Qualitative Spatial and Temporal Reasoning based on decomposition

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Qualitative Spatial and Temporal Reasoning

- Continous space and time: undecidable;
- Discretization of space: decidable;
- Representation of relations between regions and/or timeframe.

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QSTR - The Point Algebra

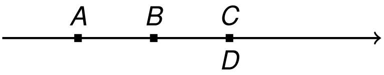


Figure: Points on a line

Definition

The set of possible relations is: $\mathscr{B} = \{<, =, >\}$

Example

Relations in Figure 1: $A < B$, $C = D$, $C > D$.

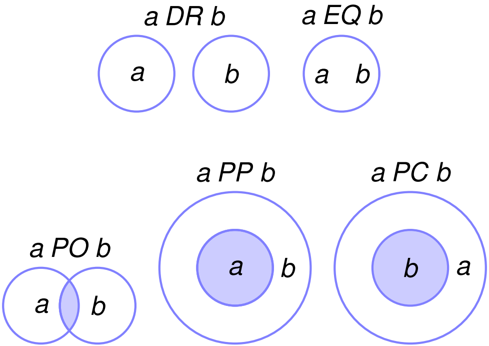


Figure: Open regions over a regular space

Definition

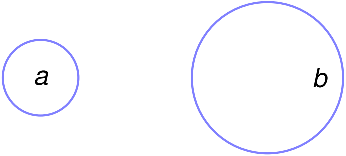
The set of possible relations is: $\mathscr{B} = \{DR, EQ, PO, PP, PC\}$

The Problem - Qualitative Constraint Networks (1)

Definition

Lets consider a set $\mathscr{C}$ of known relation. Is there any real world instantiation than satisfy all the relations in $\mathscr{C}$?

a DR b



The Problem - Qualitative Constraint Networks (2)

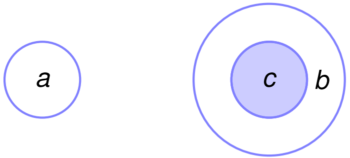
Definition

Lets consider a set $\mathscr{C}$ of known relation. Is there any real world instantiation than satisfy all the relations in $\mathscr{C}$?

a DR b

a DR c

b PC c



The Problem - Qualitative Constraint Networks (3a)

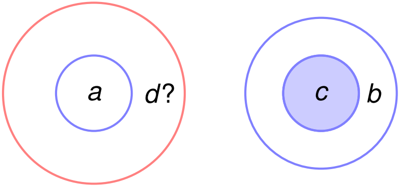
Definition

Lets consider a set $\mathscr{C}$ of known relation. Is there any real world instantiation than satisfy all the relations in $\mathscr{C}$?

a DR b

a DR c
b PC c

a PP, PO d



Definition

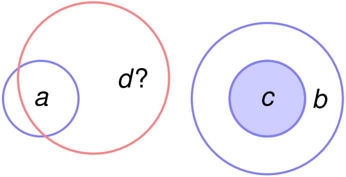
Lets consider a set $\mathscr{C}$ of known relation. Is there any real world instantiation than satisfy all the relations in $\mathscr{C}$?

$a\ DR\ b$

$a\ DR\ c$

$b\ PC\ c$

$a\ PP, PO\ d$



Definition

Lets consider a set $\mathscr{C}$ of known relation. Is there any real world instantiation than satisfy all the relations in $\mathscr{C}$?

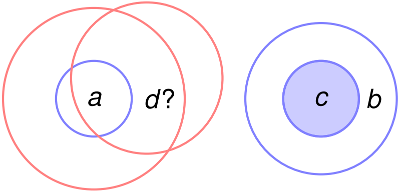
$a\ DR\ b$

$a\ DR\ c$

$b\ PC\ c$

$a\ PP, PO\ d$

$c\ EQ\ d$

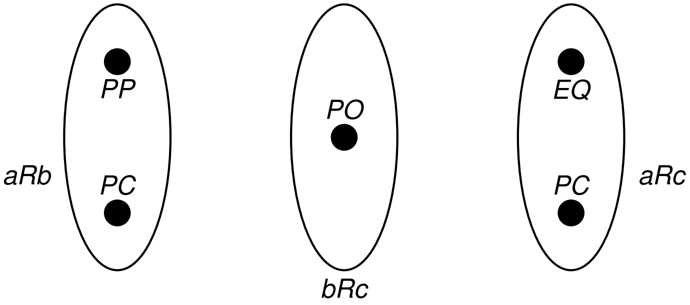


The Composition Table

$\circ$	DR_{cd}	PO_{cd}	EQ_{cd}	PP_{cd}	PC_{cd}
DR_{ac}	$*$	DR PO PP	DR	DR PO PP	DR
PO_{ac}	DR PO PC	$*$	PO	PO PP	DR PO PC
EQ_{ac}	DR	PO	EQ	PP	PC
PP_{ac}	DR	DR PO PP	PP	PP	$*$
PC_{ac}	DR PO PC	PO PC	PC	PO EQ PP PC	PC

What is usually done

Solving QCNs as CSPs with the composition table as global constraint (1)



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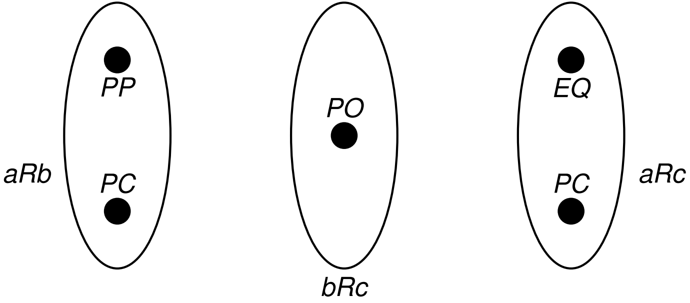
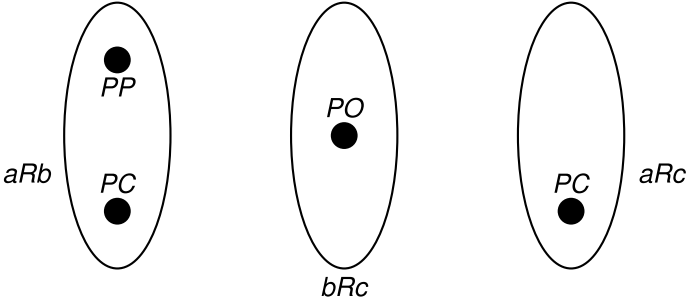


$$PP \circ PO = \{DR, PO, PP\} \quad PC \circ PO = \{PO, PC\}$$



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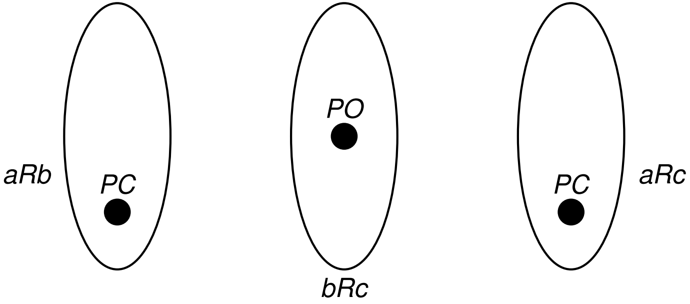


$$PP \circ PO = \{DR, PO, PP\} \quad PC \circ PO = \{PO, PC\}$$

Backtracking$(\mathscr{C}, a)$:

Input: a constraint network $\mathscr{C} = V, D, C$ and
a partial assignment a of $\mathscr{C}$

Output: a solution of $\mathscr{C}$ or "inconsistent"

if a is not consistent with $\mathscr{C}$: **return** "inconsistent"
if a is defined for all variables in V: **return** a
select some variable v_i for which a is not defined
for each value x from D_i:
    $a' := a \cup \{v_i \mapsto x\}$
    $a'' :=$ ApplyCompositionTable$(\mathscr{C}, a')$
    $a''' \leftarrow$ Backtracking$(\mathscr{C}, a'')$
    if a''' is not "inconsistent": **return** a'''
return "inconsistent"

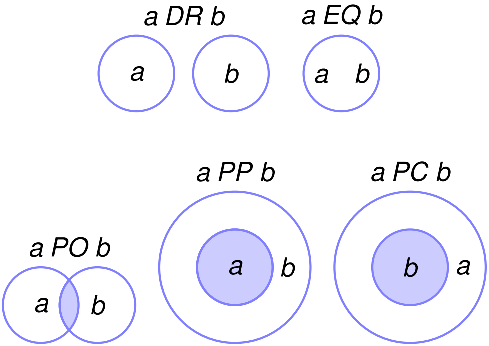
The axiomatic point of view (1)

Instead of considering the composition aspect, we deal with the semantic and divide relations in pieces:

- *a C b* := *a* and *b* share at least a common point;
- *a P b* := all points of *a* are points of *b*.

Then

	a C b	*a P b*	*b P a*
DR	×	×	×
PO	○	×	×
EQ	○	○	○
PP	○	○	×
PC	○	×	○



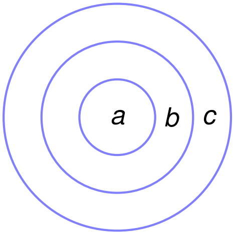
The axiomatic point of view (2)

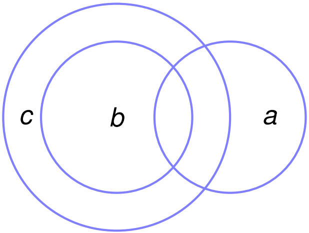
Instead of considering the composition aspect, we deal with the semantic and divide relations in pieces:

- $a\ C\ b$:= a and b share at least a common point;
- $a\ P\ b$:= all points of a are points of b.

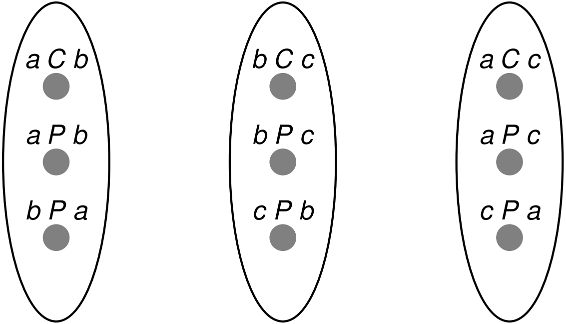
Then

$$\frac{a\ P\ b \quad b\ P\ c}{a\ P\ c} \qquad \frac{a\ C\ b \quad b\ P\ c}{a\ C\ c}$$



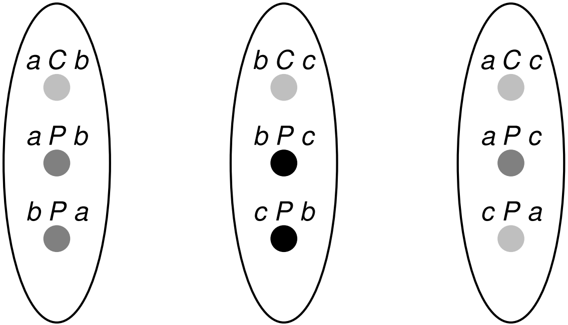


The axiomatic point of view (3)



$a\{PP, PC\}b$ $b\{PO\}c$ $a\{EQ, PC\}c$

The axiomatic point of view (3)



$a\{PP, PC\}b$ $\quad$ $b\{PO\}c$ $\quad$ $a\{EQ, PC\}c$

$PP = \circ \circ \times$

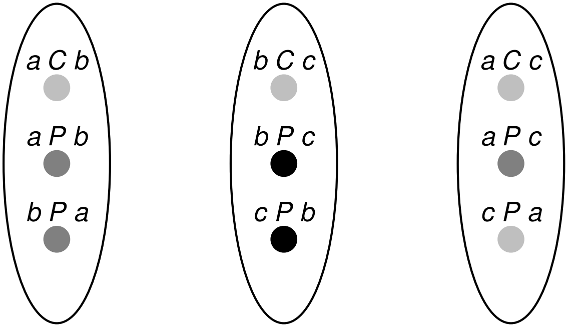
$PC = \circ \times \circ$

$PO = \circ \times \times$

$EQ = \circ \circ \circ$

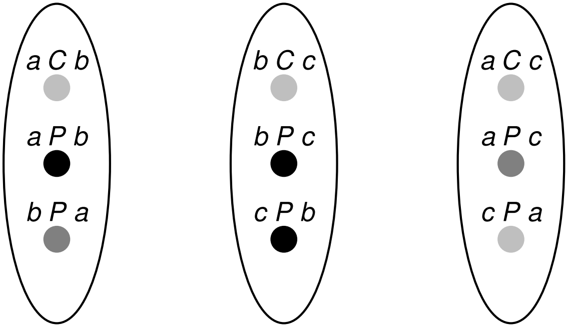
$PC = \circ \times \circ$

The axiomatic point of view (3)



$a\{PP, PC\}b$ $b\{PO\}c$ $a\{EQ, PC\}c$

$$c\,P\,a + \neg\, c\,P\,b \rightarrow \neg\, a\,P\,b$$



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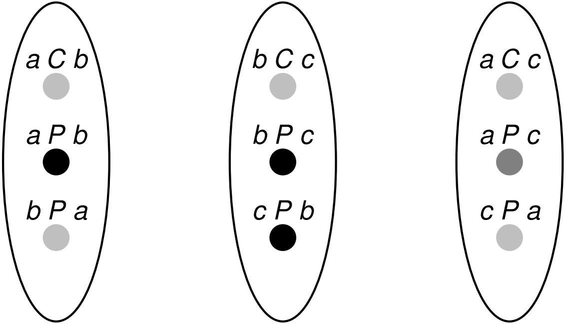
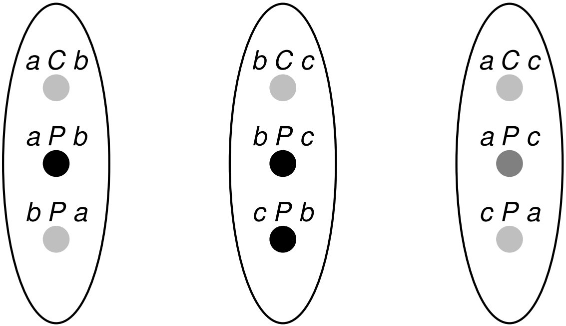
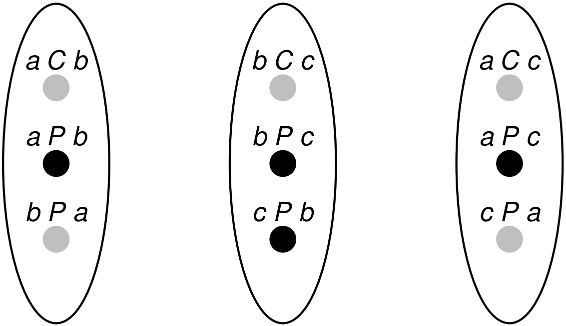


$a\{PC\}b$ $b\{PO\}c$ $a\{EQ, PC\}c$



$a\{PC\}b$ $\quad$ $b\{PO\}c$ $\quad$ $a\{EQ, PC\}c$

$$bPa + \neg\, bPc \rightarrow \neg\, aPc$$



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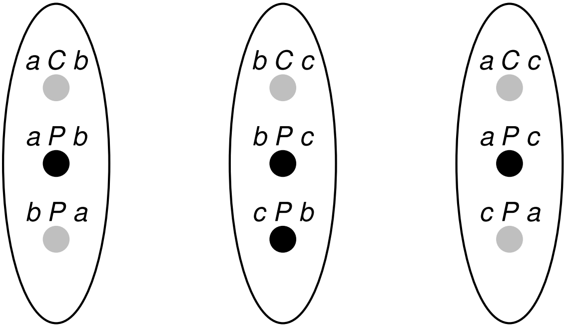

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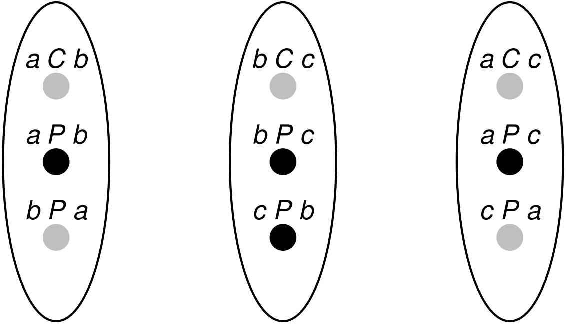

$a\{PC\}b$ $b\{PO\}c$ $a\{PC\}c$

Backtracking$(\mathscr{C}, a)$:

$a'' := \text{ApplyAxioms}(\mathscr{C}, a')$
if a is not consistent with $\mathscr{C}$: **return** "inconsistent"
if a is defined for all variables in V: **return** a
select some variable v_i which is not defined
$a' := a \cup \{v_i \mapsto \top\}$
$a'' \leftarrow \text{Backtracking}(\mathscr{C}, a')$
if a'' is not "inconsistent": **return** a''
$a' := a \cup \{v_i \mapsto \bot\}$
$a'' \leftarrow \text{Backtracking}(\mathscr{C}, a')$
if a'' is not "inconsistent": **return** a''
return "inconsistent"

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