

Principles of Knowledge Representation and Reasoning

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Exercise Sheet 9

Due: January 9th, 2013

Exercise 9.1 (CUMULATIVE AND PREFERENTIAL CONSEQUENCES, 4)

For each of the following statements (a)–(d), provide a (different!) example of a set of conditionals K and a plausible consequence $\varphi \sim \psi$ satisfying the given property (prove that your answer is correct by giving derivations and/or counterexamples as appropriate). If the property cannot hold for any set K and formulae φ and ψ , prove this.

- (a) K implies $\varphi \sim \psi$ in system **C** and in system **P**.
- (b) K implies $\varphi \sim \psi$ in system **C**, but not in system **P**.
- (c) K implies $\varphi \sim \psi$ in system **P**, but not in system **C**.
- (d) K does not imply $\varphi \sim \psi$ in either system **C** or system **P**.

Exercise 9.2 (NORMAL INTERPRETATIONS, 3)

Let \sim be an arbitrary consequence relation (i.e., a binary relation on the set of propositional logic formulae). A propositional logic interpretation I is called \sim -normal for a formula φ if for each formula ψ with $\varphi \sim \psi$, it holds $I \models \psi$.

In what follows we assume that the consequence relation \sim is closed under the rules *Reflexivity*, *Right Weakening*, and *And*. Show that for arbitrary propositional formulae φ, ψ the following statements are equivalent:

- (a) $\varphi \sim \psi$.
- (b) For each interpretation I that is \sim -normal for φ , it holds $I \models \psi$.

Hint: A set Δ of propositional logic formulae is satisfiable if and only if each of its finite subsets is satisfiable.

Exercise 9.3 (CUMULATIVE LOGICS AND SET THEORY, 4)

We call a rule (e.g. *Left Logical Equivalence* or *Right Weakening*) *set-theoretically plausible* if the following holds for arbitrary finite sets I of interpretations:

- **If** for each premise $\alpha \sim \beta$ of the rule it holds that at least 98% of the interpretations in I that satisfy α also satisfy β , **then** for the conclusion $\alpha' \sim \beta'$ of that rule it holds that at least 96% of the interpretations in I that satisfy α' also satisfy β' .

Decide which of the rules *Right Weakening*, *Cut*, *Contraposition*, and *Transitivity* are set-theoretically plausible.

Frohe Weihnachten und ein gutes neues Jahr 2013!