

# Principles of AI Planning

## 9. Interlude: Finite-domain representation

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# 1 Invariants

- Introduction
- Computing invariants
- Exploiting invariants

## Invariants

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- When we as humans reason about planning tasks, we implicitly make use of “obvious” properties of these tasks.
  - **Example:** we are never in two places at the same time
- We can express this as a logical formula  $\varphi$  that is **true in all reachable states**.
  - **Example:**  $\varphi = \neg(at\text{-}uni \wedge at\text{-}home)$
- Such formulae are called **invariants** of the task.

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How does an **automated** planner come up with invariants?

- Theoretically, testing if an arbitrary formula  $\varphi$  is an invariant is **as hard as planning** itself.
- Still, many practical invariant synthesis algorithms exist.
- To remain efficient (= polynomial-time), these algorithms only compute a **subset** of all useful invariants.
- Empirically, they tend to at least find the “obvious” invariants of a planning task.

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Most algorithms for generating invariants are based on a **generate-test-repair** paradigm:

- **Generate:** Suggest some invariant candidates, e. g., by enumerating all possible formulas  $\varphi$  of a certain size.
- **Test:** Try to prove that  $\varphi$  is indeed an invariant.  
Usually done **inductively**:
  - 1 Test that **initial state** satisfies  $\varphi$ .
  - 2 Test that if  $\varphi$  is true in the current state, it remains true after applying a single operator.
- **Repair:** If invariant test fails, replace candidate  $\varphi$  by a **weaker** formula, ideally exploiting **why** the proof failed.

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We discussed invariant synthesis in detail in previous courses on AI planning, but this year we will focus on other aspects of planning.

## Literature on invariant synthesis:

- DISCOPLAN (Gerevini & Schubert, 1998)
- TIM (Fox & Long, 1998)
- Edelkamp & Helmert's algorithm (1999)
- Rintanen's algorithm (2000)
- Bonet & Geffner's algorithm (2001)
- Helmert's algorithm (2009)

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Invariants have many uses in planning:

- **Regression search:**  
**Prune states** that violate (are inconsistent with) invariants.
- **Planning as satisfiability:**  
**Add invariants** to a SAT encoding of a planning task to get tighter constraints.
- **Reformulation:**  
Derive a **more compact** state space representation (i. e., with lower percentage of unreachable states).

We now briefly discuss the last point, since it leads to **planning tasks in finite-domain representation**, which are very important for the next chapters.

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## 2 Planning tasks in finite-domain representation

- Mutexes
- FDR planning tasks
- Relationship to propositional planning tasks
- SAS<sup>+</sup> planning tasks

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Invariants that take the form of **binary clauses** are called **mutexes** because they state that certain variable assignments cannot be simultaneously true and are hence **mutually exclusive**.

## Example (Blocksworld)

The invariant  $\neg A\text{-on-}B \vee \neg A\text{-on-}C$  states that  $A\text{-on-}B$  and  $A\text{-on-}C$  are mutex.

Often, a larger **set of literals** is mutually exclusive because every pair of them forms a mutex.

## Example (Blocksworld)

Every pair in  $\{B\text{-on-}A, C\text{-on-}A, D\text{-on-}A, A\text{-clear}\}$  is mutex.

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# Encoding mutex groups as finite-domain variables



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Let  $L = \{l_1, \dots, l_n\}$  be mutually exclusive literals over  $n$  different variables  $A_L = \{a_1, \dots, a_n\}$ .

Then the planning task can be rephrased using a single **finite-domain** (i.e., non-binary) state variable  $v_L$  with  $n + 1$  possible values in place of the  $n$  variables in  $A_L$ :

- $n$  of the possible values represent situations in which **exactly one** of the literals in  $L$  is true.
- The remaining value represents situations in which **none of the literals** in  $L$  is true.
  - **Note:** If we can prove that one of the literals in  $L$  has to be true in each state, this additional value can be omitted.

In many cases, the reduction in the number of variables can dramatically improve performance of a planning algorithm.

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## Definition (finite-domain state variable)

A **finite-domain state variable** is a symbol  $v$  with an associated **finite domain**, i. e., a non-empty finite set.

We write  $\mathcal{D}_v$  for the domain of  $v$ .

## Example

$v = \textit{above-a}$ ,  $\mathcal{D}_{\textit{above-a}} = \{\textit{b}, \textit{c}, \textit{d}, \textit{nothing}\}$

This state variable encodes the same information as the propositional variables *B-on-A*, *C-on-A*, *D-on-A* and *A-clear*.

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## Definition (finite-domain state)

Let  $V$  be a finite set of finite-domain state variables.

A **state** over  $V$  is an assignment  $s : V \rightarrow \bigcup_{v \in V} \mathcal{D}_v$  such that  $s(v) \in \mathcal{D}_v$  for all  $v \in V$ .

## Example

$$s = \{ \textit{above-a} \mapsto \textit{nothing}, \textit{above-b} \mapsto \textit{a}, \textit{above-c} \mapsto \textit{b}, \\ \textit{below-a} \mapsto \textit{b}, \textit{below-b} \mapsto \textit{c}, \textit{below-c} \mapsto \textit{table} \}$$

## Definition (finite-domain formulae)

Logical formulae over finite-domain state variables  $V$  are defined as in the propositional case, except that instead of atomic formulae of the form  $a \in A$ , there are atomic formulae of the form  $v = d$ , where  $v \in V$  and  $d \in \mathcal{D}_v$ .

## Example

The formula  $(above-a = nothing) \vee \neg(below-b = c)$  corresponds to the formula  $A-clear \vee \neg B-on-C$ .

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## Definition (finite-domain effects)

Effects over finite-domain state variables  $V$  are defined as in the propositional case, except that instead of atomic effects of the form  $a$  and  $\neg a$  with  $a \in A$ , there are atomic effects of the form  $v := d$ , where  $v \in V$  and  $d \in \mathcal{D}_v$ .

## Example

The effect

$(below-a := table) \wedge ((above-b = a) \triangleright (above-b := nothing))$

corresponds to the effect

$A-on-T \wedge \neg A-on-B \wedge \neg A-on-C \wedge \neg A-on-D \wedge (A-on-B \triangleright B-clear).$

$\rightsquigarrow$  definition of **finite-domain operators** follows from this

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## Definition (planning task in finite-domain representation)

A **deterministic planning task in finite-domain representation** or **FDR planning task** is a 4-tuple  $\Pi = \langle V, I, O, \gamma \rangle$  where

- $V$  is a finite set of **finite-domain state variables**,
- $I$  is an **initial state** over  $V$ ,
- $O$  is a finite set of **finite-domain operators** over  $V$ , and
- $\gamma$  is a formula over  $V$  describing the **goal states**.

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## Definition (induced propositional planning task)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be an FDR planning task.

The **induced propositional planning task**  $\Pi'$  is the (regular) planning task  $\Pi' = \langle A', I', O', \gamma' \rangle$ , where

- $A' = \{(v, d) \mid v \in V, d \in \mathcal{D}_v\}$
- $I'((v, d)) = 1$  iff  $I(v) = d$
- $O'$  and  $\gamma'$  are obtained from  $O$  and  $\gamma$  by replacing
  - each atomic formula  $v = d$  with the proposition  $(v, d)$ ,
  - each atomic effect  $v := d$  with the effect  $(v, d) \wedge \bigwedge_{d' \in \mathcal{D}_v \setminus \{d\}} \neg(v, d')$ .
- $\rightsquigarrow$  can define operator semantics, plans, relaxed planning graphs, ... for  $\Pi$  in terms of its induced propositional planning task

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## Definition (SAS<sup>+</sup> planning task)

An FDR planning task  $\Pi = \langle V, I, O, \gamma \rangle$  is called an **SAS<sup>+</sup> planning task** iff there are no conditional effects in  $O$  and all operator preconditions in  $O$  and the goal formula  $\gamma$  are conjunctions of atoms.

- analogue of STRIPS planning tasks for finite-domain representations
- induced propositional planning task of a SAS<sup>+</sup> planning task is STRIPS
- FDR tasks obtained by invariant-based reformulation of STRIPS planning task are SAS<sup>+</sup>

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Summary



- **Invariants** are common properties of all reachable states, expressed as logical formulas.
- A number of algorithms for **computing invariants** exist.
- These algorithms will not find **all useful invariants** (which is too hard), but try to find some useful subset within reasonable (polynomial) time.
- **Mutexes** are invariants that express that certain pairs of state variable assignments are mutually exclusive.
- Groups of mutexes can be used for **problem reformulation**, transforming a planning task into **finite-domain representation (FDR)**.
- Many planning algorithms are more efficient when working on these FDR tasks (rather than the original tasks) because they contain **fewer unreachable states**.

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