

Principles of AI Planning

7. Planning as search: relaxed planning tasks

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How to obtain a heuristic

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STRIPS (Fikes & Nilsson, 1971) used the number of state variables that differ in current state s and a STRIPS goal $a_1 \wedge \dots \wedge a_n$:

$$h(s) := |\{i \in \{1, \dots, n\} \mid s \neq a_i\}|.$$

Intuition: more true goal literals \rightsquigarrow closer to the goal

\rightsquigarrow **STRIPS heuristic (a.k.a. goal-count heuristic)** (properties?)

Note: From now on, for convenience we usually write heuristics as functions of states (as above), not nodes. Node heuristic h' is defined from state heuristic h as $h'(\sigma) := h(\text{state}(\sigma))$.

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What is wrong with the STRIPS heuristic?

- quite **uninformative**:
the range of heuristic values in a given task is small;
typically, most successors have the same estimate
- very sensitive to **reformulation**:
can easily transform any planning task into an equivalent one where $h(s) = 1$ for all non-goal states (how?)
- ignores almost all **problem structure**:
heuristic value does not depend on the set of operators!

⇒ need a better, principled way of coming up with heuristics

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General procedure for obtaining a heuristic

Solve an easier version of the problem.

Two common methods:

- **relaxation**: consider **less constrained** version of the problem
- **abstraction**: consider **smaller** version of real problem

Both have been very successfully applied in planning.

We consider both in this course, beginning with **relaxation**.

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How do we relax a problem?

Example (Route planning for a road network)

The road network is formalized as a weighted graph over points in the Euclidean plane. The weight of an edge is the **road distance** between two locations.

A relaxation **drops constraints** of the original problem.

Example (Relaxation for route planning)

Use the **Euclidean distance** $\sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}$ as a heuristic for the road distance between $\langle x_1, y_1 \rangle$ and $\langle x_2, y_2 \rangle$.
This is a **lower bound** on the road distance (\rightsquigarrow admissible).

\rightsquigarrow We drop the constraint of having to travel on roads.

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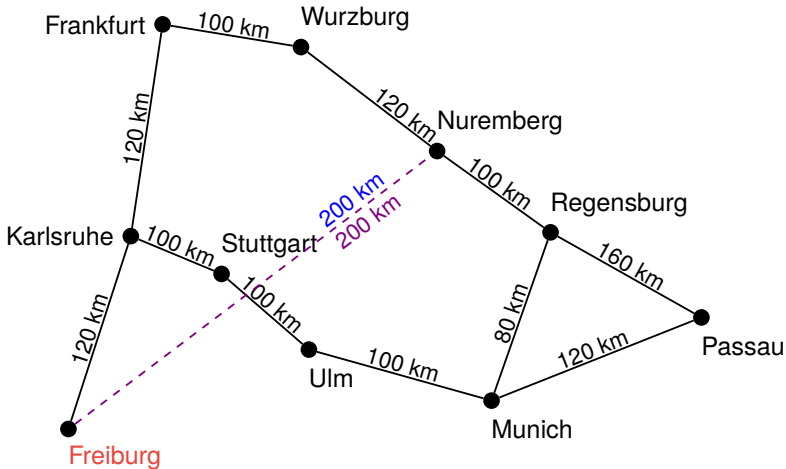
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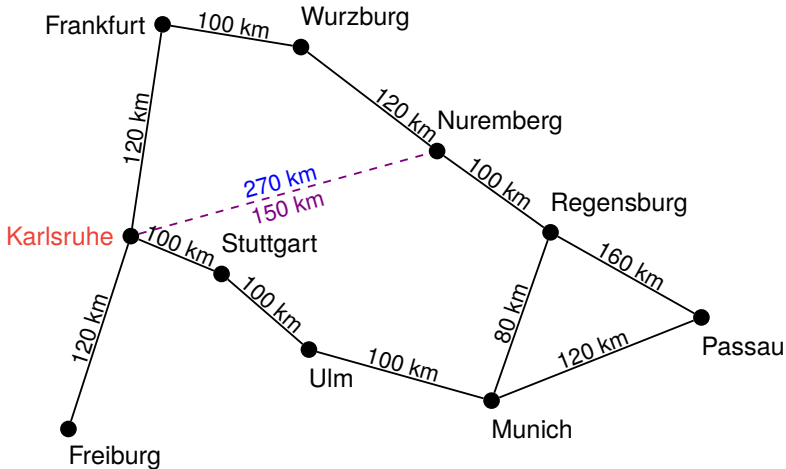
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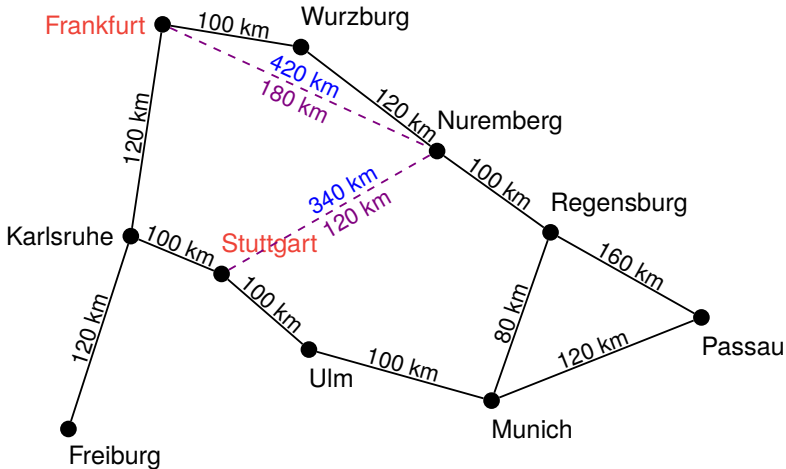
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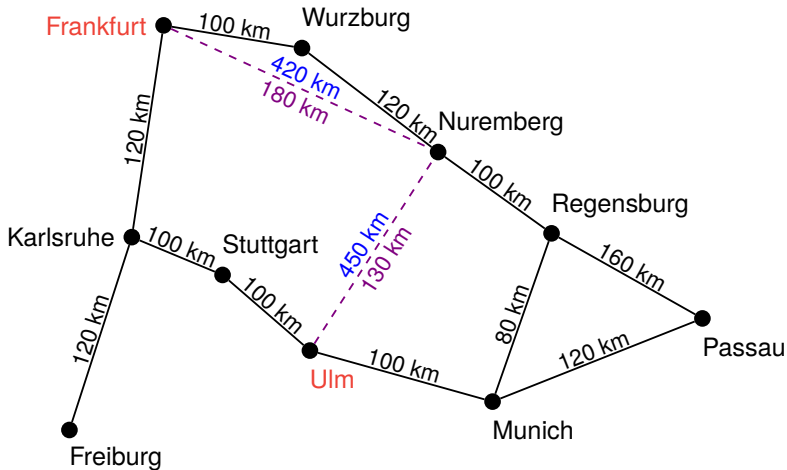
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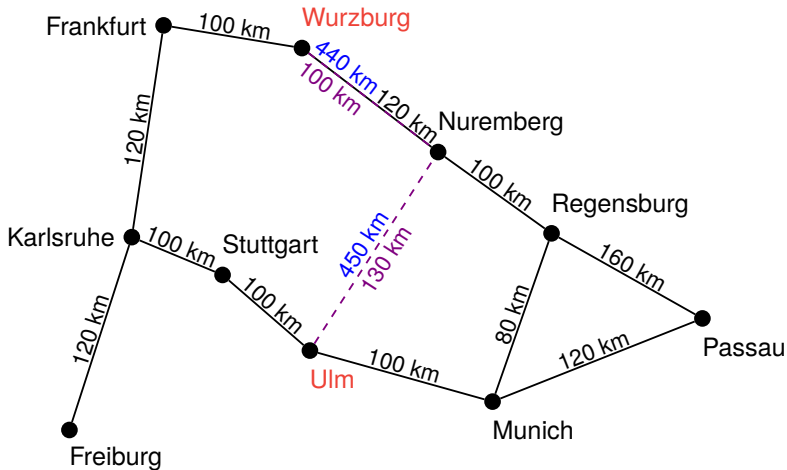
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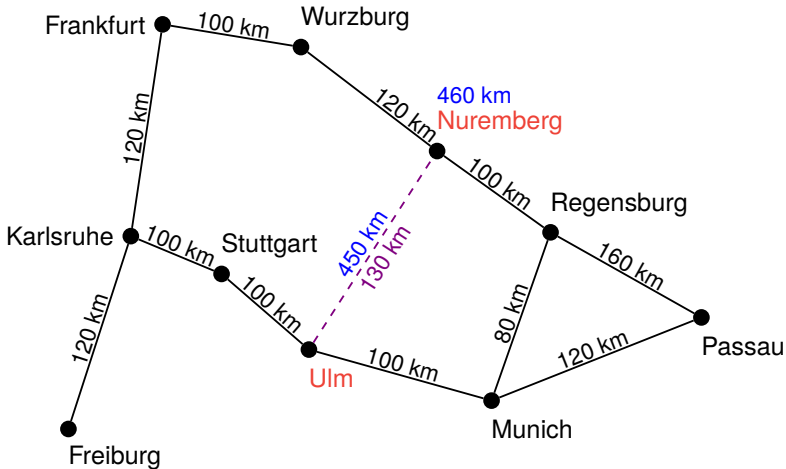
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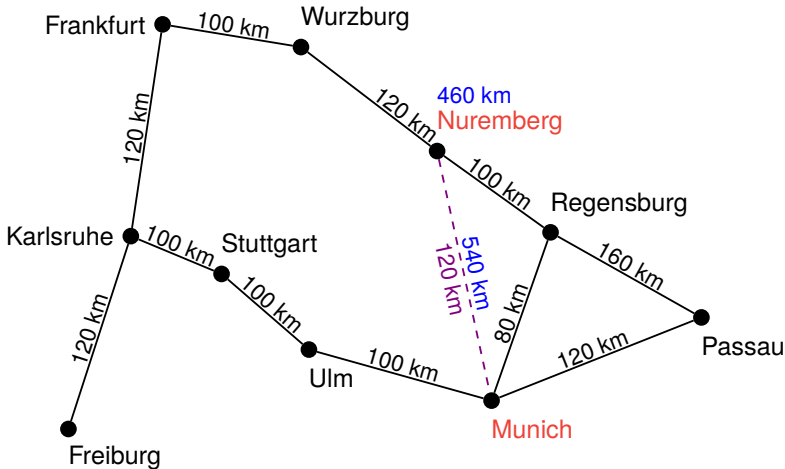
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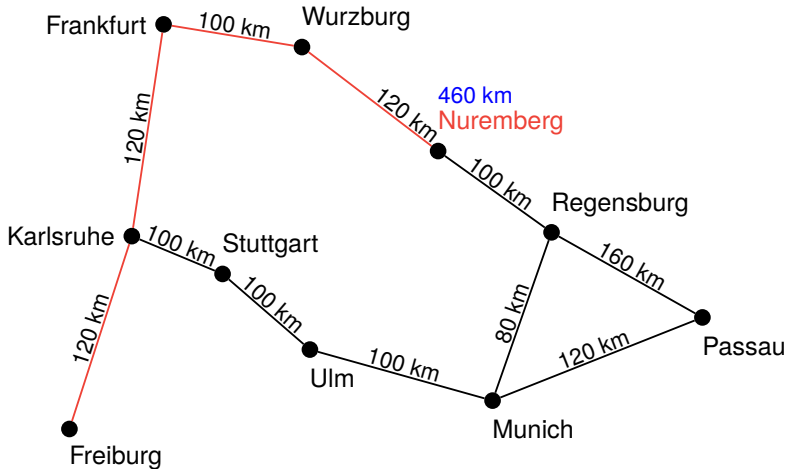
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In **positive normal form** (remember?), good and bad effects are easy to distinguish:

- Effects that make state variables true are good (**add effects**).
- Effects that make state variables false are bad (**delete effects**).

Idea for the heuristic: Ignore all delete effects.

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Definition (relaxation of operators)

The **relaxation** o^+ of an operator $o = \langle \chi, e \rangle$ in positive normal form is the operator which is obtained by replacing all negative effects $\neg a$ within e by the do-nothing effect \top .

Definition (relaxation of planning tasks)

The **relaxation** Π^+ of a planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form is the planning task $\Pi^+ := \langle A, I, \{o^+ \mid o \in O\}, \gamma \rangle$.

Definition (relaxation of operator sequences)

The **relaxation** of an operator sequence $\pi = o_1 \dots o_n$ is the operator sequence $\pi^+ := o_1^+ \dots o_n^+$.

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- Planning tasks in positive normal form without delete effects are called **relaxed planning tasks**.
- Plans for relaxed planning tasks are called **relaxed plans**.
- If Π is a planning task in positive normal form and π^+ is a plan for Π^+ , then π^+ is called a **relaxed plan for Π** .

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The **on-set** $on(s)$ of a state s is the set of true state variables in s , i.e. $on(s) = s^{-1}(\{1\})$.

A state s' **dominates** another state s iff $on(s) \subseteq on(s')$.

Lemma (domination)

Let s and s' be valuations of a set of propositional variables A and let χ be a propositional formula over A which does not contain negation symbols.

If $s \models \chi$ and s' dominates s , then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ .

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \perp$: then $s \not\models \perp$.

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Proof (ctd.)

- Base case $\chi = a \in A$: assume $s \models a$ and $on(s) \subseteq on(s')$.
With $a \in on(s)$ we get $a \in on(s')$, hence $s' \models a$.
- Inductive case $\chi = \chi_1 \wedge \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ .

$$\begin{aligned} s \models \chi &\iff s \models \chi_1 \wedge \chi_2 \\ &\iff s \models \chi_1 \text{ and } s \models \chi_2 \\ \text{I.H. (twice)} &\implies s' \models \chi_1 \text{ and } s' \models \chi_2 \\ &\iff s' \models \chi_1 \wedge \chi_2 \\ &\iff s' \models \chi. \end{aligned}$$

- Inductive case $\chi = \chi_1 \vee \chi_2$: Analogous.



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The relaxation lemma



For the rest of this chapter, we assume that all planning tasks are in positive normal form.

Lemma (relaxation)

Let s be a state, let s' be a state that dominates s , and let π be an operator sequence which is applicable in s . Then π^+ is applicable in s' and $app_{\pi^+}(s')$ dominates $app_{\pi}(s)$. Moreover, if π leads to a goal state from s , then π^+ leads to a goal state from s' .

Proof.

The “moreover” part follows from the rest by the domination lemma. Prove the rest by induction over the length of π .

Base case: $\pi = \varepsilon$

$app_{\pi^+}(s') = s'$ dominates $app_{\pi}(s) = s$ by assumption.

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Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$

By the induction hypothesis, $o_1^+ \dots o_n^+$ is applicable in s' , and $t' = \text{app}_{o_1^+ \dots o_n^+}(s')$ dominates $t = \text{app}_{o_1 \dots o_n}(s)$.

Let $o := o_{n+1} = \langle \chi, e \rangle$ and $o^+ = \langle \chi, e^+ \rangle$. By assumption, o is applicable in t , and thus $t \models \chi$. By the domination lemma, we get $t' \models \chi$ and hence o^+ is applicable in t' . Therefore, π^+ is applicable in s' .

Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals).

We get

$$\text{on}(\text{app}_{\pi}(s)) \subseteq \text{on}(t) \cup ([e]_t \cap A) \subseteq \text{on}(t') \cup [e^+]_{t'} = \text{on}(\text{app}_{\pi^+}(s')),$$

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Corollary (relaxation leads to dominance and preserves plans)

Let π be an operator sequence that is applicable in state s . Then π^+ is applicable in s and $app_{\pi^+}(s)$ dominates $app_{\pi}(s)$. If π is a plan for Π , then π^+ is a plan for Π^+ .

Proof.

Apply relaxation lemma with $s' = s$.

- \rightsquigarrow Relaxations of plans are relaxed plans.
- \rightsquigarrow Relaxations are no harder to solve than original task.
- \rightsquigarrow Optimal relaxed plans are never longer than optimal plans for original tasks.

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Corollary (relaxation preserves dominance)

Let s be a state, let s' be a state that dominates s , and let π^+ be a relaxed operator sequence applicable in s . Then π^+ is applicable in s' and $\text{app}_{\pi^+}(s')$ dominates $\text{app}_{\pi^+}(s)$.

Proof.

Apply relaxation lemma with π^+ for π , noting that $(\pi^+)^+ = \pi^+$.

- \rightsquigarrow If there is a relaxed plan starting from state s , the same plan can be used starting from a dominating state s' .
- \rightsquigarrow Making a transition to a dominating state never hurts in relaxed planning tasks.

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Monotonicity of relaxed planning tasks



We need one final property before we can provide an algorithm for solving relaxed planning tasks.

Lemma (monotonicity)

Let $o^+ = \langle \chi, e^+ \rangle$ be a relaxed operator and let s be a state in which o^+ is applicable.

Then $app_{o^+}(s)$ dominates s .

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup [e^+]_s = on(app_{o^+}(s))$.

\rightsquigarrow Together with our previous results, this means that making a transition in a relaxed planning task **never** hurts.

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Greedy algorithm for relaxed planning tasks



The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy planning algorithm for $\langle A, I, O^+, \gamma \rangle$

```
s := I
 $\pi^+ := \varepsilon$ 
forever:
  if s  $\models \gamma$ :
    return  $\pi^+$ 
  else if there is an operator  $o^+ \in O^+$  applicable in s
    with  $app_{o^+}(s) \neq s$ :
    Append such an operator  $o^+$  to  $\pi^+$ .
    s :=  $app_{o^+}(s)$ 
  else:
    return unsolvable
```

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The algorithm is **sound**:

- If it returns a plan, this is indeed a correct solution.
- If it returns “unsolvable”, the task is indeed unsolvable
 - Upon termination, there clearly is no relaxed plan from s .
 - By iterated application of the monotonicity lemma, s dominates l .
 - By the relaxation lemma, there is no solution from l .

What about **completeness** (termination) and **runtime**?

- Each iteration of the loop adds at least one atom to $on(s)$.
- This guarantees termination after at most $|A|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
 - A good implementation runs in $O(\|\Pi\|)$.

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We can apply the greedy algorithm within heuristic search:

- In a search node σ , solve the relaxation of the planning task with $state(\sigma)$ as the initial state.
- Set $h(\sigma)$ to the length of the generated relaxed plan.

Is this an **admissible** heuristic?

- Yes if the relaxed plans are **optimal** (due to the plan preservation corollary).
- However, usually they are not, because our greedy planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

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The set cover problem



To obtain an admissible heuristic, we need to generate optimal relaxed plans. Can we do this efficiently?

This question is related to the following problem:

Problem (set cover)

Given: a finite set U , a collection of subsets $C = \{C_1, \dots, C_n\}$ with $C_i \subseteq U$ for all $i \in \{1, \dots, n\}$, and a natural number K .

Question: Does there exist a set cover of size at most K , i. e., a subcollection $S = \{S_1, \dots, S_m\} \subseteq C$ with $S_1 \cup \dots \cup S_m = U$ and $m \leq K$?

The following is a classical result from complexity theory:

Theorem (Karp 1972)

The set cover problem is NP-complete.

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Theorem (optimal relaxed planning is hard)

The problem of deciding whether a given relaxed planning task has a plan of length at most K is NP-complete.

Proof.

For **membership in NP**, guess a plan and verify. It is sufficient to check plans of length at most $|A|$, so this can be done in nondeterministic polynomial time.

For **hardness**, we reduce from the set cover problem.

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Proof (ctd.)

Given a set cover instance $\langle U, C, K \rangle$, we generate the following relaxed planning task $\Pi^+ = \langle A, I, O^+, \gamma \rangle$:

- $A = U$
- $I = \{a \mapsto 0 \mid a \in A\}$
- $O^+ = \{ \langle T, \bigwedge_{a \in C_i} a \rangle \mid C_i \in C \}$
- $\gamma = \bigwedge_{a \in U} a$

If S is a set cover, the corresponding operators form a plan. Conversely, each plan induces a set cover by taking the subsets corresponding to the operators. There exists a plan of length at most K iff there exists a set cover of size K .

Moreover, Π^+ can be generated from the set cover instance in polynomial time, so this is a polynomial reduction. □

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How can we use relaxations for heuristic planning in practice?

Different possibilities:

- Implement an **optimal planner** for relaxed planning tasks and use its solution lengths as an estimate, even though it is NP-hard.
 \rightsquigarrow **h^+ heuristic**
- Do not actually solve the relaxed planning task, but compute an estimate of its difficulty in a different way.
 \rightsquigarrow **h_{\max} heuristic, h_{add} heuristic, $h_{\text{LM-cut}}$ heuristic**
- Compute a solution for relaxed planning tasks which is not necessarily optimal, but “reasonable”.
 \rightsquigarrow **h_{FF} heuristic**

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- Two general methods for coming up with heuristics:
 - **relaxation**: solve a **less constrained** problem
 - **abstraction**: solve a **small** problem
- Here, we consider the **delete relaxation**, which requires tasks in positive normal form and ignores delete effects.
- Delete-relaxed tasks have a **domination** property: it is always beneficial to make more fluents true.
- They also have a **monotonicity** property: applying operators leads to dominating states.
- Because of these two properties, **finding some relaxed plan** greedily is **easy** (polynomial).
- For an informative heuristic, we would ideally want to find **optimal relaxed plans**. This is **NP-complete**.

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