

Principles of Knowledge Representation and Reasoning

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Exercise Sheet 3

Due: November 13th, 2013

Exercise 3.1 (FORMULA GAME, 1 + 1)

The FORMULA GAME is a two-player game played on a given quantified Boolean formula (in prenex normal form) $Q_1 p_1 \dots Q_k p_k \psi$. The rules are simple: If the outermost unassigned propositional variable p_i is universally (existentially) quantified, it is the turn of player U (player E , resp.) who assigns a truth value to that variable p_i . Thus both players finally construct a truth assignment I to the variables occurring in the matrix formula ψ . Player E wins the game if $I(\psi) = 1$; otherwise, player U wins the game.

A *strategy* of player X in the game is a function that assigns to each possible state of the game in which it is X 's turn to play an allowed action (a truth assignment to the next unassigned variable). A strategy of player X is called *winning* if the player following the strategy wins, independently of how the opponent player plays.

Check whether one of the players U or E has a strategy for winning the formula game for the following formulae:

- (a) $\forall p \forall q \exists r \forall s (p \rightarrow (\neg r \rightarrow (q \wedge s)))$
- (b) $\exists p \forall q \exists r ((r \rightarrow p \vee q) \wedge (q \rightarrow \neg p) \wedge (\neg r \rightarrow \neg q))$

Exercise 3.2 (Reduction, 1+2+5)

- (a) Show that PSPACE = co-PSPACE.
- (b) Show that with regard to Turing reductions the class of all NP-hard problems coincides with the class of all co-NP-hard problems.
- (c) We consider the following two-player game \mathcal{G} played on a directed graph $\langle V, A \rangle$ with a designated start node $v_0 \in V$. Player 1 and player 2 choose in turn some arc in the graph such that each chosen arc starts in the head of the previously chosen arc. Player 1 begins with choosing an arc starting in node v_0 . A player loses the game if s/he is unable to choose an arc to a not yet visited node in the graph.

Show that the following problem is PSPACE-complete.

Instance: A directed graph $\langle V, A \rangle$, a start node v_0 .

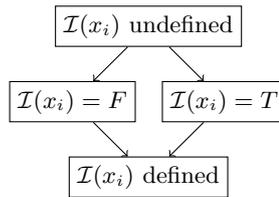
Question: Does Player 1 have a strategy for winning \mathcal{G} ?

Hint: Existence of a winning strategy in the formula game (see exercise 3.1) is known to be PSPACE-complete even for QBF of the following form:

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \dots \exists x_{2k-1} \forall x_{2k} \exists x_{2k+1} \psi,$$

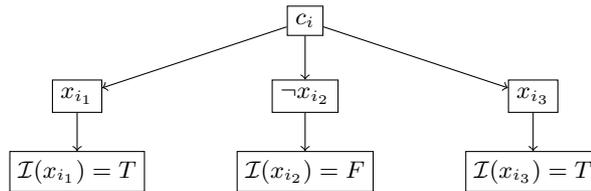
where ψ is a 3-CNF formula. For the reduction construct for a given formula of this form a directed graph. The following subgraphs will be useful:

- For each propositional variable introduce a subgraph with four nodes that represents that a variable has been assigned a truth value.



The current player will have to decide on the truth value of the next unassigned variable x_i . Note that the node corresponding to the chosen assignment may not be revisited in the game.

- Furthermore introduce nodes for each clause c_i of ψ and the literals l_{i_1}, \dots, l_{i_3} occurring in it. For example, if $c_i = x_{i_1} \vee \neg x_{i_2} \vee x_{i_3}$:



Finally discuss the size of your graph and relate the winning strategies in the games.