computing strong stubborn sets.

## Theorem

In the best case, weak stubborn sets admit **exponentially more pruning** than strong stubborn sets.

## Proof

Homework. □

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

**Weak Stubborn  
Sets**

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



**compute-DAL:** Compute a disjunctive action landmark.

## Procedure compute-DAL

```
def compute-DAL( $\gamma$ ):  
  select  $v \in vars(\gamma)$  with  $s(v) \neq \gamma(v)$   
   $L \leftarrow \{o' \in Act(s) \mid eff(o')(v) = \gamma(v)\}$   
  return  $L$ 
```

Selection of  $v \in vars(\gamma)$  arbitrary. Any variable will do.  
Selection heuristics?

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

**Algorithms**

Properties of  
Stubborn Sets

Some Experiments

Conclusion



**compute-NES:** Compute a necessary enabling set.

## Procedure compute-NES

**def** compute-NES( $o, s$ ):

  select  $v \in \text{prevars}(o)$  with  $s(v) \neq \text{pre}(o)(v)$

$N \leftarrow \{o' \in \text{Act}(s) \mid \text{eff}(o')(v) = \text{pre}(o)(v)\}$

**return**  $N$

Selection of  $v \in \text{prevars}(o)$  arbitrary. Any variable will do.

Selection heuristics?

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

**Algorithms**

Properties of  
Stubborn Sets

Some Experiments

Conclusion

**compute-interfering-operators:** Compute interfering operators.

## Procedure compute-interfering-operators (for strong SS)

```
def compute-interfering-operators(o):  
  disablers  $\leftarrow$  {o'  $\in$  O | o' disables o}  
  disablees  $\leftarrow$  {o'  $\in$  O | o disables o'}  
  conflicting  $\leftarrow$  {o'  $\in$  O | o and o' conflict}  
  return disablers  $\cup$  disablees  $\cup$  conflicting
```

## Procedure compute-interfering-operators (for weak SS)

```
def compute-interfering-operators(o):  
  disablees  $\leftarrow$  {o'  $\in$  O | o disables o'}  
  conflicting  $\leftarrow$  {o'  $\in$  O | o and o' conflict}  
  return disablees  $\cup$  conflicting
```

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

**Algorithms**

Properties of  
Stubborn Sets

Some Experiments

Conclusion

Computing (strong and weak) stubborn sets for planning can be achieved with a **fixed-point iteration** until the constraints of  $T_s$  are satisfied:

**compute-stubborn-set**: Compute (strong or weak) stubborn set.

## Precedure compute-stubborn-set

**def** compute-stubborn-set( $s$ ):

$T_s \leftarrow \text{compute-DAL}(\gamma)$

**while** no fixed-point of  $T_s$  reached **do**

  for  $o \in T_s$  applicable in  $s$ :

$T_s \leftarrow T_s \cup \text{compute-interfering-operators}(o)$

  for  $o \in T_s$  not applicable in  $s$ :

$T_s \leftarrow T_s \cup \text{compute-NES}(o, s)$

**end while**

**return**  $T_s$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion





**Observation:** stubborn sets are state-dependent, but not path-dependent.

This allows filtering the applicable operators in  $s$  in graph search algorithms like A\* that perform duplicate detection, too.

Instead of applying all applicable operators  $app(s)$  in  $s$ , only apply operators in  $T_{app(s)} := T_s \cap app(s)$ .

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Theorem

*Weak stubborn sets are completeness and optimality preserving.*

## Proof

Let  $T_{app(s)} := T_s \cap app(s)$  for a weak stubborn set  $T_s$ .

We show that for all states  $s$  from which an optimal plan consisting of  $n > 0$  operators exists,  $T_{app(s)}$  contains an operator that starts such a plan.

We show by induction that  $A^*$  restricting successor generation to  $T_{app(s)}$  is optimal.

Let  $T_s$  be a weak stubborn set and  $\pi = o_1, \dots, o_n$  be an optimal plan that starts in  $s$ .

...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion

## Theorem

*Weak stubborn sets are completeness and optimality preserving.*

## Proof

Let  $T_{app(s)} := T_s \cap app(s)$  for a weak stubborn set  $T_s$ .

We show that for all states  $s$  from which an optimal plan consisting of  $n > 0$  operators exists,  $T_{app(s)}$  contains an operator that starts such a plan.

We show by induction that  $A^*$  restricting successor generation to  $T_{app(s)}$  is optimal.

Let  $T_s$  be a weak stubborn set and  $\pi = o_1, \dots, o_n$  be an optimal plan that starts in  $s$ .

...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Theorem

*Weak stubborn sets are completeness and optimality preserving.*

## Proof

Let  $T_{app(s)} := T_s \cap app(s)$  for a weak stubborn set  $T_s$ .

We show that for all states  $s$  from which an optimal plan consisting of  $n > 0$  operators exists,  $T_{app(s)}$  contains an operator that starts such a plan.

We show by induction that  $A^*$  restricting successor generation to  $T_{app(s)}$  is optimal.

Let  $T_s$  be a weak stubborn set and  $\pi = o_1, \dots, o_n$  be an optimal plan that starts in  $s$ .

...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Theorem

*Weak stubborn sets are completeness and optimality preserving.*

## Proof

Let  $T_{app(s)} := T_s \cap app(s)$  for a weak stubborn set  $T_s$ .

We show that for all states  $s$  from which an optimal plan consisting of  $n > 0$  operators exists,  $T_{app(s)}$  contains an operator that starts such a plan.

We show by induction that  $A^*$  restricting successor generation to  $T_{app(s)}$  is optimal.

Let  $T_s$  be a weak stubborn set and  $\pi = o_1, \dots, o_n$  be an optimal plan that starts in  $s$ .

...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

As  $T_S$  contains a disjunctive action landmark,  $\pi$  must contain an operator from  $T_S$ .

Let  $o_k$  be the operator with smallest index in  $\pi$  that is also contained in  $T_S$ , i.e.,  $o_k \in T_S$  and  $\{o_1, \dots, o_{k-1}\} \cap T_S = \emptyset$ .

We observe:

1.  $o_k \in \text{app}(s)$ : otherwise by definition of weak stubborn sets, a necessary enabling set  $N$  for  $o_k$  in  $s$  would have to be contained in  $T_S$ , and at least one operator from  $N$  would have to occur before  $o_k$  in  $\pi$  to enable  $o_k$ , contradicting that  $o_k$  was chosen with smallest index.
2. ...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

As  $T_S$  contains a disjunctive action landmark,  $\pi$  must contain an operator from  $T_S$ .

Let  $o_k$  be the operator with smallest index in  $\pi$  that is also contained in  $T_S$ , i.e.,  $o_k \in T_S$  and  $\{o_1, \dots, o_{k-1}\} \cap T_S = \emptyset$ .

We observe:

1.  $o_k \in \text{app}(s)$ : otherwise by definition of weak stubborn sets, a necessary enabling set  $N$  for  $o_k$  in  $s$  would have to be contained in  $T_S$ , and at least one operator from  $N$  would have to occur before  $o_k$  in  $\pi$  to enable  $o_k$ , contradicting that  $o_k$  was chosen with smallest index.
2. ...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

As  $T_S$  contains a disjunctive action landmark,  $\pi$  must contain an operator from  $T_S$ .

Let  $o_k$  be the operator with smallest index in  $\pi$  that is also contained in  $T_S$ , i.e.,  $o_k \in T_S$  and  $\{o_1, \dots, o_{k-1}\} \cap T_S = \emptyset$ .

We observe:

1.  $o_k \in \text{app}(s)$ : otherwise by definition of weak stubborn sets, a necessary enabling set  $N$  for  $o_k$  in  $s$  would have to be contained in  $T_S$ , and at least one operator from  $N$  would have to occur before  $o_k$  in  $\pi$  to enable  $o_k$ , contradicting that  $o_k$  was chosen with smallest index.
2. ...

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion





## Proof (ctd.)

1. ...
2.  $o_k$  does not disable any of the operators  $o_1, \dots, o_{k-1}$ , and all these operators have non-conflicting effects with  $o_k$ : otherwise, as  $o_k \in \text{app}(s)$ , and by definition of weak stubborn sets, at least one of  $o_1, \dots, o_{k-1}$  would have to be contained in  $T_s$ , again contradicting the assumption.

Hence, we can move  $o_k$  to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$  is also a plan for  $\Pi$ .

It has the same cost as  $\pi$  and is hence optimal.

Thus, we have found an optimal plan of length  $n$  started by an operator  $o_k \in T_{\text{app}(s)}$ , completing the proof.  $\square$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

1. ...
2.  $o_k$  does not disable any of the operators  $o_1, \dots, o_{k-1}$ , and all these operators have non-conflicting effects with  $o_k$ : otherwise, as  $o_k \in \text{app}(s)$ , and by definition of weak stubborn sets, at least one of  $o_1, \dots, o_{k-1}$  would have to be contained in  $T_s$ , again contradicting the assumption.

Hence, we can move  $o_k$  to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$  is also a plan for  $\Pi$ .

It has the same cost as  $\pi$  and is hence optimal.

Thus, we have found an optimal plan of length  $n$  started by an operator  $o_k \in T_{\text{app}(s)}$ , completing the proof.  $\square$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

1. ...
2.  $o_k$  does not disable any of the operators  $o_1, \dots, o_{k-1}$ , and all these operators have non-conflicting effects with  $o_k$ : otherwise, as  $o_k \in \text{app}(s)$ , and by definition of weak stubborn sets, at least one of  $o_1, \dots, o_{k-1}$  would have to be contained in  $T_s$ , again contradicting the assumption.

Hence, we can move  $o_k$  to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$  is also a plan for  $\Pi$ .

It has the same cost as  $\pi$  and is hence optimal.

Thus, we have found an optimal plan of length  $n$  started by an operator  $o_k \in T_{\text{app}(s)}$ , completing the proof.  $\square$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



## Proof (ctd.)

1. ...
2.  $o_k$  does not disable any of the operators  $o_1, \dots, o_{k-1}$ , and all these operators have non-conflicting effects with  $o_k$ : otherwise, as  $o_k \in \text{app}(s)$ , and by definition of weak stubborn sets, at least one of  $o_1, \dots, o_{k-1}$  would have to be contained in  $T_s$ , again contradicting the assumption.

Hence, we can move  $o_k$  to the front:

$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$  is also a plan for  $\Pi$ .

It has the same cost as  $\pi$  and is hence optimal.

Thus, we have found an optimal plan of length  $n$  started by an operator  $o_k \in T_{\text{app}(s)}$ , completing the proof.  $\square$

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion



**Remark:** The argument to move  $o_k$  to the front also holds for strong stubborn sets: in this case,  $o_k$  is not even disabled by any of  $o_1, \dots, o_{k-1}$  (and hence,  $o_k$  is independent of  $o_1, \dots, o_{k-1}$ ), which is a stronger property than needed in the proof.

## Corollary

*Strong stubborn sets are completeness and optimality preserving.*



Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion

# Some Experiments: Overview

Optimal Planning, A\* with LM-cut Heuristic, Selected Domains



Domain (problems)	Coverage		Nodes generated	
	A*	+SSS	A*	+SSS
PARCPRINTER-08 (30)	18	<b>+12</b>	2455181	<b>&lt;1%</b>
PARCPRINTER-OPT11 (20)	13	<b>+7</b>	2454533	<b>&lt;1%</b>
WOODWORKING-OPT08 (30)	17	<b>+10</b>	26796212	<b>&lt;1%</b>
WOODWORKING-OPT11 (20)	12	<b>+7</b>	26795517	<b>&lt;1%</b>
SATELLITE (36)	7	<b>+5</b>	5116312	<b>2%</b>
ROVERS (40)	7	<b>+2</b>	1900691	<b>22%</b>
AIRPORT (50)	<b>28</b>	<b>±0</b>	545072	<b>93%</b>
OPENSTACKS-OPT08 (30)	19	<b>+2</b>	56584063	<b>51%</b>
OPENSTACKS-OPT11 (20)	14	<b>+2</b>	56456969	<b>51%</b>
DRIVERLOG (20)	13	<b>+1</b>	3679376	<b>82%</b>
SCANALYZER-08 (30)	<b>15</b>	-3	<b>14203012</b>	<b>100%</b>
SCANALYZER-OPT11 (20)	<b>12</b>	-3	<b>14202884</b>	<b>100%</b>
PARKING-OPT11 (20)	<b>3</b>	-1	<b>560914</b>	<b>100%</b>
SOKOBAN-OPT08 (30)	<b>30</b>	-1	<b>20519270</b>	<b>100%</b>
VISITALL-OPT11 (20)	<b>11</b>	-1	<b>1991169</b>	<b>100%</b>
REMAINING DOMAINS (980)	<b>544</b>	<b>±0</b>	436017004	<b>93%</b>
SUM (1396)	763	<b>+39</b>	670278179	<b>77%</b>

Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion

# Some Experiments

## Weak compared to strong stubborn sets



Motivation

Preliminaries

Stubborn  
Sets

Strong Stubborn  
Sets

Weak Stubborn  
Sets

Algorithms

Properties of  
Stubborn Sets

Some Experiments

Conclusion

Domain (problems)	Coverage		Nodes generated		# problems w. diff. gen.
	SSS	WSS	SSS	WSS	
OPENSTACKS-OPT08 (30)	<b>21</b>	$\pm 0$	152711917	<b>99.936%</b>	18
OPENSTACKS-OPT11 (20)	<b>16</b>	$\pm 0$	152642101	<b>99.936%</b>	16
PATHWAYS-NONEG (30)	<b>5</b>	$\pm 0$	162347	<b>99.702%</b>	2
PSR-SMALL (50)	<b>49</b>	$\pm 0$	18119489	<b>99.998%</b>	6
SATELLITE (36)	<b>12</b>	$\pm 0$	70299721	<b>92.804%</b>	12

⇒ In practice (or, at least, in the standard benchmark problems) there is no significant difference between weak and strong stubborn sets.



# Conclusion

Motivation

Preliminaries

Stubborn  
Sets

Conclusion





- Need for **techniques orthogonal to heuristic search**, complementing heuristics.
- One idea: **Commit to one order of operators** if they are independent. Prune other orders.
- Class of such techniques: **partial-order reduction** (POR)
- One such technique: **strong/weak stubborn sets**
- Can lead to **substantial pruning** compared to plain  $A^*$ .
- Many other POR techniques exist.
- Other pruning techniques exist as well, e.g., symmetry reduction.

Motivation

Preliminaries

Stubborn  
Sets

Conclusion