Principles of Knowledge Representation and Reasoning

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Winter Semester 2012/2013 — Department of Computer Science

Exercise Sheet 3

Due: November 14th, 2012

Exercise 3.1 (Formula Game, 1+1)

The Formula Game is a two-player game played on a given quantified Boolean formula (in prenex normal form) $Q_1 p_1 \dots Q_k p_k \psi$. The rules are simple: If the outermost unassigned variable p_i is universally (existentially) quantified, it is the turn of player U (player E resp.) who assigns a truth value to that variable p_i. Thus both players finally construct a truth assignment I to the variables occurring in the matrix formula ψ. Player E wins the game if $I(\psi) = 1$; otherwise, player U wins the game.

Check whether one of the players U or E has a strategy for winning the formula game for the following formulae:

(a) $\forall p \forall q \exists r \forall s \big((p \wedge r) \rightarrow (q \wedge s)\big)$

(b) $\forall p \exists q \exists r \big((p \rightarrow q) \wedge (q \rightarrow \neg r) \wedge (r \vee \neg p)\big)$

Exercise 3.2 (Reduction, 1+2+4)

(a) Show that PSPACE = co-PSPACE.

(b) Show that with regard to Turing reductions the class of all NP-hard problems coincides with the class of all co-NP-hard problems.

(c) We consider the following two-player game $\mathcal{G}$ played on a directed graph $\langle V, A \rangle$ with a designated start node $v_0 \in V$. Player 1 and player 2 choose in turn some arc in the graph such that each chosen arc starts in the head of the previously chosen arc. Player 1 begins with choosing an arc starting in node v_0. A player looses the game if s/he is unable to choose an arc to a not yet visited node in the graph.

Show that the following problem is PSPACE-complete.

Instance: A directed graph $\langle V, A \rangle$, a start node v_0.

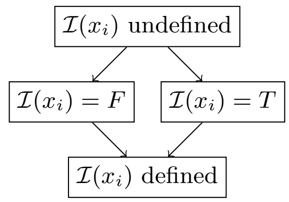
Question: Does Player 1 have a strategy for winning $\mathcal{G}$?

Hint: Existence of a winning strategy in the formula game (see exercise 3.1) is known to be PSPACE-complete even for QBF of the following form:

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \dots \exists x_{2k-1} \forall x_{2k} \exists x_{2k+1} \psi,$$

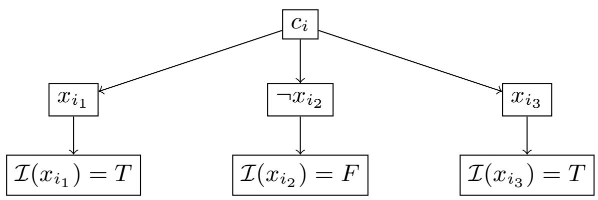
where ψ is a 3-CNF formula. For the reduction construct for a given formula of this form a directed graph. The following subgraphs will be useful:

- For each propositional variable introduce a subgraph with four nodes that represents that a variable has been assigned a truth value.



  The current player will have to decide on the truth value of the next unassigned variable x_i. Note that the node corresponding to the chosen assignment may not be revisited in the game.

- Furthermore introduce nodes for each clause c_i of ψ and the literals l_{i_1}, $\ldots, l_{i_3}$ occurring in it. For example, if $c_i = x_{i_1} \vee \neg x_{i_2} \vee x_{i_3}$:



Finally discuss the size of your graph and relate the winning strategies in the games.

Exercise 3.3 (Belief operators, 1+1+1)

Use two modal belief operators to represent the following statements:

(a) If Adam believes that Eve believes that an apple is sweet if it is red, then Adam believes that as well.

(b) Both Adam and Eve believe that, if an apple is red, the respective other believes that the apple is sweet.

Assume now that Adam sees a red apple. Given (a) and (b), does it follow that Adam believes that the apple is sweet? Provide an informal argument.