# Principles of Knowledge Representation and Reasoning

B. Nebel, S. Wölfl, J. Hué
Winter Semester 2013/2014

University of Freiburg
Department of Computer Science

## Exercise Sheet 2

**Due: November 6th, 2013**

**Exercise 2.1** (Resolution and Horn clauses, 3+1)

(a) Consider a satisfiable Horn formula $\psi$. Prove that the interpretation that makes a variable (occurring in $\psi$) true if and only if the variable is true in all models of $\psi$ is also a model of $\psi$.

(b) Apply (a) in order to show that there exists a formula which has no logically equivalent Horn formula.

**Exercise 2.2** (Predicate logic, 2+2+2)

(a) Classify the followi ng expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. The usage of symbols complies with the convention introduced with the syntax of predicate logic.

(a) $P(x, a)$

(b) $g(a, h(b, c))$

(c) $\mathcal{I} \not\models P(a, f(b))$

(d) $\mathcal{I}, \alpha \models P(f(x), f(a))$

(e) $g(f(y), a)$

(f) $Q(b)$ is falsifiable.

(g) $\forall x(P(x, y) \rightarrow Q(x)) \vee \neg P(y, x)$

(h) $\forall x \forall y(P(x, y) \wedge Q(x) \vee P(f(y), x))$

(i) $\forall x(\exists y(P(x, y) \wedge Q(x)) \vee P(x, y))$

(j) $Q(a) \vee P(a, b) \equiv P(b, a) \vee Q(b)$

(b) Consider the following theory:

$$\Theta = \left\{ \begin{array}{l} \forall x \neg P(x, x) \\ \forall x \forall y \forall z(P(x, y) \wedge P(y, z) \rightarrow P(x, z)) \\ \forall x \forall y(P(x, y) \vee x = y \vee P(y, x)) \\ \forall x \forall y(P(x, y) \rightarrow \exists z(P(x, z) \wedge P(z, y))) \\ \neg \exists y P(y, a) \end{array} \right\}$$

Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{I} \models \Theta$ (with proof). Does $\Theta$ have a model that is defined on a finite domain $D$?

(c) Proof by structural induction over terms and formulae: If $\phi$ is a formula, $\mathcal{I}$ an interpretation, and $\alpha, \alpha'$ variable maps over $\mathcal{I}$ such that $\alpha(x) = \alpha'(x)$ for each variable $x$ that occurs free in $\phi$, then

$$\mathcal{I}, \alpha \models \phi \iff \mathcal{I}, \alpha' \models \phi.$$