

Principles of AI Planning

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Exercise Sheet 7

Due: December 20th, 2011

Exercise 7.1 (Finite-domain representation, 1.5+1.5+3 points)

Consider the propositional Blocksworld planning task $\Pi = \langle A, I, O, \gamma \rangle$, with

- the set of variables

$$A = \{A\text{-clear}, B\text{-clear}, C\text{-clear}, A\text{-on-}B, A\text{-on-}C, A\text{-on-}T, \\ B\text{-on-}A, B\text{-on-}C, B\text{-on-}T, C\text{-on-}A, C\text{-on-}B, C\text{-on-}T\}$$

- $I(a) = 1$ for $a \in \{B\text{-on-}T, A\text{-on-}B, A\text{-clear}, C\text{-on-}T, C\text{-clear}\}$,
 $I(a) = 0$, else.
- O contains the actions

$$\begin{aligned} \text{move-}X\text{-}Y\text{-}Z &= \langle X\text{-on-}Y \wedge X\text{-clear} \wedge Z\text{-clear}, \\ &\quad \neg X\text{-on-}Y \wedge Y\text{-clear} \wedge X\text{-on-}Z \wedge \neg Z\text{-clear} \rangle \\ \text{move-}X\text{-Table-}Z &= \langle X\text{-on-}T \wedge X\text{-clear} \wedge Z\text{-clear}, \\ &\quad \neg X\text{-on-}T \wedge X\text{-on-}Z \wedge \neg Z\text{-clear} \rangle \\ \text{move-}X\text{-}Y\text{-Table} &= \langle X\text{-on-}Y \wedge X\text{-clear}, \\ &\quad \neg X\text{-on-}Y \wedge Y\text{-clear} \wedge X\text{-on-}T \rangle \end{aligned}$$

for pair-wise distinct $X, Y, Z \in \{A, B, C\}$

- $\gamma = B\text{-on-}C \wedge C\text{-on-}A$.

(a) The following mutex groups can be found for Π :

$$\begin{aligned} L_1 &= \{B\text{-on-}A, C\text{-on-}A, A\text{-clear}\} \\ L_2 &= \{A\text{-on-}B, C\text{-on-}B, B\text{-clear}\} \\ L_3 &= \{A\text{-on-}C, B\text{-on-}C, C\text{-clear}\} \\ L_4 &= \{A\text{-on-}B, A\text{-on-}C, A\text{-on-}T\} \\ L_5 &= \{B\text{-on-}A, B\text{-on-}C, B\text{-on-}T\} \\ L_6 &= \{C\text{-on-}A, C\text{-on-}B, C\text{-on-}T\} \end{aligned}$$

Specify a planning task Π' that is equivalent to Π and in finite-domain representation by using these mutex groups. Please name the variables in a reasonable way (e.g., analogously to the examples given in the lecture).

(b) Specify the propositional planning task Π'' that is induced by Π' .

- (c) A planning task $\Pi' = \langle V, I', O', \gamma' \rangle$ in finite-domain representation is equivalent to a propositional planning task Π if there is an isomorphism between $\Pi'' = \langle A'', I'', O'', \gamma'' \rangle$ and Π , where Π'' is the propositional planning task induced by Π .

There is an isomorphism between Π'' and Π if there are injective mappings $f : S \mapsto S''$ and $g : O \mapsto O''$ (where S is the set of reachable states in Π and S'' the set of states in Π'') with:

- $I'' = f(I)$
- For reachable states s_1 and s_2 , if $s_2 = \text{app}_o(s_1)$ then $f(s_2) = \text{app}_{g(o)}(f(s_1))$.
- For all reachable states $s \in S$ it is true that $s \models \gamma$ iff $f(s) \models \gamma''$.

Show that the planning task Π' from exercise (b) is equivalent to Π by specifying $f : S \mapsto S''$ and $g : O \mapsto O''$ and showing that they have the required properties.

Exercise 7.2 (Abstraction heuristics, 2+2 points)

A state of a 15-puzzle planning task is given as a permutation $\langle b, t_1, \dots, t_{15} \rangle$ of $\{1, \dots, 16\}$, where b denotes the empty tile (blank) and all other components denote the positions of the tiles.

Let $T^1 = \{t_1^1, \dots, t_n^1\}$, $T^2 = \{t_1^2, \dots, t_m^2\}$ with $1 \leq n, m \leq 14$ be a partitioning of $\{t_1, \dots, t_{15}\}$ (i.e., $T^1 \cup T^2 = \{t_1, \dots, t_{15}\}$ and $T^1 \cap T^2 = \emptyset$). Consider the following abstractions:

- (a) $\alpha_1(\langle b, t_1, \dots, t_{15} \rangle) = \langle b, t_1^1, \dots, t_m^1 \rangle$
- (b) $\alpha_2(\langle b, t_1, \dots, t_{15} \rangle) = \langle b, t_1^2, \dots, t_n^2 \rangle$
- (c) $\alpha_3(\langle b, t_1, \dots, t_{15} \rangle) = \langle t_1^1, \dots, t_m^1 \rangle$
- (d) $\alpha_4(\langle b, t_1, \dots, t_{15} \rangle) = \langle t_1^2, \dots, t_n^2 \rangle$

For $1 \leq i \leq 4$, the heuristic estimates of h_i are equal to lengths of optimal plans in the respective abstractions (e.g., $h_i(s) = h^*(\alpha_i(s))$). Show that:

- (a) $h_1 + h_2$ is not admissible.
- (b) $h_3 + h_4$ is admissible.

Hint: A heuristic is admissible if it is goal-aware and consistent.

Note: The exercise sheets may and should be worked on in groups of two students. Please state both names on your solution (this also holds for submissions by e-mail).