Principles of AI Planning

19. Expressive power

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Motivation

Motivation: Why Analyzing the Expressive Power?

- Expressive power is the motivation for designing new planning languages
- Often there is the question: **Syntactic sugar** or **essential feature**?

$\rightsquigarrow$ **Compiling away** or change planning algorithm?

$\rightarrow$ If a feature can be compiled away, then it is apparently only **syntactic sugar**.

- Sometimes, however, a compilation can lead to much larger planning domain descriptions or to much longer plans.

$\rightsquigarrow$ This means the planning algorithm will probably choke, i.e., it cannot be considered as a compilation

Example: DNF Preconditions

- Assume we have **DNF preconditions** in STRIPS operators
- This can be compiled away as follows
- Split each operator with a DNF precondition $c_1 \vee \ldots \vee c_n$ into n operators with the same effects and c_i as preconditions

⇝ If there exists a plan for the original planning task there is one for the new planning task and **vice versa**

→ The planning task has almost the same size

→ The shortest plans have the same size

Example: Conditional effects

- Can we compile away **conditional effects** to STRIPS?
- Example operator: $\langle a, b \rhd d \wedge \neg c \rhd e \rangle$
- Can be translated into four operators:
  $\langle a \wedge b \wedge c, d \rangle, \langle a \wedge b \wedge \neg c, d \wedge e \rangle, \ldots$
- Plan existence and plan size are identical
- Exponential blowup of domain description!

$\rightarrow$ Can this be avoided?

Propositional STRIPS and Variants

Propositional STRIPS and Variants

- In the following we will only consider propositional STRIPS and some variants of it.
- Planning task:

$$\mathcal{T} = \langle A, I, O, G \rangle.$$

- Often we refer to domain structures $\mathcal{D} = \langle A, O \rangle$.

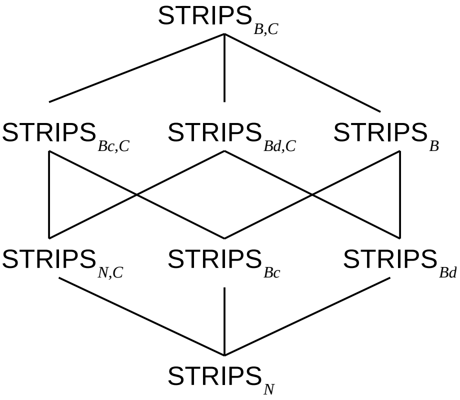
Disjunctive Preconditions: Trivial or Essential?

- Kambhampati et al [ECP 97] and Gazen & Knoblock [ECP 97]: Disjunctive preconditions are trivial – since they can be translated to basic STRIPS (DNF-preconditions)
- Bäckström [AIJ 95]: Disjunctive preconditions are probably essential – since they can not easily be translated to basic STRIPS (CNF-preconditions)
- Anderson et al [AIPS 98]: "[D]isjunctive preconditions . . . are . . . essential prerequisites for handling conditional effects" $\rightsquigarrow$ conditional effects imply disjunctive preconditions (?) (General Boolean preconditions)

More "Expressive Power"

STRIPS_N	: plain strips with negative literals
STRIPS_{Bd}	: precondition in disjunctive normal form
STRIPS_{Bc}	: precondition in conjunctive normal form
STRIPS_B	: Boolean expressions as preconditions
STRIPS_C	: conditional effects
$\text{STRIPS}_{C,N}$	: conditional effects & negative literals
...	

Ordering Planning Formalisms Partially



Computational Complexity . . .

Theorem

PLANEX is PSPACE-complete for $STRIPS_N$, $STRIPS_{C,B}$, and for all formalisms "between" the two.

Proof.

Follows from theorems proved in the previous lecture. □

Expressive Power

Measuring Expressive Power

Consider mappings between planning problems in different formalisms

- that preserve
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan “structure”
  - the solutions/plans themselves
- that are limited
  - in the **size** of the result (poly. size)
  - in the **computational resources** (poly. time)
- that transform
  - entire planning instances
  - domain structure and states in isolation

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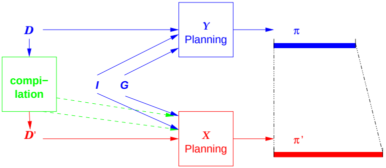
The Right Method: Compilation Schemes (Simplified)

- Transform domain structure $\mathscr{D} = \langle A, O \rangle$ (with polynomial blowup) to $\mathscr{D}'$ preserving solution existence
- Only trivial changes to states (independent of operator set)
- Resulting plans π' should not grow too much (additive constant, linear growth, polynomial growth)

$\rightsquigarrow$ Similar to knowledge compilation, with operators as the fixed part and initial states & goals as the varying part



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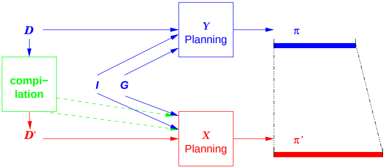

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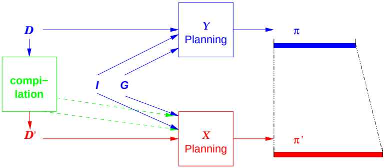

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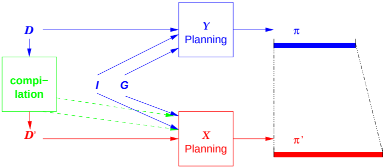

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$\mathscr{Y} \preceq \mathscr{X}$ ($\mathscr{Y}$ is compilable to $\mathscr{X}$)

iff

there exists a compilation scheme from $\mathscr{Y}$ to $\mathscr{X}$.

$\mathscr{Y} \preceq^1 \mathscr{X}$: preserving plan size exactly (modulo additive constants)

$\mathscr{Y} \preceq^c \mathscr{X}$: preserving plan size linearly (in $|\pi|$)

$\mathscr{Y} \preceq^p \mathscr{X}$: preserving plan size polynomially (in $|\pi|$ and $|\mathscr{D}|$)

$\mathscr{Y} \preceq^x_p \mathscr{X}$: polynomial-time compilability

Theorem

For all x, y, the relations $\preceq^x_y$ are transitive and reflexive.

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Back-Translatability

- Shouldn't we also require that plans in the compiled instance can be **translated back** to the original formalism?
- Yes, if we want to use this technique, one should require that!
- In all **positive cases**, there was never any problem to translate the plan back
- For the negative case, it is easier to prove non-existence
- So, in order to prove negative results, we do not need it, for positive it never had been a problem

⇝ So, similarly to the concentration on decision problems when determining complexity, we simplify things here

A (Trivial) Positive Result: STRIPS$_{Bd}$ $\preceq_p^1$ STRIPS$_N$

DNF preconditions can be "compiled away."

Assume operator $o = \langle c, e \rangle$ and

$$c = L_1 \vee \ldots \vee L_k$$

with L_i being a conjunction of literals. Create k operators $o_i = \langle L_i, e \rangle$

1. compilation is solution-preserving,
2. $\mathscr{D}'$ is only polynomially larger than $\mathscr{D}$,
3. compilation can be computed in polynomial time,
4. resulting plans do not grow at all.

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Another Positive Result: $\mathrm{STRIPS}_{C,Bc} \preceq_p^c \mathrm{STRIPS}_{C,N}$

CNF preconditions can be "compiled away" – provided we have already conditional effects.

- Evaluate the truth value of all disjunctions appearing in operators by using a special evaluation operator with conditional effects that make new "clause atoms" true
- Alternate between executing original operators (clauses replaced by new atoms) and evaluation operators

$\rightsquigarrow$ Operator sets grow only polynomially

$\rightsquigarrow$ Plans are double as long as the original plans

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Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathscr{D}$ with only one (STRIPS$_{C,B}$) operator o:

$$\langle \top, (p_1 \rhd \neg p_1) \wedge (\neg p_1 \rhd p_1) \wedge \ldots \wedge (p_k \rhd \neg p_k) \wedge (\neg p_k \rhd p_k) \rangle,$$

which "inverts" a given state. For all (I, G) with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \wedge \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a STRIPS$_{C,B}$ one-step plan.

Assume there exists a compilation preserving plan size linearly leading to a STRIPS$_B$ domain structure $\mathscr{D}'$. There are exponentially many possible initial states, but only polynomially many different c-step plans for $\mathscr{D}'$. Some STRIPS$_B$ plan π is used for different initial states I_1, I_2 (for large enough k). Let v be a variable with $I_1(v) \neq I_2(v)$.

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Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathscr{D}$ with only one (STRIPS$_{C,B}$) operator o:

$$\langle \top, (p_1 \rhd \neg p_1) \wedge (\neg p_1 \rhd p_1) \wedge \ldots \wedge (p_k \rhd \neg p_k) \wedge (\neg p_k \rhd p_k) \rangle,$$

which "inverts" a given state. For all (I, G) with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \wedge \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a STRIPS$_{C,B}$ one-step plan.
Assume there exists a compilation preserving plan size linearly leading to a STRIPS$_B$ domain structure $\mathscr{D}'$. There are exponentially many possible initial states, but only polynomially many different c-step plans for $\mathscr{D}'$. Some STRIPS$_B$ plan π is used for different initial states I_1, I_2 (for large enough k). Let v be a variable with $I_1(v) \neq I_2(v)$.
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Another Negative Result: $\mathrm{STRIPS}_{Bc} \not\preceq^{c}$ STRIPS_{N}

k-FISEX: Planning problem with fixed plan length *k* and varying initial state. Does there exist an initial state leading to a successful *k*-step plan?

1-FISEX is NP-complete for STRIPS_{Bc} (= SAT).

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$\boxed{\rightsquigarrow \mathrm{STRIPS}_{Bc} \not\preceq^{c}_{p} \mathrm{STRIPS}_{N} \text{ (if P}\neq\text{NP)}}$

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Boolean Preconditions Cannot be Compiled Away Even When Using Conditional Effects

- Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) (= NC^1)
- Conditional effects can simulate only families of circuits with fixed depth (= AC^0).
- The parity function can be expressed in the first framework (NC^1) while it cannot be expressed in the second (AC^0).

⇝ The negative result follows unconditionally!

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Boolean Circuits

- We know what Boolean circuits are (directed, acyclic graphs with different types of nodes: **and, or, not, input, output**)
- Size of circuit = number of gates
- Depth of circuit = length of longest path from input gate to output gate
- When we want to recognize formal languages with circuits, we need a sequence of circuits with an increasing number of input gates $\leadsto$ family of circuits
- Families with polynomial size and poly-log ($\log^k n$) depth
- complexity classes NC^k (Nick's class)
- $\mathrm{NC} = \bigcup_k \mathrm{NC}^k \subseteq P$, the class of problems that can be solved efficiently in parallel
- The class of languages that can be characterized by polynomially sized Boolean formulae is identical to NC^1

The classes AC^k

- The classes NC^k are defined with a fixed fan-in
- If we have unbounded fan-in, we get the classes AC^k
  - gate types: NOT, n-ary AND, n-ary OR for all $n \geq 2$
- Obviously: $NC^k \subseteq AC^k$
- Possible to show: $AC^{k-1} \subseteq NC^k$
- The parity language is in NC^1, but not in AC^0!

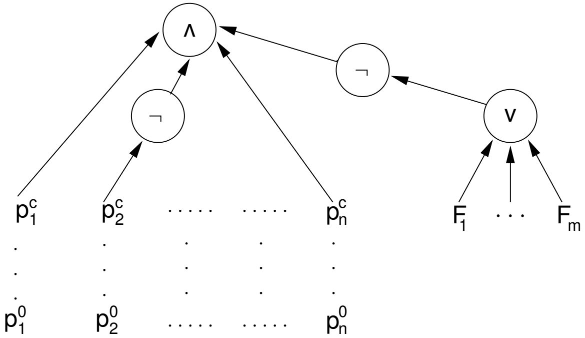
Accepting languages with families of domain structures with fixed goals

- We will view families of domain structures with fixed goals and fixed size plans as "machines" that accept languages
- Consider families of poly-sized domain structures in $STRIPS_B$ and use one-step plans for acceptance.
- Obviously, this is the same as using Boolean formulae

⇝ All languages in NC^1 can be accepted in this way

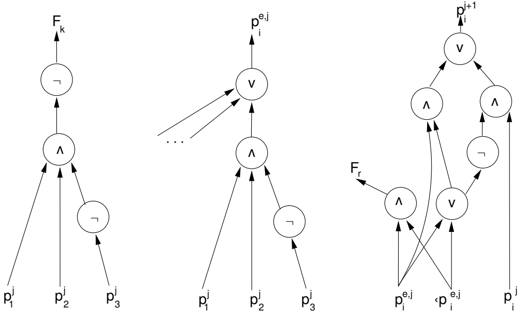
Simulating STRIPS$_{C,N}$ c-step Plans with AC^0 circuits (1)

- Represent each operator and then chain the actions together ($O(|O|^c)$ different plans):



Simulating STRIPS$_{C,N}$ c-Step Plans with AC^0 circuits (2)

- For each single action (precondition testing (a), conditional effects (b), and the computation of effects (c)



(a) (b) (c)

$\mathrm{STRIPS}_B \not\preceq^c \mathrm{STRIPS}_{C,N}$

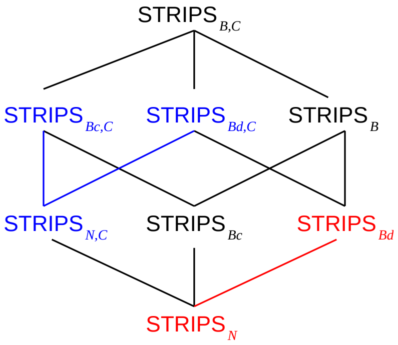
Theorem

$STRIPS_B \not\preceq^c STRIPS_{C,N}$.

Proof.

Assuming $\mathrm{STRIPS}_B \preceq^c \mathrm{STRIPS}_{C,N}$ has the consequence that the underlying compilation scheme could be used to compile a NC^1 circuit family into an AC^0 circuit family, which is impossible in the general case. □

General Results for Compilability Preserving Plan Size Linearly



All other potential positive results have been ruled out by our 3 negative results and transitivity.

Summary

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- Compilation schemes seem to be the right method to measure the **relative** expressive power of planning formalisms
- Either we get a positive result preserving plan size linearly with a polynomial-time compilation
- or we get an impossibility result

$\rightarrow$ Results are relevant for building planning systems

$\rightsquigarrow$ **CNF preconditions** do not add much when we have already conditional effects

- Note: In all cases we can get a positive result if we allow for a polynomial blow-up of the plans.