Principles of Knowledge Representation and Reasoning

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Exercise Sheet 4

Due: November 21th, 2012

Exercise 4.1 (Characteristic axioms, 2)

Proof the following statement: A frame $\mathcal{F}$ is symmetric if and only if $\mathcal{F} \models \phi \rightarrow \Box\Diamond\phi$ for each modal formula ϕ.

Exercise 4.2 (Tableau proofs, 2+2)

(a) Apply the tableau method to check whether the formula $\Box(\Box(a \rightarrow b) \rightarrow (\Diamond a \rightarrow \Diamond(a \wedge b)))$ is valid in K. If it is not valid, specify a counterexample as constructed from the tableau.

(b) Apply the tableau method to check whether the formula $\Box(a \rightarrow \Box\Box b) \wedge \Diamond(\Box(a \rightarrow \Box b) \rightarrow \Diamond(a \wedge \Diamond(\Diamond\neg b \vee \neg a)))$ is unsatisfiable in $S4$. If it is not unsatisfiable, specify a satisfying model as constructed from the tableau.

Exercise 4.3 (Tableau solver for propositional logic, 6)

Write a solver that answer the SAT problem for propositional logic using the tableau rules. You can use any programming language you like (given that it is usable under Ubuntu 12). Source code **must be submitted** on time to: `westpham@informatik.uni-freiburg.de`. The solver should take as input propositional formulae with the format given on the exercice sheet 2[1], i.e., we restrict ourselves to postfix notation[2]. You can re-use the code you made for parsing input formulae.

A tableau solver is a backtracking solver which applies the following rules:

$$\frac{w \models a \vee b}{w \models a \mid w \models b} \qquad \frac{w \models a \wedge b}{w \models a, w \models b} \qquad \frac{w \models \neg a}{w \not\models a}$$

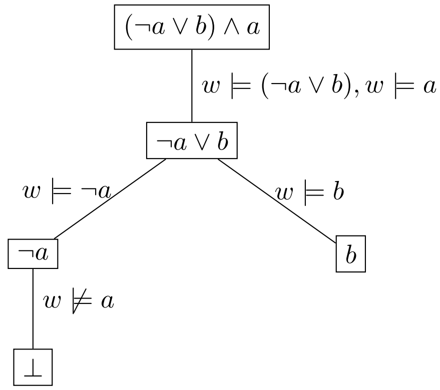
$$\frac{w \not\models a \vee b}{w \not\models a, w \not\models b} \qquad \frac{w \not\models a \wedge b}{w \not\models a \mid w \not\models b} \qquad \frac{w \not\models \neg a}{w \models a}$$

These rules are applied when they can be used to deduce some formulae which have not been deduced yet. If a contradiction arises (when $w \models a$ and $w \not\models a$ appear in the same branch), then the formulas are unsatisfiable. If no new formula can be created and there is no contradiction, then the formulas are satisfiable. A branch is created whenever an $\vee$ is met.

For example, with the formula $(\neg a \vee b) \wedge a$ the following may be a possible tableau sequence:

[1] `http://www.informatik.uni-freiburg.de/~ki/teaching/ws1213/krr/`

[2] `http://en.wikipedia.org/wiki/Postfix_notation`



First, $(\neg a \vee b) \wedge a$ deduces $w \models \neg a \vee b$ and $w \models a$. Then, for $\neg a \vee b$ one can decide to deduce $w \models \neg a$ and thus $w \not\models a$ which would trigger an inconsistency. This last deduction ($w \models \neg a$) is then cancelled and we come back to the last disjunction where $w \models b$ is deduced. At that point, there is no new formula that can be deduced and one can claim the formula is satisfiable.