

# Principles of AI Planning

## 5. Planning as search: progression and regression

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October 31st, 2014



# Planning as (classical) search

## Search

Introduction  
Classification

Progression

Regression

Summary

# What do we mean by search?



- **Search** is a very generic term.
- ↪ Every algorithm that tries out various alternatives can be said to “search” in some way.
- Here, we mean **classical search** algorithms.
  - **Search nodes** are **expanded** to generate **successor nodes**.
  - **Examples:** breadth-first search, A\*, hill-climbing, ...
- To be brief, we just say **search** in the following (not “classical search”).

# Do you know this stuff already?



- We **assume prior knowledge** of basic search algorithms:
  - uninformed vs. informed
  - systematic vs. local
- There will be a small refresher in the next chapter.
- **Background:** Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)

Search

Introduction  
Classification

Progression

Regression

Summary



- **search**: one of the **big success stories** of AI
- many planning algorithms based on classical AI search (we'll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)

Search

Introduction  
Classification

Progression

Regression

Summary

# Satisficing or optimal planning?



Must carefully distinguish two different problems:

- **satisficing planning**: any solution is OK (although shorter solutions typically preferred)
- **optimal planning**: plans must have shortest possible length

Both are often solved by search, but:

- details are **very different**
- almost **no overlap** between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners

Search

Introduction  
Classification

Progression

Regression

Summary



How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 1: Search direction

- **progression**: forward from initial state to goal
- **regression**: backward from goal states to initial state
- **bidirectional search**

Search

Introduction

Classification

Progression

Regression

Summary



How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 2: Search space representation

- search nodes are associated with **states**  
( $\rightsquigarrow$  **state-space search**)
- search nodes are associated with **sets of states**

Search

Introduction

Classification

Progression

Regression

Summary





How to apply search to planning?  $\rightsquigarrow$  **many choices to make!**

## Choice 3: Search algorithm

- **uninformed search:**  
depth-first, breadth-first, iterative depth-first, ...
- **heuristic search (systematic):**  
greedy best-first,  $A^*$ , Weighted  $A^*$ , IDA\*, ...
- **heuristic search (local):**  
hill-climbing, simulated annealing, beam search, ...

Search

Introduction

Classification

Progression

Regression

Summary



How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 4: Search control

- **heuristics** for informed search algorithms
- **pruning techniques**: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, ...

Search

Introduction

Classification

Progression

Regression

Summary



Search

Introduction

Classification

Progression

Regression

Summary

## FF (Hoffmann & Nebel, 2001)

- search direction: forward search
- search space representation: single states
- search algorithm: enforced hill-climbing (informed local)
- heuristic: FF heuristic (inadmissible)
- pruning technique: helpful actions (incomplete)

↪ one of the best satisficing planners



## Fast Downward Stone Soup (Helmert et al., 2011)

- search direction: forward search
- search space representation: single states
- search algorithm: A\* (informed systematic)
- heuristic: multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, ...)
- pruning technique: none

↪ one of the best optimal planners

Search

Introduction

Classification

Progression

Regression

Summary

# Our plan for the next lectures



## Choices to make:

- 1 search direction: progression/regression/both  
~> **this chapter**
- 2 search space representation: states/sets of states  
~> **this chapter**
- 3 search algorithm: uninformed/heuristic; systematic/local  
~> **next chapter**
- 4 search control: heuristics, pruning techniques  
~> **following chapters**

Search

Introduction

Classification

Progression

Regression

Summary



# Progression

[Search](#)

[Progression](#)

[Overview](#)

[Example](#)

[Regression](#)

[Summary](#)



**Progression:** Computing the successor state  $app_o(s)$  of a state  $s$  with respect to an operator  $o$ .

**Progression planners** find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and **progress it** through an operator, generating a new state
- solution found when a goal state generated

**pro:** very easy and efficient to implement

Search

Progression

Overview

Example

Regression

Summary



Two alternative search spaces for progression planners:

## 1 search nodes correspond to states

- when the same state is generated along different paths, it is not considered again (**duplicate detection**)
- **pro**: save time to consider same state again
- **con**: memory intensive (must maintain **closed list**)

## 2 search nodes correspond to operator sequences

- different operator sequences may lead to identical states (**transpositions**); search does not notice this
- **pro**: can be very memory-efficient
- **con**: much wasted work (often exponentially slower)

↪ first alternative usually preferable in planning  
(**unlike** many classical search benchmarks like 15-puzzle)

Search

Progression

Overview

Example

Regression

Summary



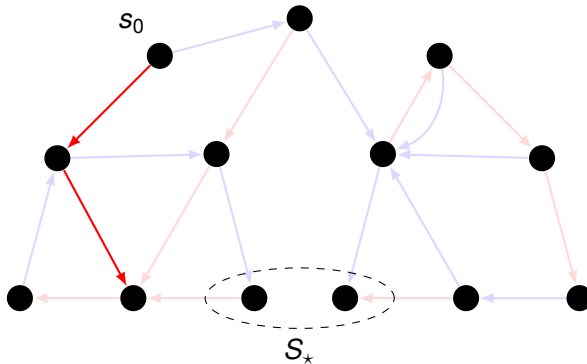




# Progression planning example (depth-first search)



**Example** where search nodes correspond to operator sequences  
(no duplicate detection)



Search

Progression

Overview

Example

Regression

Summary

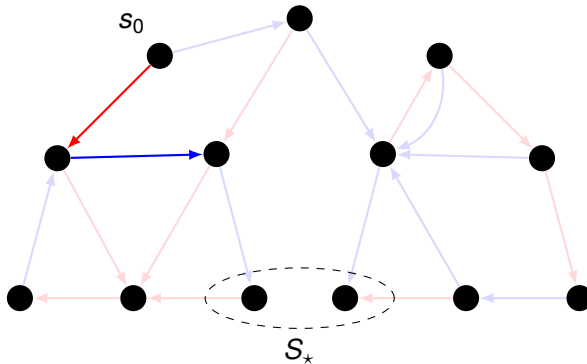




# Progression planning example (depth-first search)



Example where search nodes correspond to operator sequences  
(no duplicate detection)



Search

Progression

Overview

Example

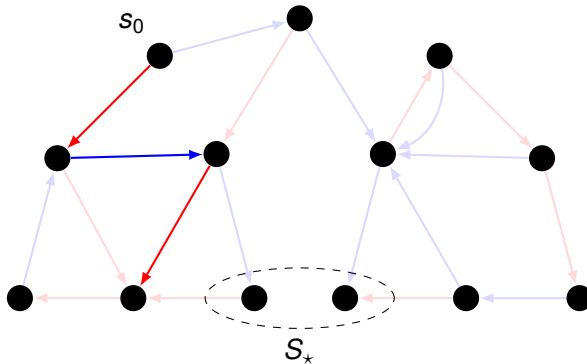
Regression

Summary

# Progression planning example (depth-first search)



**Example** where search nodes correspond to operator sequences  
(no duplicate detection)



Search

Progression

Overview

Example

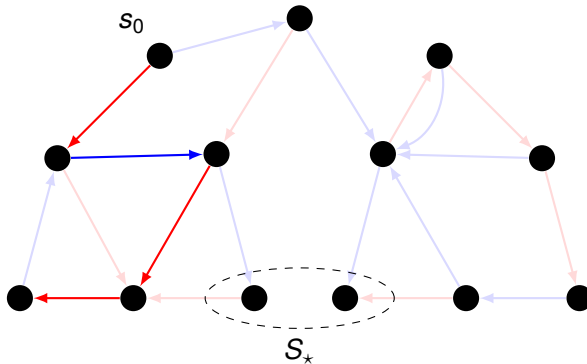
Regression

Summary

# Progression planning example (depth-first search)



Example where search nodes correspond to operator sequences  
(no duplicate detection)



Search

Progression

Overview

Example

Regression

Summary

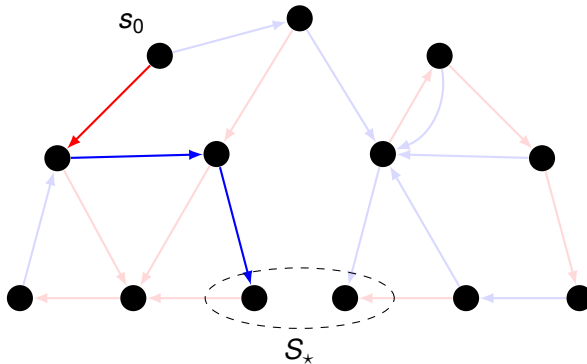




# Progression planning example (depth-first search)



Example where search nodes correspond to operator sequences  
(no duplicate detection)



Search

Progression

Overview

Example

Regression

Summary



# Regression

[Search](#)

[Progression](#)

**[Regression](#)**

[Overview](#)

[Example](#)

[STRIPS](#)

[General case](#)

[Practical issues](#)

[Summary](#)



Going through a transition graph in forward and backward directions is **not symmetric**:

- forward search starts from a **single** initial state; backward search starts from a **set** of goal states
- when applying an operator  $o$  in a state  $s$  in forward direction, there is a **unique successor state**  $s'$ ; if we applied operator  $o$  to end up in state  $s'$ , there can be **several possible predecessor states**  $s$

↪ most natural representation for backward search in planning associates **sets of states** with search nodes

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



**Regression:** Computing the possible predecessor states  $regr_o(G)$  of a set of states  $G$  with respect to the last operator  $o$  that was applied.

**Regression planners** find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated state set and **regress it** through an operator, generating a new state set
- solution found when a generated state set includes the initial state

**Pro:** can handle many states simultaneously

**Con:** basic operations complicated and expensive

[Search](#)

[Progression](#)

[Regression](#)

[Overview](#)

[Example](#)

[STRIPS](#)

[General case](#)

[Practical issues](#)

[Summary](#)



Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

identify state sets with **logical formulae** (again):

- **search nodes correspond to state sets**
- each state set is represented by a **logical formula**:  
 $\varphi$  represents  $\{s \in \mathcal{S} \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-hard or coNP-hard

# Regression planning example (depth-first search)



Search

Progression

Regression

Overview

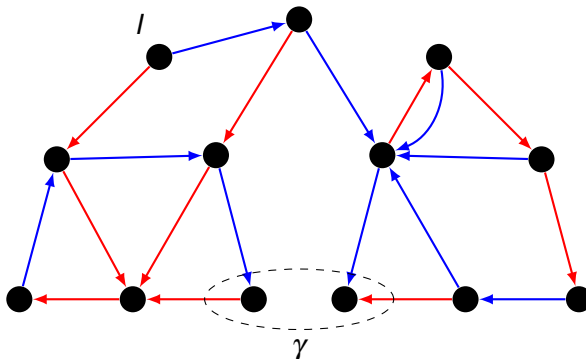
**Example**

STRIPS

General case

Practical issues

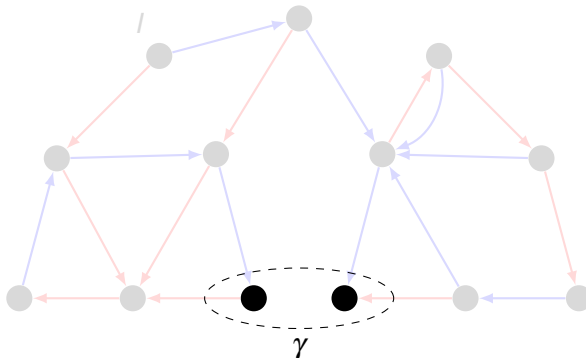
Summary



# Regression planning example (depth-first search)



$\gamma$



- Search
- Progression
- Regression
  - Overview
  - Example**
  - STRIPS
  - General case
  - Practical issues
- Summary

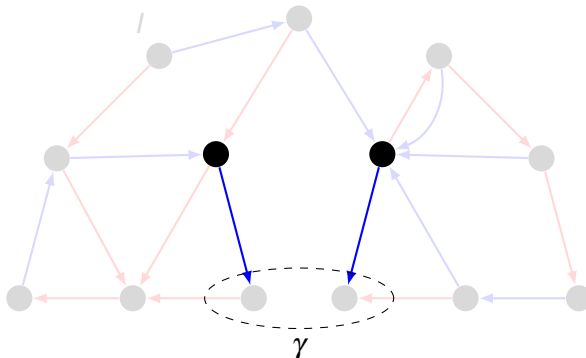


# Regression planning example (depth-first search)



$$\varphi_1 = \text{regr} \rightarrow (\gamma)$$

$$\varphi_1 \longrightarrow \gamma$$



- Search
- Progression
- Regression
  - Overview
  - Example**
  - STRIPS
  - General case
  - Practical issues
- Summary



# Regression planning example (depth-first search)

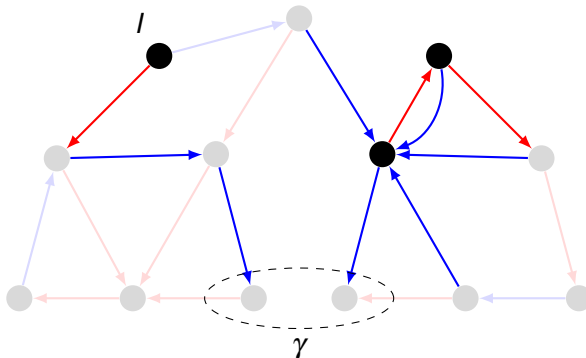


$$\varphi_1 = \text{regr} \rightarrow (\gamma)$$

$$\varphi_2 = \text{regr} \rightarrow (\varphi_1)$$

$$\varphi_3 = \text{regr} \rightarrow (\varphi_2), I \models \varphi_3$$

$$\varphi_3 \xrightarrow{\text{red}} \varphi_2 \xrightarrow{\text{blue}} \varphi_1 \xrightarrow{\text{blue}} \gamma$$



- Search
- Progression
- Regression
  - Overview
  - Example
  - STRIPS
  - General case
  - Practical issues
- Summary



## Definition (STRIPS planning task)

A planning task is a **STRIPS planning task** if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression **for STRIPS planning tasks** is very simple:

- Goals are conjunctions of atoms  $a_1 \wedge \dots \wedge a_n$ .
- **First step**: Choose an operator that makes none of  $a_1, \dots, a_n$  false.
- **Second step**: Remove goal atoms achieved by the operator (if any) and add its preconditions.

↪ Outcome of regression is again conjunction of atoms.

**Optimization**: only consider operators making some  $a_i$  true

Search

Progression

Regression

Overview

Example

**STRIPS**

General case

Practical issues

Summary

## Definition (STRIPS regression)

Let  $\varphi = \varphi_1 \wedge \dots \wedge \varphi_n$  be a conjunction of atoms, and let  $o = \langle \chi, e \rangle$  be a STRIPS operator which adds the atoms  $a_1, \dots, a_k$  and deletes the atoms  $d_1, \dots, d_l$ .

The **STRIPS regression** of  $\varphi$  with respect to  $o$  is

$$sregr_o(\varphi) := \begin{cases} \perp & \text{if } a_i = d_j \text{ for some } i, j \\ \perp & \text{if } \varphi_i = d_j \text{ for some } i, j \\ \chi \wedge \wedge(\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{otherwise} \end{cases}$$

**Note:**  $sregr_o(\varphi)$  is again a conjunction of atoms, or  $\perp$ .

Search

Progression

Regression

Overview

Example

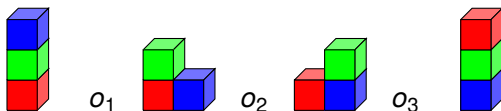
STRIPS

General case

Practical issues

Summary

# STRIPS regression example



**Note:** Predecessor states are in general not unique.  
This picture is just for illustration purposes.

$$\begin{aligned}
 o_1 &= \langle \text{blue on green} \wedge \text{blue clr}, & \neg \text{blue on green} \wedge \text{blue onT} \wedge \text{green clr} \rangle \\
 o_2 &= \langle \text{green on red} \wedge \text{green clr} \wedge \text{blue clr}, & \neg \text{blue clr} \wedge \neg \text{green on red} \wedge \text{green on blue} \wedge \text{red clr} \rangle \\
 o_3 &= \langle \text{red onT} \wedge \text{red clr} \wedge \text{green clr}, & \neg \text{green clr} \wedge \neg \text{red onT} \wedge \text{red on green} \rangle
 \end{aligned}$$

$$\gamma = \text{red on green} \wedge \text{green on blue}$$

$$\varphi_1 = \text{sregr}_{o_3}(\gamma) = \text{red onT} \wedge \text{red clr} \wedge \text{green clr} \wedge \text{green on blue}$$

$$\varphi_2 = \text{sregr}_{o_2}(\varphi_1) = \text{green on red} \wedge \text{green clr} \wedge \text{blue clr} \wedge \text{red onT}$$

$$\varphi_3 = \text{sregr}_{o_1}(\varphi_2) = \text{blue on green} \wedge \text{blue clr} \wedge \text{green on red} \wedge \text{red onT}$$

- Search
- Progression
- Regression
  - Overview
  - Example
  - STRIPS**
  - General case
  - Practical issues
- Summary



- With disjunctions and conditional effects, things become more tricky. How to regress  $a \vee (b \wedge c)$  with respect to  $\langle q, d \triangleright b \rangle$ ?
- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.
- We present a general method for doing regression for any formula and any operator.
- Now we extensively use the idea of representing sets of states as formulae.

[Search](#)

[Progression](#)

[Regression](#)

[Overview](#)

[Example](#)

[STRIPS](#)

[General case](#)

[Practical issues](#)

[Summary](#)

## Definition (effect precondition)

The **effect precondition**  $EPC_l(e)$  for literal  $l$  and effect  $e$  is defined as follows:

$$\begin{aligned}EPC_l(l) &= \top \\EPC_l(l') &= \perp \text{ if } l \neq l' \quad (\text{for literals } l') \\EPC_l(e_1 \wedge \dots \wedge e_n) &= EPC_l(e_1) \vee \dots \vee EPC_l(e_n) \\EPC_l(\chi \triangleright e) &= EPC_l(e) \wedge \chi\end{aligned}$$

**Intuition:**  $EPC_l(e)$  describes the situations in which effect  $e$  causes literal  $l$  to become true.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



Search

Progression

Regression

Overview

Example

STRIPS

**General case**

Practical issues

Summary

## Example

$$EPC_a(b \wedge c) = \perp \vee \perp \equiv \perp$$

$$EPC_a(a \wedge (b \triangleright a)) = \top \vee (\top \wedge b) \equiv \top$$

$$EPC_a((c \triangleright a) \wedge (b \triangleright a)) = (\top \wedge c) \vee (\top \wedge b) \equiv c \vee b$$

# Effect preconditions: connection to change sets



## Lemma (A)

*Let  $s$  be a state,  $l$  a literal and  $e$  an effect.  
Then  $l \in [e]_s$  if and only if  $s \models EPC_l(e)$ .*

## Proof.

Induction on the structure of the effect  $e$ .

Base case 1,  $e = l$ :  $l \in [l]_s = \{l\}$  by definition, and  $s \models EPC_l(l) = \top$  by definition. Both sides of the equivalence are true.

Base case 2,  $e = l'$  for some literal  $l' \neq l$ :  $l \notin [l']_s = \{l'\}$  by definition, and  $s \not\models EPC_l(l') = \perp$  by definition. Both sides are false.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

# Effect preconditions: connection to change sets



## Lemma (A)

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

## Proof (ctd.)

Inductive case 1,  $e = e_1 \wedge \dots \wedge e_n$ :

$I \in [e]_s$  iff  $I \in [e_1]_s \cup \dots \cup [e_n]_s$  (Def  $[e_1 \wedge \dots \wedge e_n]_s$ )

iff  $I \in [e']_s$  for some  $e' \in \{e_1, \dots, e_n\}$

iff  $s \models EPC_I(e')$  for some  $e' \in \{e_1, \dots, e_n\}$  (IH)

iff  $s \models EPC_I(e_1) \vee \dots \vee EPC_I(e_n)$

iff  $s \models EPC_I(e_1 \wedge \dots \wedge e_n)$ . (Def  $EPC$ )

Inductive case 2,  $e = \chi \triangleright e'$ :

$I \in [\chi \triangleright e']_s$  iff  $I \in [e']_s$  and  $s \models \chi$  (Def  $[\chi \triangleright e']_s$ )

iff  $s \models EPC_I(e')$  and  $s \models \chi$  (IH)

iff  $s \models EPC_I(e') \wedge \chi$

iff  $s \models EPC_I(\chi \triangleright e')$ . (Def  $EPC$ )



Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Inductive case 1,  $e = e_1 \wedge \dots \wedge e_n$ :

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iff  $s \models EPC_I(e')$  for some  $e' \in \{e_1, \dots, e_n\}$  (IH)

iff  $s \models EPC_I(e_1) \vee \dots \vee EPC_I(e_n)$

iff  $s \models EPC_I(e_1 \wedge \dots \wedge e_n)$ . (Def  $EPC$ )

Inductive case 2,  $e = \chi \triangleright e'$ :

$I \in [\chi \triangleright e']_s$  iff  $I \in [e']_s$  and  $s \models \chi$  (Def  $[\chi \triangleright e']_s$ )

iff  $s \models EPC_I(e')$  and  $s \models \chi$  (IH)

iff  $s \models EPC_I(e') \wedge \chi$

iff  $s \models EPC_I(\chi \triangleright e')$ . (Def  $EPC$ )



Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Remark: *EPC* vs. effect normal form

Notice that in terms of  $EPC_a(e)$ , any operator  $\langle \chi, e \rangle$  can be expressed in effect normal form as

$$\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \triangleright a) \wedge (EPC_{\neg a}(e) \triangleright \neg a)) \right\rangle,$$

where  $A$  is the set of all state variables.

Search

Progression

Regression

Overview

Example

STRIPS

**General case**

Practical issues

Summary



The formula  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$  expresses the **value of state variable  $a \in A$  after applying  $o$**  in terms of **values of state variables before applying  $o$** .

Either:

- $a$  became true, or
- $a$  was true before and it did not become false.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Example

Let  $e = (b \triangleright a) \wedge (c \triangleright \neg a) \wedge b \wedge \neg d$ .

variable $x$	$EPC_x(e) \vee (x \wedge \neg EPC_{\neg x}(e))$
$a$	$b \vee (a \wedge \neg c)$
$b$	$\top \vee (b \wedge \neg \perp) \equiv \top$
$c$	$\perp \vee (c \wedge \neg \perp) \equiv c$
$d$	$\perp \vee (d \wedge \neg \top) \equiv \perp$

[Search](#)

[Progression](#)

[Regression](#)

[Overview](#)

[Example](#)

[STRIPS](#)

[General case](#)

[Practical issues](#)

[Summary](#)



## Lemma (B)

Let  $a$  be a state variable,  $o = \langle \chi, e \rangle$  an operator,  $s$  a state, and  $s' = \text{app}_o(s)$ .

Then  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$  if and only if  $s' \models a$ .

## Proof.

( $\Rightarrow$ ): Assume  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$ .

Do a case analysis on the two disjuncts.

- 1 Assume that  $s \models \text{EPC}_a(e)$ . By Lemma A, we have  $a \in [e]_s$  and hence  $s' \models a$ .
- 2 Assume that  $s \models a \wedge \neg \text{EPC}_{\neg a}(e)$ . By Lemma A, we have  $\neg a \notin [e]_s$ . Hence  $a$  remains true in  $s'$ .

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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( $\Rightarrow$ ): Assume  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$ .

Do a case analysis on the two disjuncts.

- 1 Assume that  $s \models \text{EPC}_a(e)$ . By Lemma A, we have  $a \in [e]_s$  and hence  $s' \models a$ .
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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Lemma (B)

Let  $a$  be a state variable,  $o = \langle \chi, e \rangle$  an operator,  $s$  a state, and  $s' = \text{app}_o(s)$ .

Then  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$  if and only if  $s' \models a$ .

## Proof.

( $\Rightarrow$ ): Assume  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$ .

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Proof (ctd.)

( $\Leftarrow$ ): We showed that if the formula is **true** in  $s$ , then  $a$  is **true** in  $s'$ . For the second part, we show that if the formula is **false** in  $s$ , then  $a$  is **false** in  $s'$ .

- So assume  $s \not\models EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ .
  - Then  $s \models \neg EPC_a(e) \wedge (\neg a \vee EPC_{\neg a}(e))$  (de Morgan).
  - Case distinction:  $a$  is true or  $a$  is false in  $s$ .
    - 1 Assume that  $s \models a$ . Now  $s \models EPC_{\neg a}(e)$  because  $s \models \neg a \vee EPC_{\neg a}(e)$ .  
Hence by Lemma A  $\neg a \in [e]_s$  and we get  $s' \not\models a$ .
    - 2 Assume that  $s \not\models a$ . Because  $s \models \neg EPC_a(e)$ , by Lemma A we get  $a \notin [e]_s$  and hence  $s' \not\models a$ .
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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary





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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



We base the definition of regression on formulae  $EPC_I(e)$ .

## Definition (general regression)

Let  $\varphi$  be a propositional formula and  $o = \langle \chi, e \rangle$  an operator.  
The **regression of  $\varphi$  with respect to  $o$**  is

$$\text{regr}_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$$

where

- 1  $\varphi_r$  is obtained from  $\varphi$  by replacing each  $a \in A$  by  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ , and
- 2  $\kappa = \bigwedge_{a \in A} \neg(EPC_a(e) \wedge EPC_{\neg a}(e))$ .

The formula  $\kappa$  expresses that operators are only applicable in states where their change sets are consistent.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



- $\text{regr}_{\langle a, b \rangle}(b) \equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge \top \equiv a$
- $\text{regr}_{\langle a, b \rangle}(b \wedge c \wedge d)$   
 $\equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge (\perp \vee (c \wedge \neg \perp)) \wedge (\perp \vee (d \wedge \neg \perp)) \wedge \top$   
 $\equiv a \wedge c \wedge d$
- $\text{regr}_{\langle a, c \triangleright b \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg \perp)) \wedge \top \equiv a \wedge (c \vee b)$
- $\text{regr}_{\langle a, (c \triangleright b) \wedge (b \triangleright \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg b)) \wedge \neg(c \wedge b)$   
 $\equiv a \wedge c \wedge \neg b$
- $\text{regr}_{\langle a, (c \triangleright b) \wedge (d \triangleright \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg d)) \wedge \neg(c \wedge d)$   
 $\equiv a \wedge (c \vee b) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d)$   
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[Search](#)

[Progression](#)

[Regression](#)

[Overview](#)

[Example](#)

[STRIPS](#)

[General case](#)

[Practical issues](#)

[Summary](#)

# Regression example: binary counter



$$\begin{aligned} & (\neg b_0 \triangleright b_0) \wedge \\ & ((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

$$EPC_{b_2}(e) = \neg b_2 \wedge b_1 \wedge b_0$$

$$EPC_{b_1}(e) = \neg b_1 \wedge b_0$$

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$$EPC_{\neg b_0}(e) = (\neg b_1 \wedge b_0) \vee (\neg b_2 \wedge b_1 \wedge b_0) \equiv (\neg b_1 \vee \neg b_2) \wedge b_0$$

Regression replaces state variables as follows:

$$b_2 \quad \text{by} \quad (\neg b_2 \wedge b_1 \wedge b_0) \vee (b_2 \wedge \neg \perp) \equiv (b_1 \wedge b_0) \vee b_2$$

$$\begin{aligned} b_1 \quad \text{by} \quad & (\neg b_1 \wedge b_0) \vee (b_1 \wedge \neg(\neg b_2 \wedge b_1 \wedge b_0)) \\ & \equiv (\neg b_1 \wedge b_0) \vee (b_1 \wedge (b_2 \vee \neg b_0)) \end{aligned}$$

$$b_0 \quad \text{by} \quad \neg b_0 \vee (b_0 \wedge \neg((\neg b_1 \vee \neg b_2) \wedge b_0)) \equiv \neg b_0 \vee (b_1 \wedge b_2)$$

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Theorem (correctness of $regr_o(\varphi)$ )

Let  $\varphi$  be a formula,  $o$  an operator and  $s$  a state.

Then  $s \models regr_o(\varphi)$  iff  $o$  is applicable in  $s$  and  $app_o(s) \models \varphi$ .

## Proof.

Let  $o = \langle \chi, e \rangle$ . Recall that  $regr_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$ , where  $\varphi_r$  and  $\kappa$  are as defined previously.

If  $o$  is inapplicable in  $s$ , then  $s \not\models \chi \wedge \kappa$ , both sides of the “iff” condition are false, and we are done. Hence, we only further consider states  $s$  where  $o$  is applicable. Let  $s' := app_o(s)$ .

We know that  $s \models \chi \wedge \kappa$  (because  $o$  is applicable), so the “iff” condition we need to prove simplifies to:

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Search

Progression

Regression

Overview

Example

STRIPS

**General case**

Practical issues

Summary





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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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To show:  $s \models \varphi_r$  iff  $s' \models \varphi$ .

We show that for all formulae  $\psi$ ,  $s \models \psi_r$  iff  $s' \models \psi$ , where  $\psi_r$  is  $\psi$  with every  $a \in A$  replaced by  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ .

The proof is by structural induction on  $\psi$ .

Induction hypothesis  $s \models \psi_r$  if and only if  $s' \models \psi$ .

Base cases 1 & 2  $\psi = \top$  or  $\psi = \perp$ : trivial, as  $\psi_r = \psi$ .

Base case 3  $\psi = a$  for some  $a \in A$ :

Then  $\psi_r = EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ .

By Lemma B,  $s \models \psi_r$  iff  $s' \models \psi$ .

Search

Progression

Regression

Overview

Example

STRIPS

**General case**

Practical issues

Summary



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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



## Proof (ctd.)

Inductive case 1  $\psi = \neg\psi'$ :

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r \\ \text{iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 2  $\psi = \psi' \vee \psi''$ :

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\psi' \vee \psi'')_r \text{ iff } s \models \psi'_r \vee \psi''_r \\ \text{iff } s \models \psi'_r \text{ or } s \models \psi''_r \\ \text{iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi'' \\ \text{iff } s' \models \psi' \vee \psi'' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 3  $\psi = \psi' \wedge \psi''$ : Very similar to inductive case 2, just with  $\wedge$  instead of  $\vee$  and “and” instead of “or”.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

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Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

## Proof (ctd.)

Inductive case 1  $\psi = \neg\psi'$ :

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r \\ \text{iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 2  $\psi = \psi' \vee \psi''$ :

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\psi' \vee \psi'')_r \text{ iff } s \models \psi'_r \vee \psi''_r \\ \text{iff } s \models \psi'_r \text{ or } s \models \psi''_r \\ \text{iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi'' \\ \text{iff } s' \models \psi' \vee \psi'' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 3  $\psi = \psi' \wedge \psi''$ : Very similar to inductive case 2, just with  $\wedge$  instead of  $\vee$  and “and” instead of “or”.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that  $regr_o(\varphi)$  does not represent the empty set (which would mean that search is in a dead end).  
For example,  $regr_{\langle a, \neg p \rangle}(p) \equiv a \wedge \perp \equiv \perp$ .
- Test that  $regr_o(\varphi)$  does not represent a subset of  $\varphi$  (which would make the problem harder than before).  
For example,  $regr_{\langle b, c \rangle}(a) \equiv a \wedge b$ .

Both of these problems are **NP-hard**.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

The formula  $regr_{o_1}(regr_{o_2}(\dots regr_{o_{n-1}}(regr_{o_n}(\varphi))))$  may have size  $O(|\varphi||o_1||o_2|\dots|o_{n-1}||o_n|)$ , i. e., the product of the sizes of  $\varphi$  and the operators.

$\rightsquigarrow$  worst-case **exponential** size  $O(m^n)$

## Logical simplifications

- $\perp \wedge \varphi \equiv \perp$ ,  $\top \wedge \varphi \equiv \varphi$ ,  $\perp \vee \varphi \equiv \varphi$ ,  $\top \vee \varphi \equiv \top$
- $a \vee \varphi \equiv a \vee \varphi[\perp/a]$ ,  $\neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a]$ ,  
 $a \wedge \varphi \equiv a \wedge \varphi[\top/a]$ ,  $\neg a \wedge \varphi \equiv \neg a \wedge \varphi[\perp/a]$
- idempotency, absorption, commutativity, associativity, ...

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



**Problem** very big formulae obtained by regression

**Cause** **disjunctivity** in the (NNF) formulae  
(formulae **without disjunctions** easily convertible to  
small formulae  $l_1 \wedge \dots \wedge l_n$  where  $l_i$  are literals and  $n$   
is at most the number of state variables.)

**Idea** handle disjunctivity when generating search trees

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

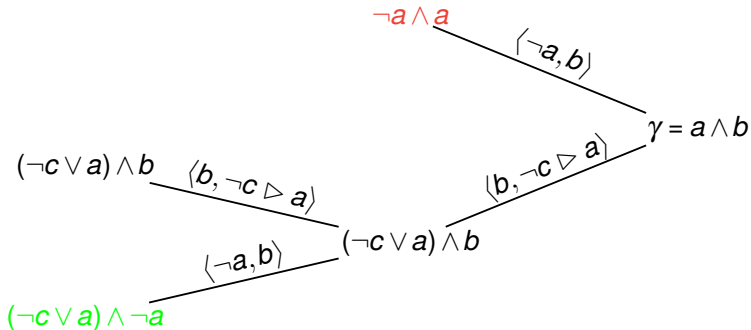
Summary

# Unrestricted regression: search tree example



**Unrestricted regression:** do not treat disjunctions specially

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$ .



Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary

# Full splitting: search tree example

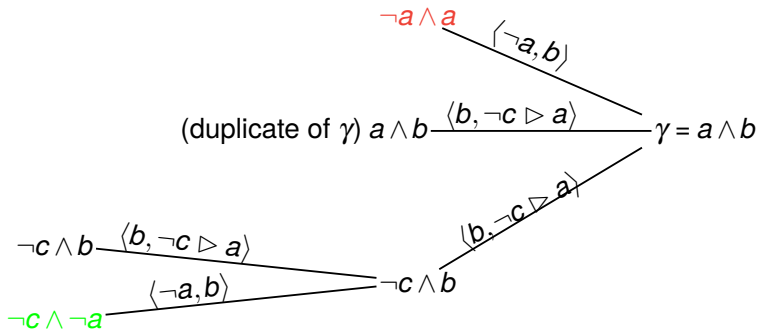


**Full splitting:** always remove all disjunctivity

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$ .

$(\neg c \vee a) \wedge b$  in DNF:  $(\neg c \wedge b) \vee (a \wedge b)$

$\rightsquigarrow$  split into  $\neg c \wedge b$  and  $a \wedge b$



Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary





## Alternatives:

- 1 Do nothing (**unrestricted regression**).
- 2 Always eliminate all disjunctivity (**full splitting**).
- 3 Reduce disjunctivity if formula becomes too big.

## Discussion:

- **With unrestricted regression** the formulae may have **size that is exponential** in the number of state variables.
- **With full splitting** search tree can be **exponentially bigger** than without splitting.
- The third option lies between these two extremes.

Search

Progression

Regression

Overview

Example

STRIPS

General case

Practical issues

Summary



- (Classical) **search** is a very important planning approach.
- Search-based planning algorithms differ along many dimensions, including
  - **search direction** (forward, backward)
  - **what each search node represents**  
(a state, a set of states, an operator sequence)
- **Progression search** proceeds forwards from the initial state.
  - If we use duplicate detection, each search node corresponds to a unique **state**.
  - If we do not use duplicate detection, each search node corresponds to a unique **operator sequence**.



- **Regression search** proceeds backwards from the goal.
  - Each search node corresponds to a **set of states** represented by a **formula**.
  - Regression is simple for **STRIPS** operators.
  - The theory for **general regression** is more complex.
  - When applying regression in practice, additional considerations such as when and how to perform **splitting** come into play.